

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1793

L

Unique Paper Code : 2352011201

Name of the Paper : Linear Algebra

Name of the Course : B.Sc. (H) Mathematics

Semester : II / DSC

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting any **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of calculators is not allowed.

1. (a) If x and y are vectors in $\mathbb{R}^n$, then prove that

$$\|x + y\| \geq \left| \|x\| - \|y\| \right|$$

Also verify it for the vectors $x = [2, -1, 3, 2]$ and $y = [4, 3, 2, 1]$ in $\mathbb{R}^4$.

(5+2.5)

- (b) Solve the following system of linear equations using Gaussian Elimination method :

$$3x_1 + x_2 + 7x_3 + 2x_4 = 13$$

$$2x_1 - 4x_2 + 14x_3 - x_4 = -10$$

$$5x_1 + 11x_2 - 7x_3 + 8x_4 = 59$$

$$2x_1 + 5x_2 - 4x_3 - 3x_4 = 39$$

Also give the complete solution set if the system is consistent. (6+1.5)

- (c) Using the Gauss-Jordan method, find the minimal integer values for the variables that will balance the following chemical equation :



2. (a) Find the reduced row echelon form B of the following matrix A , keeping track of the row operations used :

$$A = \begin{bmatrix} 4 & 0 & -20 \\ -2 & 0 & 11 \\ 3 & 1 & -15 \end{bmatrix} \quad (7.5)$$

- (b) Determine whether the vector $x = [5, 17, -20]$ is in the row space of the matrix

$$A = \begin{bmatrix} 3 & 1 & -2 \\ 4 & 0 & 1 \\ -2 & 4 & -3 \end{bmatrix}$$

If so, then express $[5, 17, -20]$ as a linear combination of the rows of A . (5+2.5)

- (c) (i) Define Eigenvalue and Eigenspace for a square matrix A .
(ii) Find all the eigenvalues and eigenspaces for A .

$$A = \begin{bmatrix} -4 & 8 & -12 \\ 6 & -6 & 12 \\ 6 & -8 & 14 \end{bmatrix} \quad (2+5.5)$$

3. (a) Define a subspace of a vector space over $\mathbb{R}$. Which of the following are subspaces of $M_{2 \times 2}(\mathbb{R})$? Give reasons to support your answer.

(i) $W_1 = \{A \in M_{2 \times 2} : \text{Trace}(A) = 0\}$

(ii) $W_2 = \{A \in M_{2 \times 2} : A^T = 4I\}$

(iii) $W_3 = \{A \in M_{2 \times 2} : |A| = 0\}$

(1.5+2+2+2)

- (b) Define sum of two subspaces W_1 and W_2 of a vector space $V(\mathbb{R})$. Show that $W_1 + W_2$ is the smallest subspace of V containing both W_1 and W_2 .

(2+5.5)

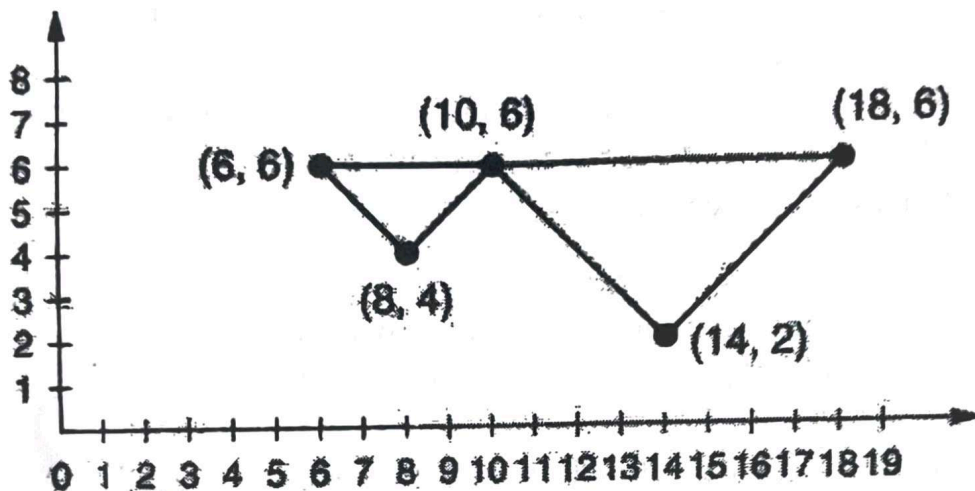
- (c) Define Linear Span of a set S in a vector space V . Is the linear span of S a subspace of V ? Determine if the vector $(2, -1, 1)$ is in the linear span of $S = \{(1, 0, 2), (-1, 1, 1)\}$ (2+2+3.5)
4. (a) Define a linearly independent subset of a vector space. For $k = 0, 1, 2, \dots, n$, $p_k(x) = x^k + x^{k+1} + \dots + x^n$. Show that $\{p_0(x), p_1(x), \dots, p_n(x)\}$ is linearly independent subset of the space of all polynomials. (2+5.5)
- (b) Let $W = \{(a_1, a_2, a_3, a_4, a_5) : a_2 = a_4, a_1 + a_3 + a_5 = 0\}$ be a subset of $\mathbb{R}^5$. Is W a subspace of $\mathbb{R}^5$? Find a basis for W . What is $\dim_{\mathbb{R}} W$? (2+4+1.5)
- (c) Let $S = \{(1, 3, -4, 2), (2, 2, -4, 0), (1, -3, 2, -4), (-1, 0, 1, 0)\}$ be a subset of $\mathbb{R}^4$. Show that S is linearly dependent. Write a vector in S as a linear combination of other vectors. (5.5+2)
5. (a) Define a linear transformation. Is there any linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ such that $T(1, 0, 3) = (1, 1)$ and $T(-2, 0, -6) = (2, 1)$? (2+5.5)
- (b) Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be defined by $T(a, b) = (a - b, a, 2a + b)$
Let β be the standard ordered basis for $\mathbb{R}^2$ and $\gamma = \{(1, 1, 0), (0, 1, 1), (2, 2, 3)\}$. Compute $[T]_{\beta}^{\gamma}$. (7.5)
- (c) (i) Let V and W be vector spaces and let $T: V \rightarrow W$ be a linear transformation. Then show that T is one-to-one if and only if $N(T) = \{0\}$.
(ii) Let $T: P_2(\mathbb{R}) \rightarrow P_3(\mathbb{R})$ be a linear transformation defined by
- $$T(f(x)) = 2f'(x) + \int_0^x 3f(t)dt.$$
- Show that T is one-one. (3+4.5)
6. (a) Let V and W be vector spaces and let $T: V \rightarrow W$ be a linear. Define inverse of T . If T is invertible linear transformation then show that $T^{-1}: W \rightarrow V$ is linear. (2+5.5)

- (b) Define isomorphic vector spaces. Which of the following spaces are isomorphic? Justify your answer.

(i) $M_{2 \times 2}(\mathbb{R})$ and $P_3(\mathbb{R})$

(ii) $V = \{A \in M_{2 \times 2}(\mathbb{R}) : \text{tr}(A) = 0\}$ and $\mathbb{R}^4$ (2.5+2.5+2+2.5)

- (c) For the graphic in given figure, use ordinary coordinates to find the new vertices after performing each indicated operations :



- (i) Rotation about the origin through $\theta = 90^\circ$
- (ii) Reflection about the line $y = \frac{1}{2}x$.
- (iii) Translation along the vector $[-3, 5]$ (2.5+2.5+2.5)

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Your Roll No. :

Sl. No. of Q. Paper : 2058 I

Unique Paper Code : 2352011202

Name of the Paper : DSC-5: Calculus

Name of the Course : B.Sc. (H) Mathematics
(NEP-UGCF 2022)

Semester : II

Time : 3 Hours Maximum Marks : 90

Instructions for Candidates :

- (a) Write your Roll No. on the top immediately on receipt of this question paper.
- (b) Attempt **all** questions by selecting **three** parts from each question.
- (c) The parts of each question are to be attempted together.
- (d) **All** questions carry equal marks.
- (e) Use of Calculator is **not** allowed.

1. (a) Let $f:A \rightarrow \mathbb{R}$ and let c be a cluster point of A . Show that $\lim_{x \rightarrow c} f = L$ if and only if for every sequence $\langle x_n \rangle$ in A that converges to c such that $x_n \neq c$ for all $n \in \mathbb{N}$, the sequence $\langle f(x_n) \rangle$ converges to L .

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- (b) Show that $\lim_{x \rightarrow 0} \sin \frac{1}{x}$ does not exist in $\mathbb{R}$.
- (c) Using $\epsilon - \delta$ definition, show that $\lim_{x \rightarrow 1} \frac{x}{1+x} = \frac{1}{2}$.
- (d) Find $\lim_{x \rightarrow 0} \frac{\sqrt{1+2x} - \sqrt{1+3x}}{x + 2x^2}$ where $x > 0$.
2. (a) Suppose that the functions f and g have limits in $\mathbb{R}$ as $x \rightarrow \infty$ and that $f(x) \leq g(x)$ for all $x \in (0, \infty)$. Prove that $\lim_{x \rightarrow \infty} f \leq \lim_{x \rightarrow \infty} g$.
- (b) Prove by verifying the $\epsilon - \delta$ property that the function $f(x) = x^2$ is continuous at $x_0 = 2$.
- (c) State the Intermediate Value Theorem and show that $x = \cos x$ for some x in $\left(0, \frac{\pi}{2}\right)$.
- (d) Show that the function $f(x) = \frac{1}{x^2}$ is not uniformly continuous on $(0, 1)$.
3. (a) Prove that every uniformly continuous function on a set A is continuous on A . Is the converse true? Justify.

- (b) If f and g are functions that are differentiable at the point a , then prove that $(fg)'(a) = f(a)g'(a) + f'(a)g(a)$.
- (c) Let $f(x) = |x| + |x+1|$ for $x \in \mathbb{R}$. Determine the set of points at which the function f is not differentiable. Justify your answer.
- (d) Prove $|\cos x - \cos y| \leq |x - y|$ for all $x, y \in \mathbb{R}$.
4. (a) Suppose that $f: [0, 2] \rightarrow \mathbb{R}$ is continuous on $[0, 2]$, differentiable on $(0, 2)$ and that $f(0) = 0, f(1) = 1, f(2) = 1$.
- (i) Show that $f'(c_1) = 1$ for some $x \in (0, 1)$.
- (ii) Show that $f'(c_2) = 0$ for some $x \in (1, 2)$.
- (iii) Show that $f'(c) = \frac{1}{10}$ for some $x \in (0, 2)$.
- (b) State and prove Mean Value Theorem.
- (c) Show that $e^x \geq 1 + x$ for all $x \in \mathbb{R}$.
- (d) Prove that if f and g are differentiable on $\mathbb{R}$, if $f(0) = g(0)$ and if $f'(x) \leq g'(x)$ for all $x \in \mathbb{R}$, then $f(x) \leq g(x)$ for all $x \geq 0$.

5. (a) If $I_n = \frac{d^n}{dx^n}(x^n \log x)$, prove that $I_n = nI_{n-1} + (n-1)!$ and hence prove that

$$I_n = n! \left[\frac{1}{n} + \frac{1}{n-1} + \frac{1}{n-2} + \dots + \frac{1}{2} + 1 + \log x \right].$$

- (b) If $y = e^{a \sin^{-1} x}$, prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + a^2)y_n = 0.$$

- (c) State the Taylor's theorem for a function defined on some open interval I of real numbers. Also, find the Taylor's series expansion of e^{2x} , for $x \in \mathbb{R}$ about zero.

- (d) Sketch a graph of the equation $y = \frac{2x^2 - 8}{x^2 - 16}$.

6. (a) Find $\lim_{x \rightarrow 0} (1 + \sin x)^x$.

- (b) Find the asymptotes of the rational

function, $y = \frac{x^3 - x^2 - 8}{x - 1}$

- (c) Examine the nature of origin on the curve.

$$(2x + y)^2 - 6xy(2x + y) - 7x^3 = 0$$

- (d) Trace the curve $r = 1 - \cos \theta$.

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Your Roll No.....

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Sr. No. of Question Paper : 2077

Unique Paper Code : 2352011203

Name of the Paper : Ordinary Differential Equations

Name of the Course : B.Sc. (H) Mathematics

Semester : II

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the questions are compulsory.
3. Attempt any **two** parts from each question.
4. Each question carries **7.5** marks.
5. Use of a non-programmable scientific calculator is allowed.

1. (a) Determine the constant A such that the differential equation is exact, and hence solve the resulting equation

$$(x^2 + 3xy)dx + (Ax^2 + 4y)dy = 0.$$

- (b) Solve the initial-value problem

$$(6x + 4y + 1)dx + (4x + 2y + 2)dy = 0, \quad y\left(\frac{1}{2}\right) = 3.$$

- (c) A cake is removed from an oven at 210°F and left to cool at room temperature, which is 70°F. After 30 minutes, the temperature of the cake is 140°F. In how much time will the cake's temperature reach 100°F ?

2. (a) In a certain culture of bacteria, the number of bacteria increased sixfold in 10 h. Assuming the bacteria continue to grow exponentially at a constant rate, how long did it take for the population to double?

- (b) Solve the differential equation by reducing the order

$$xy'' + y' = 4x.$$

- (c) The world's total population reached 6 billion persons in mid-1999, and was then increasing at the rate of about 212 thousand persons each day. Assuming that natural population growth at this rate continues, then

(i) What is the annual growth rate k ?

(ii) What will be the world population in mid-2050?

3. (a) Show that e^x and e^{2x} are linearly independent solutions of the differential equation $y'' - 3y' + 2y = 0$. What is the general solution? Find the particular solution satisfying the initial conditions $y(0) = 1$, $y'(0) = 1$.
- (b) Solve the differential equation using the method of variation of parameters.

$$\frac{d^2y}{dx^2} + y = \sec^2 x$$

- (c) Find the general solution of the differential equation using the method of undetermined coefficients.

$$\frac{d^2y}{dx^2} + y = 2e^x + \cos x$$

4. (a) Use the operator method to find the general solution of the following linear system :

$$\frac{dx}{dt} + 2x - 3y = t$$

$$\frac{dy}{dt} - 3x + 2y = e^{2t}.$$

- (b) Find the general solution of the differential equation, assuming that $x > 0$

$$x^2 \frac{dy}{dx} + y = 3x^2.$$

- (c) A body with mass $\frac{1}{2}$ kg is attached to the end of a spring that is stretched 2 m by a force of 100 N. It is set in motion with an initial position $x_0 = 1$ m and initial velocity $v_0 = -5$ m/s. Find the amplitude, period, and frequency of the resulting motion.

5. (a) Derive the epidemic model of influenza

$$\frac{dS}{dt} = -\beta SI, \quad \frac{dI}{dt} = \beta SI - \gamma I, \quad \frac{dR}{dt} = \gamma I$$

where β and γ are positive constants, $S(t)$, $I(t)$, $R(t)$ denote the number of susceptibles, the number of infectives and the number of recovered at time t respectively.

- (b) Define equilibrium points and find the equilibrium solutions of the Predator-Prey model :

$$\frac{dX}{dt} = \beta_1 X - c_1 XY, \quad \frac{dY}{dt} = c_2 XY - \alpha_2 Y,$$

where X and Y denote prey and predator density, respectively. Positive constants c_1 , c_2 are interaction parameters, β_1 is the per capita birth rate of prey and α_2 is the per capita death rate of the predator. Also, draw a sketch of the trajectory direction in the phase plane.

- (c) The fish population $P(t)$ in a fish farm, follows a density-dependent growth model given by the logistic equation

$$\frac{dP}{dt} = 0.5P \left(1 - \frac{P}{2000} \right)$$

where t is measured in months, and initially at $t = 0$ the pond is stocked with 400 fish.

- (i) Identify the intrinsic growth rate (r) and the carrying capacity (K) of the pond from the equation.
- (ii) Solve the logistic equation to find the expression for $P(t)$.
- (iii) Calculate the fish population after 10 months.

6. (a) Consider the following differential equation of a lake pollution model.

$$\frac{dC}{dt} = \frac{F}{V} C_{in} - \frac{F}{V} C,$$

where C is the concentration of the pollutant in the lake at time t , C_{in} is the concentration of pollutant entering the lake, $F = 4 \times 10^6 \text{ m}^3/\text{month}$ is the flow (in and out), and the lake has constant volume $V = 28 \times 10^6 \text{ m}^3$. If only fresh water enters the lake.

- (i) How long would it take for the lake with a pollution concentration 10^7 parts/ m^3 to drop below the safety threshold 4×10^6 parts/ m^3 ?
 - (ii) How long will it take to reduce the pollution level to 5% of its current level?
- (b) A neutralisation chamber in a pharmaceutical plant maintains a steady operational volume of 1,000 litres of liquid waste. At the beginning of a cleaning cycle ($t = 0$ minutes), the chamber contains an initial mass of m_0 kg of a specific chemical residue. To treat the waste, a neutralizing agent is pumped into the chamber at a constant flow rate of 10 L/min. Due to an automated ramp-up system, the concentration of the neutralising agent in the inflow, $C_{in}(t)$, is not constant but follows the function: $C_{in}(t) = 2e^{-t/200}$. To maintain a constant volume within the chamber, the thoroughly processed (well-mixed) solution is discharged at the same rate of 10 L/min.
- (i) Formulate a first-order differential equation for the total mass of the chemical, $m(t)$, present in the chamber at any time t .
 - (ii) Find the general solution for $m(t)$, given the initial condition $m(0) = m_0$.
 - (iii) Determine the mass of the chemical in the chamber after 50 minutes if $m_0 = 5 \text{ kg}$.
- (c) Find the equilibrium solution of the logistic equation

$$\frac{dX}{dt} = rX \left(1 - \frac{X}{K} \right)$$

where r and K are the reproduction rate and carrying capacity of the population, respectively. Discuss the stability of the equilibrium solution.

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Your Roll No. :
Sl. No. of Q. Paper : **1813** **I**
Unique Paper Code : 2352012401
Name of the Paper : Sequences and Series
of Functions (DSC)
Name of the Course : **B.A/B.Sc.(H)**
Semester : IV

Time : 3 Hours

Maximum Marks : 90

Instructions for Candidates :

- Write your Roll No. on the top immediately on receipt of this question paper.
 - Attempt **all** questions by selecting any **two** parts from each question.
 - All** questions carry equal marks.
 - Use of calculator is **NOT** allowed.
1. (a) State and prove Cauchy's criterion for uniform convergence for sequence of functions.
- (b) Show that the sequence $(n^2x^2e^{-nx})$ converges uniformly on the interval $[1, \infty)$, but that it does not converge uniformly in $[0, \infty)$.
- (c) (i) Show that if the sequences (f_n) , (g_n) converge uniformly on the set A to f and g , respectively, then the sequence $(f_n + g_n)$ converges uniformly to $f + g$.
- (ii) Discuss the uniform convergence of the

sequence of functions $\left(\frac{x^2 + nx}{n} \right)$ on $\mathbb{R}$.

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2. (a) Let (f_n) be a sequence of continuous functions on a set $A \subseteq \mathbb{R}$ and suppose that (f_n) converges uniformly on A to a function $f : A \rightarrow \mathbb{R}$. Prove that f is continuous on A .
- (b) Let $f_n(x) = \frac{nx}{1+nx^2}$ for $x \in A = [0, \infty)$. Show that each f_n is bounded on A , but the pointwise limit f of the sequence (f_n) is not bounded on A . Does the sequence (f_n) converge uniformly to f on A ?
- (c) (i) Let $f_n(x) = \frac{x^n}{n}$, for $x \in [0, 1]$. show that the sequence (f_n) of differentiable functions converges uniformly to a differentiable function f on $[0, 1]$, and that the sequence (f'_n) converges on $[0, 1]$, but that $g(1) \neq f'(1)$.
- (ii) If $a > 0$, show that $\lim_{n \rightarrow \infty} \int_a^{\pi} \frac{\sin nx}{nx} dx = 0$
3. (a) State and prove Weierstrass M-test for uniform convergence of the series of functions. Hence, show that $\sum b_n \sin nx$ is a uniformly convergent series given that $\sum b_n$ is an absolutely convergent series.
- (b) Let f_n be a real valued function on $[a, b]$ that has derivative f'_n on $[a, b]$ for each $n \in \mathbb{N}$. Suppose that the series $\sum f_n$ converges for at least one point of $[a, b]$ and the series of derivatives $\sum f'_n$ converges uniformly on $[a, b]$. Show that there

exists a real valued function f on $[a, b]$ such that $\sum f_n$ converges uniformly to f on $[a, b]$. Also, that f has a derivative on $[a, b]$ and $f' = \sum f'_n$

- (c) Discuss the pointwise convergence of $\sum_{n=1}^{\infty} \frac{1}{x^n + 1}, x \geq 0$. Also, show that the given series is uniformly convergent on $[2, \infty)$ but not uniformly convergent on $(1, \infty)$.

4. (a) Prove that $\sum_{n=1}^{\infty} \sin\left(\frac{x}{n^2}\right)$ converges for all $x \in \mathbb{R}$.

Also, show that the series converges uniformly on $[-a, a]$, $a > 0$ but convergence is not uniform on $\mathbb{R}$.

- (b) Let (f_n) be a sequence of integrable functions on $[a, b]$ and suppose that $\sum_{n=1}^{\infty} f_n$ converges uniformly to f on $[a, b]$. Show that f is integrable

on $[a, b]$ and $\int_a^b f = \sum_{n=1}^{\infty} \int_a^b f_n$.

- (c) Prove that $\sum_{n=1}^{\infty} \frac{1}{n^2 x^2}$ is convergent for all $x \neq 0$.

Also, show that the convergence is not uniform for $|x| < 1$ but is uniform for $|x| \geq 1$.

5. (a) Define power series and its radius of convergence. Evaluate the radius of convergence

for the series $\sum \frac{n!^2}{(2n)!} x^n$

(b) show that

$$\sum_{n=0}^{\infty} (-1)^n x^{2n} = \frac{1}{1+x^2} \text{ for } x \in (-1, 1)$$

(c) let $f(x) = \sum_{n=0}^{\infty} a_n x^n$ be a power series with finite positive radius of convergence R . Prove that if the series converge at $x = R$, then f is continuous at $x = R$. If the series converges at $x = -R$, then f is continuous at $x = -R$

6. (a) Let f be a continuous function on $[a, b]$. Show that there exists a sequence (p_n) of polynomials such that $p_n \rightarrow f$ uniformly on $[a, b]$ and $p_n(a) = f(a)$ for all n .

(b) If $x \geq 0$ and $x \in \mathbb{N}$, show that

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \dots + (-x)^{n-1} + \frac{(-x)^n}{1+x}$$

Hence show that

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^n \frac{x^n}{n} + \int_0^x \frac{(-t)^n}{1+t} dt$$

(c) Show that if $x > 0$ then

$$1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} \leq \cos x \leq 1 - \frac{x^2}{2} + \frac{x^4}{24}$$

Use this inequality to establish a lower bound for π .

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Your Roll No. :
Sl. No. of Q. Paper : **1932** **I**
Unique Paper Code : 2352012402
Name of the Paper : Multivariate Calculus-
DSC
Name of the Course : **B.Sc. (H) Mathematics**
Semester : IV

Time : 3 Hours

Maximum Marks : 90

Instructions for Candidates :

- Write your Roll No. on the top immediately on receipt of this question paper.
- All** questions are compulsory and are of **15** marks each.
- Attempt any **two** parts from each Questions. Each part is of **7.5** marks.

1. (a) Consider the function:

$$f(x, y) = \begin{cases} \frac{2x^2 - x^2y^2 + 2y^2}{x^2 + y^2} & \text{for } (x, y) \neq (0, 0) \\ A & \text{for } (x, y) = (0, 0) \end{cases}$$

Find the value of A, which will make f continuous at the (0,0).

- The temperature at a point (x, y) on a given metal plate in the xy plane is determined according to the formula $T(x, y) = x^3 + 2xy^2 + y$ degrees. Compute the rate at which the temperature changes with distance if we start at (2, 1) and move
 - parallel to the vector j
 - parallel to the vector i

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- (c) A juice can is 12 cm tall and has a radius of 3 cm. A manufacturer is planning to reduce the height of the can by 0.2 cm and the radius by 0.3 cm. Use a total differential to estimate the percentage decrease in volume that occurs when the new cans are introduced. (Round to the nearest percent.)
2. (a) Let $w = f(t)$ be a differentiable function of t , where $t = (x^2 + y^2 + z^2)^{1/2}$. show that

$$\left(\frac{dw}{dt}\right)^2 = \left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2$$

- (b) Find the directional derivative of $f(x, y) = e^{x^2} y^2$ at $P_0(1, -1)$ in the direction toward $Q(2, 3)$.
- (c) Find the absolute extrema of the function $f(x, y) = xy - 2x - 5y$ on the closed bounded set S , where S is the triangular region with vertices $(0, 0)$, $(7, 0)$ and $(7, 7)$.
3. (a) (i) Find the volume of the solid bounded below by the rectangle R in the xy -plane and above by the graph:

$$z = \frac{xy}{\sqrt{x^2 + y^2 + 1}}; \quad R: 0 \leq x \leq 1, 0 \leq y \leq 1$$

- (ii) Sketch the region of integration and compute the double integral:

$$\int_0^{\frac{\pi}{3}} \int_0^{y^2} \frac{1}{y} \sin \frac{x}{y} dx dy$$

- (b) (i) Compute double integral by converting to polar coordinates:

$$\int_0^3 \int_x^{\sqrt{9-x^2}} \cos(x^2 + y^2) dy dx$$

- (ii) Use a double integral to find the area bounded by $r = \cos 2\theta$.

- (c) (i) Evaluate Triple integral:

$$\iiint_D x dV$$

where, D is bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 1$.

- (ii) Compute the iterated triple integral.

$$\int_{-1}^2 \int_0^\pi \int_1^4 yz \cos xy dz dx dy$$

4. (a) Find the volume of the solid bounded by cylinders $y = z^2$ and $y = 2 - z^2$ and the planes $x = 1$ and $x = -2$
 (b) Evaluate triple integral

$$\iiint_D \frac{dx dy dz}{\sqrt{x^2 + y^2 + z^2}}$$

where, D is the solid sphere $x^2 + y^2 + z^2 \leq 3$

- (c) Use suitable change of variable find the area of the region R bounded by the hyperbolas $xy = 1$ and $xy = 4$ and the lines $y = x$ and $y = 4x$.

5. (a) Evaluate $\int_C F dR$, where $F = (y^2 - z^2)i + (2yz)j - x^2k$ and C is the curve defined parametrically by $x = t^2, y = 2t$ and $z = t$, for $0 \leq t \leq 1$.

- (b) Show that the vector field $F(x,y) = (x+2y)i + (2x+y)j$ is conservative and find a scalar potential f for F . Evaluate the line integral $\int_C F \cdot dR$, where C is any smooth path connecting $A(0,0)$ to $B(1,1)$.

- (c) Use Green's theorem to find the work done by the force field

$$F(x,y) = (3y-4x)i + (4x-y)j$$

When an object moves once counterclockwise around the ellipse $4x^2 + y^2 = 4$.

6. (a) Evaluate $\iint_S (x^2 + y^2) dS$, where S is the surface of the hemisphere $z = \sqrt{1 - x^2 - y^2}$.

- (b) Let S be the portion the plane $x+y+z = 1$ that lies in the first octant, and let C be the boundary of S , traversed counterclockwise. Verify Stokes' theorem for the surface S and the vector field $F =$

$$F = -\frac{3}{2}y^2i - 2xyj + yzk.$$

- (c) Use the divergence theorem to evaluate.

$$\iint_S F \cdot N dS$$

Where, $F = xyzi + xyzj + xyzk$, S is the surface of the box $0 \leq x \leq 1, 0 \leq y \leq 2, 0 \leq z \leq 3$, with outward unit normal vector N .

This question paper contains 4 printed pages]

Roll No.

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S. No. of Question Paper : **2039**

Unique Paper Code : **2352012403**

Name of the Paper : **DSC-Numerical Analysis**

Name of the Course : **B.Sc. (H) Mathematics**

Semester : **IV**

Duration : **3 Hours**

Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the six questions are compulsory.

Attempt any *two* parts from each question.

Marks are indicated against each question.

Choice is given within the question.

Use of scientific calculator is allowed.

1. (a) Define the order of convergence of a sequence p_n which converges to a number p . Show that the sequence generated by the formula

$$p_{n+1} = \frac{p_n^3 + 3p_n a}{3p_n^2 + a}$$

converges to $\sqrt{a}$. Determine the order of convergence and the asymptotic error constant.

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P.T.O.

(b) Verify that the equation $x^3 + x^2 - 3x - 3 = 0$ has a root on the interval $(1, 2)$. Perform the bisection method to determine p_4 , the fourth approximation to the location of the root.

(c) Perform four iterations of the method of false position to approximate the root of the equation $x^4 - 18x^2 + 45 = 0$ in the interval $(1, 2)$. 15

2. (a) Let g be a continuous function on the interval $[a, b]$ such that $g([a, b]) \subset [a, b]$. Suppose g is differentiable function on (a, b) and satisfies

$$|g'(x)| \leq k < 1, \forall x \in (a, b)$$

A sequence p_n is generated by the fixed point iteration $p_n = g(p_{n-1}), n \geq 1$.

Show that the sequence $\{p_n\}$ converges to a fixed point p of g for any initial approximation $p_0 \in [a, b]$.

(b) Use Newton's method to approximate the root of the equation $x^5 + 2x - 1 = 0$ in the interval $(0, 1)$. Use error tolerance 10^{-4} , and initial approximation $p_0 = 0.5$.

(c) Perform the Secant method to determine p_4 , the fourth approximation to the location of the root of the equation $\tan(\pi x) - \pi - 6 = 0$ with starting approximations $p_0 = 0, p_1 = 0.48$. 15

3. (a) Find an LU decomposition of the matrix

$$A = \begin{bmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{bmatrix}$$

and use it to solve the system $AX = [14 \ 36 \ 7]^T$.

- (b) Perform three iterations of Jacobi method to solve the system of equations, for the given coefficient matrix and right hand side vector, starting with the initial vector $x^{(0)} = 0$,

$$\begin{bmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{bmatrix}, \begin{bmatrix} 10 \\ -14 \\ 33 \end{bmatrix}$$

- (c) State and prove existence and uniqueness theorem for Interpolating Polynomial. 15

4. (a) Find the Lagrange form of interpolating polynomial for the given data set and hence interpolate at $x = 1.5$.

x	-1	0	1	2
$f(x)$	5	1	1	11

- (b) If $f(x) = \frac{1}{x}$ then evaluate the n^{th} divided difference $f[x_0, x_1, x_2, \dots, x_n]$.
- (c) Obtain the piecewise Linear Interpolating Polynomial for the function $f(x)$ defined by the given data set and hence estimate the value of $f(1.5)$

x	0	1	2	3	4
$f(x)$	1	2	5	10	16

15

5. (a) Prove the forward difference approximation formula $f'(x_0) \approx \frac{-3f(x_0) + 4f(x_0 + h) + f(x_0 + 2h)}{2h}$ and determine the approximate value of the derivative of $f(x) = 2x^2 - 1$ at $x_0 = 1$, with the step length $h = 0.1, 0.01$ and 0.001 , and absolute error.
- (b) Verify that the second-order central difference approximation for the second derivative provides the exact value of the second derivative, regardless the value of h , for the functions $f(x) = 1$, $f(x) = x$, $f(x) = x^2$, and $f(x) = x^3$, but not for the function $f(x) = x^4$.
- (c) Derive the expression of the Simpson's rule to approximate definite integral and hence approximate the value of the integral $\int_0^1 e^{-x} dx$. 15
6. (a) Define the degree of precision of the quadrature rule. Further, calculate the values of the coefficients A_0 , A_1 , and A_2 so that the quadrature formula $I(f) = \int_{-1}^1 f(x) dx = A_0 f\left(\frac{-1}{2}\right) + A_1 f(0) + A_2 f\left(\frac{1}{2}\right)$ has degree of precision at least 2.
- (b) Use the Modified Euler Method to determine the approximate solution of the initial value problem $\frac{dx}{dt} = \frac{t}{x}$, $x(1) = 1$, $1 \leq t \leq 2$, taking the step size as $h = 0.5$.
- (c) Use the Runge-Kutta Method fourth order to determine the approximate solution of the initial value problem $\frac{dx}{dt} = 1 + \frac{x}{t}$, $x(1) = 1$, $1 \leq t \leq 2$, taking the step size as $h = 0.5$. 15

[This question paper contains 2 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 13219
Unique Paper Code : 32351403
Name of the Paper : Ring Theory and Linear Algebra-I
Name of the Course : B.Sc. (H) Mathematics DSC
Semester : IV
Duration : 3 Hours

L

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. Parts of the questions to be attempted together.
4. **All** questions carry equal marks.
5. Use of Calculator not allowed.

1. (a) State and prove Eisenstein's criterion for irreducibility.
(b) In $\mathbb{Z}[\sqrt{-5}]$, prove that $1+3\sqrt{-5}$ is irreducible but not prime.
(c) State and prove the division algorithm for $F[x]$ where F is a field.

(6,6,6)

2. (a) Prove that the ideal $\langle x \rangle$ in $\mathbb{Q}[x]$ is maximal.
(b) State and prove the test for divisibility by 11.
(c) Prove that $8x^3-6x+1$ is irreducible over $\mathbb{Q}$.

(6, 6, 6)

3. (a) Prove that any intersection of subspaces of a vector space V is a subspace of V .
Also, prove or disprove that the union of subspaces of a vector space is a subspace.

- (b) Define the linear span of a set. Check whether the set

$$S = \{x^2+3x-2, 2x^2+5x-3, -x^2-4x+4\} \text{ spans } P_2(\mathbb{R}).$$

- (c) Define the linear dependence and linear independence of finite set of vectors.

Determine whether the following sets are linearly dependent or linearly independent.

(i) $S_1 = \{(1, -1, 2), (1, -2, 1), (1, 1, 4)\}$ in $\mathbb{R}^3$

(ii) $S_2 = \{(x^3 + 2x^2, x^2 + 3x + 1, -x^2 + 2x - 1)\}$ in $P_3(\mathbb{R})$.

(6,6,6)

4. (a) Define a basis of a vector space

(i) Check whether the set $S = \{(1, 2, -1), (1, 0, 2), (2, 1, 1)\}$ is a basis of $\mathbb{R}^3$.

(ii) Show that the following set is a basis of $M_{2 \times 2}(\mathbb{R})$

$$S = \left\{ \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \right\}.$$

(b) If a vector space V is generated by a finite set S , then prove that some subset of S is a basis of V .

(c) State the Replacement Theorem. Let V be a vector space with dimension n then prove that any linearly independent subset of V that contains exactly n vectors is a basis for V .

(6, 6, 6)

5. (a) Let V be a vector space and $\beta = \{u_1, u_2, \dots, u_n\}$ be a subset of V . Prove that β is a basis for V iff each v in V can be uniquely expressed as a linear combination of vectors of β .

(b) Prove that $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$

$$T(a_1, a_2, a_3) = (a_1 - a_2, 2a_3)$$

is a linear transformation. Also, find its null space and range space.

(c) Consider the linear transformations

$$T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$$

$$\text{and } U: \mathbb{R}^2 \rightarrow \mathbb{R}^3$$

$$T(a_1, a_2) = (a_1 + 3a_2, 0, 2a_1 - 4a_2) \quad U(a_1, a_2) = (a_1 - a_2, 2a_1, 3a_1 + 2a_2)$$

Find $[T]_\beta^\alpha$, $[U]_\beta^\alpha$ and $[T+U]_\beta^\alpha$ where β and α are standard basis of $\mathbb{R}^2$ and $\mathbb{R}^3$, respectively.

6. (a) Let V and W be finite dimensional vector spaces with ordered basis β and γ respectively. Let $T: V \rightarrow W$ be a linear transformation. Prove that

T is invertible iff $[T]_\beta^\gamma$ is invertible. Also, prove that $[T^{-1}]_\gamma^\beta = ([T]_\beta^\gamma)^{-1}$.

(b) Let $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ and $\beta = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}$ be an ordered basis of $\mathbb{R}^2$. Find $[L_A]_\beta$ and also find an invertible matrix Q such that

$$[L_A]_\beta = Q^{-1}AQ$$

where L_A is a left multiplication transformation.

(c) Let V , W and Z be vector spaces and let $T: V \rightarrow W$, $U: W \rightarrow Z$ be linear transformations. Prove that

(i) If UT is one-one then T is one-one.

(ii) If UT is onto then U is onto.

(7.5, 7.5, 7.5)

85
[This question paper contains 2 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 278

L

Unique Paper Code : 2353010015

Name of the Paper : Research Methodology

Name of the Course : **Bachelor of Science (Honours Course) / Bachelor of Arts / Bachelor of Science (Programme) – DSE**

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
 2. Attempt all the **six** questions, selecting any **two** parts from each question.
 3. Parts of the question are to be attempted together.
 4. All the **six** questions carry equal marks.
-
1. (a) List and elaborate the general principles of mathematical writing.
(b) What are the various ways for practicing good mathematical judgement? Explain in detail.
(c) What is the format of a mathematical research paper? Suppose that there is a journal called 'XYZ', which publishes research papers in Mathematics. What is the unique international identifier, associated with this journal 'XYZ' called?
 2. (a) What are the three questions that should be considered, when formulating a definition in a mathematics research paper? Explain the importance of each of these questions.
(b) State the important points that should be kept in mind, when citing papers/ references in a research paper.
(c) What are the key points that should be taken into consideration, before and after submission of a manuscript to a journal, for possible publication?
 3. (a) Explain the use of each of the following LaTeX commands: \label, \ref, and \cite. Give examples for each of the above commands.

- (b) Write a code in Latex to obtain the following output :

Example of a Theorem Environment

We can now state and use our custom theorem environment.

Theorem 1 (Pythagoras Theorem). *Let a and b be the legs of a triangle of right angles and c be the hypotenuse. Then*

$$c^2 = a^2 + b^2.$$

We can also give a proof using the built-in proof environment:

- (c) What are the *equation* and the *align* environments in LaTeX? Explain the use of each of the two environments. State when, each one should be used.
4. (a) Write PSTricks code to draw a triangle and label its vertices A, B, and C.
- (b) Discuss the important features of a good poster, including content organization, visual design, and readability.
- (c) Make a Beamer presentation on the topic 'Importance of Mathematical Writing' with exactly five slides including a title slide, three content slides, and a thank you slide.
5. (a) List any four features of each of the following web resources for Mathematics: AMS and MAA.
- (b) What is meant by JCR? How is the metric 'impact factor' of journal, defined as per JCR? Give the purpose and challenges of this journal metric.
- (c) Write a short note on Google Scholar metrics. What are the two main indicators on the basis of which, publications are ranked by Google Scholar metrics. Explain these indicators in brief.
6. (a) What is Scopus? What are the key features of Scopus? What is the importance of Scopus, with respect to the development of research in Mathematics?
- (b) State the research ethics as standardized by COPE, concerning all parties involved in the conduct of research.
- (c) List out the steps/instructions for using the plagiarism checker 'Turnitin' software, for finding out the percentage of similarity of a given research paper.

[This question paper contains 2 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1752

L

Unique Paper Code : 2352013602

Name of the Paper : Advanced Linear Algebra

Name of the Course : B.Sc. (H) Mathematics (NEP-UGCF 2022)

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Attempt any TWO parts from each question.
4. All questions carry equal marks.
5. Use of Calculator is not allowed.

1. (a) Let β and β' be two ordered bases for a finite-dimensional vector space V .
(i) Define change of coordinate matrix Q that changes β' -coordinates into β -coordinates.
(ii) Show that for any $v \in V$, $[v]_{\beta} = Q[v]_{\beta'}$.
(iii) Let $\beta = \{(1,1), (1,-1)\}$ and $\beta' = \{(2,4), (3,1)\}$ be two ordered bases of $\mathbb{R}^2$. Taking $v = (2,3)$, verify that $[v]_{\beta} = Q[v]_{\beta'}$.
(b) Let $T: P_1(\mathbb{R}) \rightarrow \mathbb{R}^2$ be a linear transformation given by
$$T(p(x)) = (p(0), p(2)), \forall p(x) \in P_1(\mathbb{R}).$$

Let β and γ be standard ordered bases for $P_1(\mathbb{R})$ and $\mathbb{R}^2$ respectively. Verify that $[T^t]_{\gamma}^{\beta} = ([T]_{\beta})^t$.
(c) Define annihilator W^0 of a subset W of a vector space V . Show that W^0 is a subspace of V . Prove that if W is a subspace of V , then $\dim(W) + \dim(W^0) = \dim(V)$.
2. (a) Define eigenvalues, eigenvectors and characteristic polynomial of a linear operator T defined on an n -dimensional vector space V . Let T be the operator on $\mathbb{R}^2$ that rotates the plane through an angle θ ($0 < \theta < \pi$). Obtain the characteristic polynomial of T and hence deduce that T has no eigenvalues and eigenvectors.
(b) Show that the characteristic polynomial of any diagonalizable linear operator splits. Is the converse true? Justify.
(c) Let $W_1, W_2, \dots, W_k$ be subspaces of a finite-dimensional vector space V . Show that $V = W_1 \oplus W_2 \oplus \dots \oplus W_k$ if and only if each vector $v \in V$ can be uniquely written as $v = v_1 + v_2 + \dots + v_k$ for some $v_i \in W_i$.

3. (a) For which of the following linear operator T on the vector space V , determine whether the given subspace W is T -invariant subspace of V :
- (i) $V = P_2(\mathbb{R})$, $T(f(x)) = f'(x)$, and $W = \{f \in P_2(\mathbb{R}) : f(0) = 1\}$.
- (ii) $V = \mathbb{R}^3$, $T(a, b, c) = (a + b, b + c, a + c)$, and $W = \{(t, t, t) : t \in \mathbb{R}\}$.
- (b) Let T be a linear operator on a finite dimensional vector space V and let W be a T -invariant subspace of V . Prove that if the characteristic polynomial of T splits, then the characteristic polynomial of T_W also splits.
- (c) Compute the minimal polynomial of the matrix A ,
- $$A = \begin{pmatrix} 3 & -1 & 0 \\ 0 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}.$$
4. (a) Let $x = (2, 1 + i, -i)$ and $y = (2 + i, 2, 1 - 2i)$ be vectors in C^3 . Compute $\langle x, y \rangle$, $\|x\|$, $\|y\|$ and $\|x + y\|$. Then verify the Cauchy-Schwarz inequality and the triangle inequality for above vectors x and y .
- (b) Prove that if V is an inner product space, then $|\langle x, y \rangle| = \|x\|\|y\|$ if and only if one of the vectors x or y is a multiple of the other.
- (c) Using Gram-Schmidt process, obtain an orthonormal basis β for $\text{span}(S)$ where $S = \{(1, 0, 1), (0, 1, 1), (1, 1, 0)\}$. Also, compute the Fourier coefficients of $x = (1, 1, 1)$ with respect to β .
5. (a) Write the definition of the orthogonal complement of a set S in an inner product space V . Let $W = \text{span}\{(i, 0, 1)\}$ in C^3 . Find orthonormal bases for W and $W^\perp$.
- (b) For each of the following inner product spaces V and linear operators T on V , evaluate T^* at the given vector in V :
- (i) $V = \mathbb{R}^2$, $T(a, b) = (a + b, a - b)$, $x = (1, 2)$.
- (ii) $V = P_1(\mathbb{R})$ with $\langle f(x), g(x) \rangle = \int_{-1}^1 f(t)g(t)dt$, $T(f) = f'$, $f(t) = 1 + 2t$.
- (c) Find the minimal solution for the following system of linear equations
- $$\begin{aligned} x + 2y + z &= 4 \\ x - y + 2z &= -11 \\ x + 5y &= 19. \end{aligned}$$
6. (a) Let T be a linear operator on a finite dimensional complex inner product space V . Prove that T is normal if and only if there exists an orthonormal basis for V consisting of eigenvectors of T .
- (b) For the given matrix A , find an orthogonal matrix P and a diagonal matrix D such that $P^*AP = D$, where
- $$A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}.$$
- (c) Let T be a linear operator on an inner product space V . Prove that T is an orthogonal projection if and only if T has an adjoint T^* and $T^2 = T^* = T$.

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1772

L

Unique Paper Code : 2352013603

Name of the Paper : DSC-18 Complex Analysis

Name of the Course : **Bachelor of Science (Honours Course)**
Mathematics

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
2. All questions carry equal marks.
3. Attempt **two** parts from each question.

1. (a) (i) Solve $e^z = 3$, where z is any complex number.

(ii) Find the values of $\text{Log}(1+i)^2$ and $\text{Log}(-1+i)^2$ and justify the truth of $\text{Log}(z_1 \cdot z_2) = \text{Log}(z_1) + \text{Log}(z_2)$ for $z_1, z_2 \in \mathbb{C}$.

- (b) Prove that if $f'(z) = 0$ everywhere in a domain D , then $f(z)$ must be constant throughout D .

- (c) Show that each of the following functions is nowhere analytic :

(i) $f(z) = xy + iy$

(ii) $f(z) = 2xy + i(x^2 - y^2)$

2. (a) Prove that if the function $f(z) = u(x, y) + iv(x, y)$ be defined throughout some ϵ -neighbourhood of a point $z_0 = x_0 + iy_0$, and the following conditions hold true :

(i) The first order derivatives of the functions u and v with respect to x and y exist everywhere in the neighbourhood;

(ii) Those partial derivatives are continuous at (x_0, y_0) and satisfy the Cauchy- Riemann equations at (x_0, y_0) ,

Then $f'(z)$ exists and $f'(z) = u_x + iv_x$ at $z = z_0$.

(b) Find the image of the domain $x > 0, y > 0, xy < 0$ under the mapping $w = z^2$.

(c) If z_0 and w_0 are points in the z and w planes, respectively, then prove that

$$(i) \lim_{z \rightarrow \infty} f(z) = w_0 \text{ if and only if } \lim_{z \rightarrow 0} f\left(\frac{1}{z}\right) = w_0.$$

$$(ii) \lim_{z \rightarrow \infty} f(z) = \infty \text{ if and only if } \lim_{z \rightarrow 0} \frac{1}{f\left(\frac{1}{z}\right)} = 0.$$

3. (a) Prove that if $w(t)$ is piecewise continuous complex-valued function defined on an interval $a \leq t \leq b$, the :

$$\left| \int_a^b w(z) dt \right| \leq \int_a^b |w(t)| dt.$$

Find an upper bound for $\left| \int_C \frac{z+4}{z^3-1} dz \right|$, where C is the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$.

(b) Suppose that a function $f(z)$ is continuous in a domain D . Prove that if $f(z)$ has an antiderivative $F(z)$ throughout D , then the value of the integral of $f(z)$ along contours lying entire y in D and extending from any fixed point z_1 to any other fixed point z_2 is

$$\int_{z_1}^{z_2} f(z) dz = F(z_2) - F(z_1)$$

Use it to find the value if $\int_0^{\pi+2i} \cos\left(\frac{z}{2}\right) dz$.

(c) State Cauchy-Goursat theorem. Use it to show that

$$\int_C f(z) dz = 0, \text{ when}$$

(i) $f(z) = \frac{1}{z^2 + 2z + 2}$ and C is the unit circle $|z| = 1$.

(ii) $f(z) = \frac{5z + 7}{z^2 - 4z + 3}$ and C is the unit circle $|z - 2| = 2$.

4. (a) For a function f given to be analytic inside and on a positively oriented circle C_R , centered at z_0 and with radius R , if M_R denotes the maximum value of $|f(z)|$ on C_R , then prove that $|f^n(z_0)| \leq \frac{n! M_R}{R^n}$.

Hence, show that if f is an entire function such that $|f(z)| \leq A|z|$ for all z , where A is a fixed positive number, then $f(z) = a_1 z$, where a_1 is a complex constant.

- (b) Evaluate $\int_C \frac{3z+1}{z(z-2)^2} dz$, where C is the figure-eight contour formed by two circles with centres at $(0,0)$ and $(2,0)$ respectively with radii 0.5 and 1.5 respectively, described in the positive direction,
- (c) State and prove Liouville's theorem. Apply this to show that if $f(z)$ is entire and the function $u(x, y) = \operatorname{Re}[f(z)]$ is bounded above by u_0 at all points (x, y) in the xy plane, then $u(x, y)$ must be constant throughout the plane.

5. (a) Give all the Laurent series expansions in the powers of z for the function

$$f(z) = \frac{1}{z^2(1-z)} \text{ and specify the domains in which those expansions are valid.}$$

- (b) Suppose that $z_n = x_n + iy_n$ ($n = 1, 2, 3, \dots$) and $S = X + iY$. Then, prove that $\sum_{n=1}^{\infty} z_n = S$ iff $\sum_{n=1}^{\infty} x_n = X$ and $\sum_{n=1}^{\infty} y_n = Y$. Also show that if $\sum_{n=1}^{\infty} z_n = S$ then $\sum_{n=1}^{\infty} \overline{z_n} = \overline{S}$.

- (c) State Taylor's theorem and derive Taylor's series representations for the following :

$$(i) \frac{z}{(z^4 + 9)} = \sum_{n=0}^{\infty} \frac{(-1)^n}{3^{2n+2}} z^{4n+1} \text{ where, } |z| < \sqrt{3}$$

$$(ii) \frac{\sin(z^2)}{z^4} = \frac{1}{z^2} - \frac{z^2}{3!} + \frac{z^6}{5!} - \frac{z^{10}}{7!} \dots \dots \dots, \text{ when } z \neq 0.$$

6. (a) State Cauchy's residue theorem and use it to evaluate the integral

$$\int \frac{dz}{z^3(z+4)}$$

over the circle $|z + 2| = 3$ oriented anticlockwise.

- (b) Define the terms: isolated and non-isolated singular points. Write the principal part of each of the following functions at their respective isolated singular points and determine whether that point is a pole, a removable singular point, or an essential singular point.

$$(i) z \exp\left(\frac{1}{z}\right)$$

$$(ii) \frac{1 - \cos z}{z^2}$$

$$(iii) \frac{1}{(2-z)^3}.$$

- (c) Use residues to evaluate $\int_0^{2\pi} \frac{d\theta}{5 + 4\sin \theta}.$

[This question paper contains 2 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1714

L

Unique Paper Code : 2352014801

Name of the Paper : Field Theory and Galois Extension

Name of the Course : B.Sc. (Hons.) Mathematics

Semester : VIII

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Attempt any two parts from each question.
4. All questions carry equal marks.
5. Use of non-programmable scientific calculator is permitted.
6. All symbols have usual meaning as per prescribed textbook.

1. (a) If $M : L$ and $L : K$ are extensions. Then show that

$$[M : K] = [M : L] [L : K]$$

- (b) If $f \in K[x]$, then there exists a splitting field extension $L : K$ for f , with $[L : K] \leq n!$

- (c) If $L : K$ is an extension of field K then $[L : K] < \infty$ if and only if there exist finitely many algebraic elements $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$ in L such that $L = K(\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n)$.

2. (a) If $L : K$ is an extension of field K and $\alpha \in L$, then α is algebraic over K if and only if $[K(\alpha) : K] < \infty$. Further deduce that $[K(\alpha) : K] = \deg(m_\alpha)$ for minimal polynomial m_α .

- (b) If $f \in K[x]$ is irreducible of degree n , show that there is a simple extension $K(\alpha) : K$ such that $[K(\alpha) : K] = n$ and $f(\alpha) = 0$.

- (c) (i) Let $f = x^2 + 2x + 3$ be in $\mathbb{Z}[x]$, find its splitting field in $\mathbb{C}(x)$. What if $f \in \mathbb{Z}_7[x]$?

- (ii) Can we construct a regular seven-sided polygon with ruler and compasses? Justify.

3. (a) Show that a finite extension $L : K$ is normal if and only if $L : K$ is a splitting field for some $g \in K[x]$.

- (b) Define $L : K$ as a separable extension. Show that if $L : K$ is separable and M is an intermediate field ($L : M : K$), then $L : M$ is separable and $M : K$ is separable.

- (c) Show that if $L:K$ is an extension and A is a subset of $\text{Aut}(L)$, then
- $\gamma\phi(A) \supseteq A$
 - $\phi\gamma(K) \supseteq K$
 - $\phi\gamma\phi(A) = \phi(A)$
 - $\gamma\phi\gamma(K) = \gamma(K)$.
4. (a) Suppose $L:K$ is finite. If $L:K$ is a Galois extension, show that $|\gamma(K)| = [L:K]$ and $K = \phi\gamma(K)$. Otherwise, $|\gamma(K)| < [L:K]$ and K is a proper subfield of $\phi\gamma(K)$. State any theorems you use clearly.
- (b) Find the Galois group of $x^4 - 2$ over the field $\mathbb{Z}_7$.
- (c) Suppose $L:K$ is finite. Let $G = \Gamma(L:K)$ and let $K_0 = \phi(G)$. If $L:M:K$, let $\gamma(M) = \Gamma(L:M)$. Show that ϕ is one-to-one from the set of subgroups of G onto fields M , intermediate between L and K , and γ is its inverse map. State any theorems you use clearly.
5. (a) Define the discriminant of a polynomial. Prove that the discriminant is zero if and only if the polynomial has a repeated root. Compute the discriminant of the polynomial $f(x) = x^3 - 3x + 1$.
- (b) Suppose $L:K$ is a splitting field extension for $x^m - 1$ over K . Define m -th-cyclotomic polynomial Φ_m . Prove that $x^m - 1 = \prod_{d|m} \Phi_d$.
- (c) Show that the mapping $\Phi(a) = a^p$ is an automorphism of a finite field F of order p^n (p , a prime) and generates $\Gamma(F, \mathbb{Z}_p)$.
6. (a) Show that an algebraic extension $L:K$ is simple if there are only finitely many intermediate fields.
- (b) Show that $\mathbb{Q}(\sqrt{2}, \sqrt{3}): \mathbb{Q}$ is a simple extension.
- (c) Find the Galois groups of $x^5 + 1$ over $\mathbb{Q}$.

[This question paper contains 2 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 7382

L

Unique Paper Code : 2353010024

Name of the Paper : DSE 6 (iv) Geometry of Curves and Surfaces

Name of the Course : B.Sc. (Honors) Mathematics

Semester : VIII

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt all **six (06)** questions by doing any **two** parts from each question.
3. All questions carry equal marks.
4. Notations used are standard.

Q1 (a) Write the parametric equation of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ traced in the anticlockwise direction and then determine its curvature.

(b) Prove that if the curvature of a plane curve p is constant $\kappa > 0$, then p describes a circle with radius $\frac{1}{\kappa}$.

(c) Prove that every oval (i.e. convex closed curve) has at least four vertices.

Q2 (a) Define the curvature of a space curve $p(s)$ and then state the Frenet-Serret's formula. Also state the assumption required on the curvature, if any.

(b) Calculate the curvature and torsion for the space curve $q(t) = (t, t^2, t^3)$.

(c) If a simple closed curve $p(s)$ in the space is a non-trivial knot, then show that its total curvature is at least 4π .

Q3 (a) For each of the following surfaces, write the name of surface and its equation in cartesian form

(i) $x = au \cos v, y = bu \sin v, z = u^2$

(ii) $x = a \left(\frac{u-v}{u+v} \right), y = b \left(\frac{uv+1}{u+v} \right), z = c \left(\frac{uv-1}{u+v} \right)$

(b) Prove that the necessary and sufficient condition for a curve $\vec{p}(s) = (u(s), v(s))$ on a surface $\vec{p}(u, v)$ to be a geodesic is that $(u(s), v(s))$ is a solution to the following system of differential equations

$$\frac{d^2 u}{ds^2} + \Gamma_{uu}^u \frac{du}{ds} \frac{du}{ds} + 2\Gamma_{uv}^u \frac{du}{ds} \frac{dv}{ds} + \Gamma_{vv}^u \frac{dv}{ds} \frac{dv}{ds} = 0,$$

$$\frac{d^2 v}{ds^2} + \Gamma_{uu}^v \frac{du}{ds} \frac{du}{ds} + 2\Gamma_{uv}^v \frac{du}{ds} \frac{dv}{ds} + \Gamma_{vv}^v \frac{dv}{ds} \frac{dv}{ds} = 0$$

(c) Prove that the surface

$$x = 3u + 3uv^2 - u^3, \quad y = v^3 - 3v - 3u^2v, \quad z = 3(u^2 - v^2),$$

is a minimal surface. Also, calculate its Gaussian curvature K .

Q4 (a) What is Ruled Surface? Prove that for a ruled surface, $K \leq 0$ everywhere.

(b) Define the exterior multiplication. Prove that when the coordinates (u, v) are changed to the polar coordinates (r, θ) by using the equations $u = r \cos \theta$, $v = r \sin \theta$, then

$$du \wedge dv = r dr \wedge d\theta.$$

(c) Let $\vec{p}(s)$ be a space curve with curvature $k > 0$ everywhere. Show that the torsion $\tau = 0$ everywhere if and only $\vec{p}(s)$ lies in a plane.

Q5 (a) Let the differential operators X and Y acting on functions be defined as

$$X = \xi^1 \frac{\partial}{\partial u^1} + \xi^2 \frac{\partial}{\partial u^2}; \quad Y = \eta^1 \frac{\partial}{\partial u^1} + \eta^2 \frac{\partial}{\partial u^2}.$$

Express the vector field $[X, Y]$ using ξ^i, η^i ($i = 1, 2$). Prove that for three vector fields X, Y, Z , the following equations holds:

$$[[X, Y], [Z]] + [[Y, Z], [X]] + [[Z, X], [Y]] = 0$$

(b) For a differential 2-form $\varphi = f du^1 \wedge du^2$, let $\varphi(x, y) = f(\xi^1 \eta^2 - \xi^2 \eta^1)$. Show that if φ and ψ are 1-forms, then

$$(i) (\varphi \wedge \psi)(X, Y) = \varphi(X)\psi(Y) - \psi(X)\varphi(Y).$$

$$(ii) (d\varphi)(X, Y) = X(\varphi(Y)) - Y(\varphi(X)) - \varphi([X, Y]).$$

(c) Define the covariant derivative $\frac{dX}{dt}$ of a tangent vector $X = \xi^1 e_1 + \xi^2 e_2$ along the curve $\vec{p}(t) = \vec{p}(u(t), v(t))$ lying on the surface $\vec{p}(u, v)$. Show that it is independent of the second fundamental form.

Q6 (a) Prove that every geodesic of a sphere is a great circle.

(b) Determine the connection form ω_2^1 and the curvature K when a Riemannian metric is given by

$$du du + G dv dv$$

with respect to the parallel coordinate system.

(c) Let $C: u = 0$ be a geodesic in the construction of the geodesic parallel coordinate system and $g = \sqrt{G}$ in the Riemannian metric $du du + G dv dv$. Prove that $g_u(0, v) = 0$.

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[This question paper contains 4 printed pages.]

Your Roll No.....

L

Sr. No. of Question Paper : 7383
Unique Paper Code : 2353010025
Name of the Paper : Integral Equations and Calculus of Variations
Name of the Course : **Bachelor of Science (Honours Course)**
Mathematics
Semester : VIII
Duration : 3 Hours
Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions carry equal marks.
3. Attempt any **two** parts from each question.

1. (a) Show that

$$y(x) = \cos x$$

satisfies

$$\phi(x) = 1 - \int_0^x (x-t)\phi(t) dt.$$

- (b) Convert the initial value problem

$$y''(x) = y(x), y(0) = 1, y'(0) = 0$$

into an integral equation.

- (c) Solve the integral equation by successive approximations:

$$\phi(x) = x - \int_0^x (x-t)\phi(t) dt, \quad \phi_0(x) = 0.$$

2. (a) Solve using Laplace transform :

$$\phi(x) = e^{2x} + \int_0^x e^{t-x} \phi(t) dt.$$

(b) Solve the integro-differential equation:

$$\phi''(x) - 2\phi'(x) + \phi(x) + 2 \int_0^x \cos(x-t) \phi''(t) dt + 2 \int_0^x \sin(x-t) \phi'(t) dt = \cos(x)$$

$$\phi'(0) = \phi(0) = 0.$$

(c) Solve the generalized Abel equation :

$$f(x) = \int_0^x (x-t)^{\frac{1}{2}} \phi(t) dt.$$

3. (a) Show that the function $\varphi(x) = \sqrt{x}$ is a solution of the Fredholm-type integral equation

$$\varphi(x) - \int_0^1 K(x,t) \varphi(t) dt = \sqrt{x} + \frac{x}{15} (4x^{\frac{3}{2}} - 7)$$

where the kernel is of the form

$$K(x,t) = \begin{cases} \frac{x(2-t)}{2}, & 0 \leq x \leq t \\ \frac{t(2-x)}{2}, & t \leq x \leq 1 \end{cases}$$

(b) Using Fredholm determinates, find the resolvent kernel of the kernel

$$K(x,t) = 2x - 1; \quad 0 \leq x \leq 1, \quad 0 \leq t \leq 1.$$

(c) Determine the iterated kernels $K_1(x,t)$ and $K_2(x,t)$ if $K(x,t) = e^{\min(x,t)}$, $a = 0$, $b = 1$.

4. (a) Solve the integral equation

$$\varphi(x) - \lambda \int_0^1 K(x,t) \varphi(t) dt = e^x$$

where

$$K(x, t) = \begin{cases} \frac{\sinh(x) \sinh(t-1)}{\sinh(1)}, & \text{if } 0 \leq x \leq t \\ \frac{\sinh(t) \sinh(x-1)}{\sinh(1)}, & \text{if } t \leq x \leq 1 \end{cases}$$

- (b) Define the degenerate kernel of the Fredholm integral equation. Then, solve the integral equation

$$\varphi(x) - \lambda \int_0^{2\pi} (\sin(x) \cos(t) - \sin(2x) \cos(2t) + \cos(3t) \sin(3x)) \varphi(t) dt = \cos(x)$$

- (c) Define the characteristic number and eigenfunction for the Fredholm integral equation. Determine the characteristic numbers and eigenfunction of the following integral equation

$$\varphi(x) - \lambda \int_0^{\pi} (\cos^2(x) \cos(2t) + \cos(3x) \cos^3(t)) \varphi(t) dt = 0.$$

5. (a) Prove that the necessary condition for the functional

$$J[y] = \int_{x_1}^{x_2} f(x, y, y') dx,$$

to be an extremum is that

$$\frac{d}{dx} \left\{ f - y' \frac{\partial f}{\partial y'} \right\} - \frac{\partial f}{\partial x} = 0.$$

- (b) On which curve the given functional be extremized

$$J[y] = \int_0^{\pi} (y'^2 - y^2 + 2xy) dx, \quad y(0) = 0, \quad y\left(\frac{\pi}{2}\right) = 0.$$

(14)

Unique Paper Code : **2353010024**

Name of the Paper : **DSE 6-(iv) : Geometry of Curves and Surfaces**

Name of the Course : **B.Sc. (Honors) Mathematics**

Semester : VIII

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all six (6) questions by doing any *two* parts from each question.

All questions carry equal marks.

Notations used are standard.

1. (a) Show that the curve $x = a \cosh t$, $y = a \sinh t$ becomes the hyperbola $x^2 - y^2 = 1$ and then determine its curvature.
- (b) Prove that curvature of a curve $\mathbf{p}(s)$ is identically 0 if and only if $\mathbf{p}(s)$ is a line.
- (c) Define the terms vertex of a plane curve and rotation index of a closed curve. Give an example of a plane curve $\mathbf{p}(s)$ with rotation index 0 and a plane curve $\bar{\mathbf{p}}(s)$ with rotation index 4. Is it possible to gradually deform $\mathbf{p}(s)$ to $\bar{\mathbf{p}}(s)$? Justify your answer.

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2. (a) Assume that the curvature of a space curve $\mathbf{p}(s)$ is everywhere positive. Then show that the torsion is zero everywhere if and only if $\mathbf{p}(s)$ lies on a plane.

- (b) Find the curvature and torsion of the helix

$$x = a \cos t, y = a \sin t, z = bt \quad (a > 0).$$

- (c) Define the term total curvature of a closed space curve. Derive the total curvature formula for a space curve $\mathbf{q}(t)$ ($a \leq t \leq b$), where t is any parameter (not necessarily the length parameter).

3. (a) For each of the following surfaces, write the name of the surface and its equation in cartesian form :

(i) $x = a(u + v), y = b(u - v), z = 4uv.$

(ii) $x = a \left(\frac{u - v}{u + v} \right), y = b \left(\frac{uv - 1}{u + v} \right), z = c \left(\frac{uv - 1}{u + v} \right).$

- (b) Prove that a surface is a plane if its second fundamental form $\mathbf{II}$ vanishes everywhere.

- (c) Find the umbilic points of an ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ ($a > b > c > 0$).

4. (a) For Right helicoid $x = u \cos v, y = u \sin v, z = f(v)$, determine which part of the sphere is covered by Gauss' spherical map.

- (b) Define the exterior differentiation d for 0-form, 1-form and 2-form.

Also, prove that

$$dd\theta = 0$$

when θ is of degree 0.

- (c) Derive the second structure equation of the surface for Exterior Differential Forms

$$d\omega_2^1 = K\theta^1 \wedge \theta^2, \quad K = b_{11}b_{22} - b_{12}b_{21}.$$

5. (a) Prove that every geodesic of a sphere is a great circle.
 (b) Determine the connection form ω_2^1 and the curvature K when a Riemannian metric is given by

$$dudv + Gdvdv$$

with respect to the parallel coordinate system.

- (c) Let $C : u = 0$ be a geodesic in the construction of the geodesic parallel coordinate system and $g = \sqrt{G}$ in the Riemannian metric $dudv + Gdvdv$. Prove that $g_u(0, v) = 0$.

- i. (a) State Gauss-Bonnet Theorem. Verify it for a sphere with parametric equations.

$$x = a \cos u \cos v, \quad y = a \cos u \sin v, \quad z = a \sin v.$$

- (b) Prove that :

$$\int_A K\theta^1 \wedge \theta^2 + \int_{\partial A} k_g ds = \sum_{i=1}^4 l_i - 2\pi$$

when A is a quadrilateral and l_1, l_2, l_3, l_4 are the angles at its vertices.

- (c) Suppose that A Riemannian metric is defined on a rectangular domain in the (u, v) - plane and its Gaussian curvature is negative or zero. Prove that two geodesics can never intersect at two points such that they together enclose a simply connected domain.

[This question paper contains 4 printed pages.]

Your Roll No.....

L

Sr. No. of Question Paper : 8061

Unique Paper Code : 2353010025

Name of the Paper : Integral Equations and Calculus of Variations

Name of the Course : Bachelor of Science (Honours Course) Mathematics

Semester : VIII

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions carry equal marks.
3. Attempt any two parts from each question.

1. (a) Find resolvent kernel for the following integral equation

$$\phi(x) = f(x) + \int_0^x (x-t)\phi(t) dt.$$

- (b) Solve by successive approximations :

$$\phi(x) = \frac{x^3}{3} - 2x - \int_0^x \phi(t) dt, \quad \phi_0(x) = x^2.$$

- (c) Solve the integral equation

$$\phi(x) = 1 - 2x - 4x^2 + \int_0^x [3 - 6(x-t) - 4(x-t)^2]\phi(t) dt.$$

2. (a) Solve convolution equation

$$\phi(x) = \cos(x) - \int_0^x (x-t) \cos(x-t) \phi(t) dt.$$

- (b) Solve the following integral equation of first kind by first reducing it to integral equation of second kind

$$\int_0^x a^{x-t} \phi(t) dt = f(x), \quad f(0) = 0.$$

- (c) Solve the integral equation

$$\frac{1}{2} \int_0^x (x-t)^2 \phi(t) dt = \cos(x) - 1 + \frac{x^2}{2}.$$

3. (a) Show that the function $\phi(x) = xe^{-x}$ is a solution of the Fredholm-type integral equation

$$\phi(x) - 4 \int_0^1 e^{-(x+t)} \phi(t) dt = (x-1)e^{-x}.$$

- (b) Using Fredholm determinates, find the resolvent kernel of the kernel

$$K(x, t) = \sin(x) - \sin(t); \quad 0 \leq x \leq 2\pi, \quad 0 \leq t \leq 2\pi.$$

- (c) Find the iterated kernels $K_1(x, t)$ and $K_2(x, t)$ if $K(x, t) = e^x \cos(t)$, $a = 0$, $b = \pi$.

4. (a) Solve the integral equation

$$\phi(x) - \lambda \int_0^1 K(x, t) \phi(t) dt = x$$

where

$$K(x, t) = \begin{cases} x(t-1), & \text{if } 0 \leq x \leq t \\ t(x-1), & \text{if } t \leq x \leq 1 \end{cases}$$

- (b) Solve the given Fredholm integral equation

$$\varphi(x) - \lambda \int_0^{\pi} \sin(x-t) \varphi(t) dt = \cos(x)$$

where λ is a parameter.

- (c) Find the characteristic numbers and eigenfunctions of the integral equation

$$\varphi(x) - \lambda \int_0^{\pi} K(x, t) \varphi(t) dt = 0$$

where

$$K(x, t) = \begin{cases} \cos(x) \sin(t), & \text{if } 0 \leq x \leq t \\ \cos(t) \sin(x), & \text{if } t \leq x \leq \pi \end{cases}$$

5. (a) Among all the curves joining two given points (x_0, y_0) and (x_1, y_1) , find the one which generates the surface of minimum area when rotated about the x -axis.
- (b) Among all the curves lying on the sphere $x^2 + y^2 + z^2 = a^2$ and passing through two given points (x_0, y_0, z_0) and (x_1, y_1, z_1) , find the one which has the least length.
- (c) Find the shortest distance of point $P(0,0,4)$ from line $x = y = z$.
6. (a) Find the path on which a particle in the absence of friction, will slide from one point to another in the shortest time under the action of gravity.

- (b) Among all curves of length 1 in the upper half-plane passing through the points $(-a, 0)$ and $(a, 0)$, find the one which together with the interval $[-a, a]$ encloses the largest area.
- (c) Find the shortest distance of point $P(0,2,3)$ from line $x/2 = y/3 = z/4$.

This question paper contains 4 printed pages]

Roll No.

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S. No. of Question Paper : 8737

Unique Paper Code : 2353010024

Name of the Paper : DSE 6 (iv) Geometry of Curves and Surfaces

Name of the Course : B.Sc. (Honors) Mathematics

Semester : VIII

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all six (06) questions by doing any two parts from each question.

All questions carry equal marks.

Notations used are standard.

1. (a) Write the parametric equation of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ traced in the clockwise direction and then determine its curvature.
- (b) Prove that the curvature does not change when a curve p is rotated and displaced in parallel.
- (c) Define the term total curvature of a plane curve. Prove that the total curvature of a closed plane curve $p(s)$ is at least 2π .

2. (a) If the torsion of a space curve $\mathbf{p}(s)$ is zero everywhere, then show that the curve lies on a plane.

- (b) Calculate the curvature and torsion for the space curve :

$$\mathbf{q}(t) = \left(\frac{1}{\sqrt{3}} \cos t + \frac{1}{\sqrt{2}} \sin t, \frac{1}{\sqrt{3}} \cos t, \frac{1}{\sqrt{3}} \cos t - \frac{1}{\sqrt{2}} \sin t \right).$$

- (c) Define the notion of isotopic space curves, a knot and a non-trivial knot.

3. (a) Calculate the Gaussian curvature and Mean curvature of the surface of a hyperboloid of one sheet :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1.$$

- (b) Define principal curvatures k_1 and k_2 at a point on a surface $\vec{p}(u, v)$.

Prove that the two principal directions at a point $\vec{p}_0$ on the surface

$\vec{p}(u, v)$ are orthogonal to each other if $k_1 \neq k_2$.

- (c) Calculate the unit normal vector $\hat{e}$, the first and second fundamental forms, the Gaussian curvature K and the mean curvature H for a surface of the form $z = f(x, y)$.

4. (a) What is a ruled surface ? Write the hyperboloid of one sheet

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \text{ in the form of a ruled surface } \vec{q}(u) + vt(u).$$

- (b) Define exterior multiplication for 0-forms, 1-forms, and 2-forms. Prove that :

$$\alpha \wedge \beta = -\beta \wedge \alpha,$$

where $\alpha = a_1 du + a_2 dv$ and $\beta = b_1 du + b_2 dv$.

- (c) Prove that the first structure equation for exterior differential forms of the surface $\vec{p}(u, v) = (x(u, v), y(u, v), z(u, v))$ is :

$$d\theta^i = \sum_{j=1}^2 \theta^j \wedge \omega_i^j \quad (i=1,2).$$

5. (a) Prove that the inner product $\langle X, Y \rangle = \xi^1 \eta^1 + \xi^2 \eta^2$ and the length $|X| = \langle X, X \rangle^{1/2}$ of the vectors are invariant by parallel translations.
- (b) Determine the parallel translation along the latitude line on a sphere with center at the origin and radius a .
- (c) Let $\hat{e}_1 = v \frac{\partial}{\partial u}$ and $\hat{e}_2 = v \frac{\partial}{\partial v}$ be an orthonormal system in the upper half complex plane $w = u + iv, v > 0$. Determine :
- (i) parallel vector fields along a curve $u = a \cos t, v = a \sin t, 0 < t < \pi$.
- (ii) parallel vector fields along a line $u = t, v = a, 0 < t < \pi$.
6. (a) Prove that for a right circular cylinder, a curve on it is a geodesic if and only if it becomes a line when developed.

- (b) Prove that a geodesic of the upper half plane with Poincare metric is either a half circle with its diameter on the x -axis, or a line perpendicular to the x -axis.
- (c) Show that the geodesic equation can be written as :

$$\frac{D\bar{X}}{dt} + \lambda \bar{X} = 0 \quad \text{or} \quad \frac{d^2 u^i}{ds^2} + \sum_{j,k=1}^2 \Gamma_{jk}^i \frac{du^j}{ds} \frac{du^k}{ds} = 0 \quad (i = 1, 2)$$

where λ is a function on the curve.

[This question paper contains 4 printed pages.]

Your Roll No.....

L

Sr. No. of Question Paper : 8738

Unique Paper Code : 2353010025

Name of the Paper : Integral Equations and Calculus of Variations

Name of the Course : Bachelor of Science (Honours Course) Mathematics

Semester : VIII

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions carry equal marks.
3. Attempt any two parts from each question.

1. (a) Find resolvent kernel for the following kernel

$$K(x, t) = e^{x^2 - t^2}$$

- (b) Solve by successive approximations :

$$\phi(x) = \frac{x^2}{2} + x - \int_0^x \phi(t) dt, \quad \phi_0(x) = 1.$$

- (c) Solve the integral equation

$$\phi(x) = e^{2x} + \int_0^x e^{t-x} \phi(t) dt$$

2. (a) Solve convolution equation

$$\phi(x) = e^x + \int_0^x \phi(t) dt.$$

- (b) Solve integro-differential equation

$$\phi''(x) + \phi(x) + \int_0^x \sinh(x-t) \phi(t) dt + \int_0^x \cosh(x-t) \phi'(t) dt = \cosh(x),$$

$$\phi'(0) = \phi(0) = 0.$$

- (c) Solve the integral equation

$$\int_0^x \frac{\phi(t)}{(x-t)^\alpha} dt = x^n, \quad (0 < \alpha < 1).$$

3. (a) Show that the function $\phi(x) = \cos(2x)$ is a solution of the Fredholm-type integral equation

$$\phi(x) - 3 \int_0^\pi K(x, t) \phi(t) dt = \cos(x)$$

where the kernel is of the form

$$K(x, t) = \begin{cases} \sin(x) \cos(t), & 0 \leq x \leq t \\ \sin(t) \cos(x), & t \leq x \leq 1 \end{cases}$$

- (b) Using Fredholm determinates, find the resolvent kernel of the kernel

$$K(x, t) = x^2 t - x t^2; \quad 0 \leq x \leq 1, \quad 0 \leq t \leq 1.$$

- (c) Define the iterated kernels for the Fredholm integral equation. Find the iterated kernels $K_1(x, t)$ and $K_2(x, t)$ if $K(x, t) = e^{|x-t|}$, $a = 0$, $b = 1$.

4. (a) Solve the integral equation

$$\phi(x) - \lambda \int_0^1 K(x, t) \phi(t) dt = x$$

where

$$K(x, t) = \begin{cases} (x+1)(t-3), & \text{if } 0 \leq x \leq t \\ (t+1)(x-3), & \text{if } t \leq x \leq 1 \end{cases}$$

- (b) Define the degenerate kernel of the Fredholm integral equation. Then, solve the integral equation

$$\varphi(x) - \lambda \int_0^1 xt \varphi^2(t) dt = 0$$

where λ is a parameter.

- (c) State the extremal properties of the characteristic number and eigenfunctions of the Fredholm integral equation. Find the maximum of

$$|K\varphi, \varphi| = \left| \int_0^\pi \int_0^\pi K(x, t) \varphi(x) \varphi(t) dx dt \right|$$

provided

$$(\varphi, \varphi) = \int_0^\pi \varphi^2(x) dx = 1$$

where

$$K(x, t) = \cos(x) \cos(2t) + \cos(t) \cos(2x) + 1.$$

5. (a) Find the solution of the Euler Equation corresponding to the functional

$$J[y] = \int_1^2 \frac{\sqrt{1+y'^2}}{x} dx, y(1) = 0, y(2) = 1.$$

- (b) Find the extremal of the functional

$$J[y] = \iint \left[\left(\frac{\partial z}{\partial y} \right)^2 + \left(\frac{\partial z}{\partial x} \right)^2 \right] dx dy.$$

- (c) Find the curve passing through the points (x_0, y_0) and (x_1, y_1) , which when rotated about the x -axis gives a minimum surface area.

6. (a) Find the extremals of the functional

$$J[r] = \int_{\phi_0}^{\phi_1} \sqrt{r^2 + r'^2} d\phi, \text{ where } r = r(\phi).$$

- (b) Find the shortest distance of point $P(0,0,5)$ from surface $z = x^2 + y^2$.
- (c) Among all curves of length 1 in the upper half-plane passing through the points $(-a, 0)$ and $(a, 0)$, find the one which together with the interval $[-a, a]$ encloses the largest area.

[This question paper contains 2 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 2360

L

Unique Paper Code : 2352571201

Name of the Paper : Elementary Linear Algebra

Name of the Course : B.A. / B.Sc. (Programme) with Mathematics as
Non-Major / Minor

Semester : II – DSC

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **any 5** parts from Question 1 and any **2** parts from Question 2-6.
3. Each Question carries **15** marks.
4. Use of calculator is **NOT** allowed.

1. Attempt any 5 parts : (All parts carry 3 marks)

- i). Are the vectors $x = [10, -8, 3, 0, 27]$ and $y = \left[\frac{5}{6}, -\frac{2}{3}, \frac{3}{4}, 0, -\frac{5}{2}\right]$ parallel? Justify.
- ii). Is $[1, 1, 1]$ a linear combination of $[2, 0, 3]$ and $[0, 2, 1]$? Justify.
- iii). Show that for unit vectors a and b , if $(a - b) \cdot (a - b) \geq 0$, then $a \cdot b \leq 1$.
- iv). Let x, y, z be vectors in $\mathbb{R}^n$. Is it true that $(x \cdot y) \cdot z = x \cdot (y \cdot z)$? Justify.
- v). Find a unit vector in the direction of vector $[2, -3, 1, 2]$.
- vi). $(-4, -1, 6), (2, 3, -2), (8, -1, 6)$ are vertices of an isosceles triangle. True or False? Justify.
- vii). Let x, y be vectors in $\mathbb{R}^n$ and d is a scalar. Is it true that $(dx) \cdot y = x \cdot (dy)$? Justify.

2. a. Find the quadratic equation $y = ax^2 + bx + c$ that passes through the points $(2, 0), (3, 5)$ and $(-1, -3)$.

- b. Use Gauss Jordan method to find complete solution set for the following homogeneous system of equations. Also give one particular solution.

$$2x_1 + 4x_2 - x_3 + 5x_4 + 2x_5 = 0$$

$$3x_1 + 3x_2 - x_3 + 3x_4 = 0$$

$$-5x_1 - 6x_2 + 2x_3 - 6x_4 - x_5 = 0$$

- c. Find the rank of the matrix $A = \begin{bmatrix} 2 & 1 & -2 & 0 \\ 2 & -4 & 3 & 1 \\ 3 & 15 & -13 & -2 \end{bmatrix}$.

3. a. Use diagonalization method to calculate A^8 for

$$A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

- b. Let $V = \mathbb{R}^2$ with the operations of addition and scalar multiplication defined as

$$[x, y] \oplus [w, z] = [x + w + 1, y + z - 2]$$

and

$$a \odot [x, y] = [ax + a - 1, ay - 2a + 2]$$

respectively. Prove all the properties of a vector space involving only vector addition.

- c. Prove or disprove that the set of all 2×2 symmetric matrices is a subspace of M_{22} under usual matrix operations.

4. a. Use simplified span method to find a simplified general form for all vectors in $\text{span}(S)$, where

$$S = \{x^3 - 1, x^2 - x, x - 1\}$$

is a subset of P_3 , set of all polynomials of degree ≤ 3 .

- b. Show that

$$T = \left\{ \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 2 \\ -6 & 1 \end{bmatrix}, \begin{bmatrix} 4 & -1 \\ -5 & 2 \end{bmatrix}, \begin{bmatrix} 3 & -3 \\ 0 & 0 \end{bmatrix} \right\}$$

is a linearly independent subset of M_{22} . Does it form a basis of M_{22} ?

- c. Find the basis and dimension for the subspace W of $\mathbb{R}^3$ spanned by the set
 $S = \{[3, 2, 1], [1, 2, 0], [-1, 2, -1]\}$
 of three vectors.

5. a. Check whether the following function is a linear transformation or not

$$T: P(\mathbb{R}) \rightarrow P(\mathbb{R}) \text{ defined by } T(f(x)) = x + \int_0^x f(t) dt$$

Also check that T is onto or not.

- b. Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation such that $T(1, 0) = (1, 4)$ and $T(1, 1) = (2, 5)$. Find a formula for $T(x, y)$ and also find $T(2, 3)$.

- c. Let T be a linear transformation from a vector space V to a vector space W . Prove that Kernel of T is a subspace of V and Range of T is a subspace of W .

6. a. State the Dimension Theorem for a linear transformation and verify it for linear transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined by $T(a, b) = (a + b, 0, 2a - b)$

- b. Consider the linear transformation

$$T: \mathbb{R}^3 \rightarrow \mathbb{R}^3 \text{ defined by}$$

$$T(v) = Av, \text{ where}$$

$$A = \begin{bmatrix} 1 & -1 & 5 \\ -2 & 3 & 13 \\ 3 & -3 & 15 \end{bmatrix}$$

Find the basis for kernel of T .

- c. Define an Isomorphism between two vector spaces and check whether the map $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(a, b, c) = (3a - 2c, b, 3a + 4b)$ is an isomorphism or not?

[This question paper contains 2 printed pages.]

Your Roll No.....
L

Sr. No. of Question Paper : 2675

Unique Paper Code : 2352571201

Name of the Paper : Elementary Linear Algebra

Name of the Course : B.A. / B.Sc. (Programme) with Mathematics as
Non-Major / Minor

Semester : II – DSC

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **any 5** parts from Question 1 and any 2 parts from Question 2-6.
 - Each Question carries **15** marks.
 - Use of calculator is **NOT** allowed.

1. Attempt any 5 parts: : (All parts carry 3 marks)

- i) Are the vectors $\mathbf{x} = [-3, 1, 2]$ and $\mathbf{y} = [6, -2, 4]$ parallel? Justify.
- ii) Is $[5, -1, 2]$ a linear combination of $[-1, 0, 3]$ and $[2, -1, -1]$? Justify.
- iii) Let $\mathbf{a}$ and $\mathbf{b}$ be vectors in $\mathbb{R}^n$, show that if $\|\mathbf{a} + \mathbf{b}\|^2 = \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2$, then $\mathbf{a} \cdot \mathbf{b} = 0$.
- iv) Let $\mathbf{x}, \mathbf{y}, \mathbf{z}$ be vectors in $\mathbb{R}^n$. Is it true that $\mathbf{x} \cdot \mathbf{y} - \mathbf{x} \cdot \mathbf{z} = \mathbf{x} \cdot (\mathbf{y} - \mathbf{z})$? Justify.
- v) Find a unit vector in the direction of vector $[3, -5, 6]$.
- vi) $(7, -3, 6), (11, -5, 3), (10, -7, 8)$ are vertices of an isosceles triangle. True or False? Justify.
- vii) If two vectors $\mathbf{x}$ and $\mathbf{y}$ in $\mathbb{R}^n$ are parallel, then they have the same direction. True or False? Justify.

2. a. Find the quadratic equation $y = ax^2 + bx + c$ that passes through the points $(1, 0), (-1, 12)$ and $(2, 9)$.

- b. Use Gauss Jordan method to find complete solution set for the following homogeneous system of equations. Also give one particular solution.

$$2x_1 - 6x_2 + 3x_3 - 21x_4 = 0$$

$$4x_1 - 5x_2 + 2x_3 - 24x_4 = 0$$

$$-x_1 + 3x_2 - x_3 + 10x_4 = 0$$

$$-2x_1 + 3x_2 - x_3 + 13x_4 = 0$$

c. Find the rank of the matrix $B = \begin{bmatrix} -1 & 4 & 19 \\ 5 & 1 & -11 \\ 4 & -5 & -32 \\ 2 & 1 & -2 \\ 1 & -2 & -11 \end{bmatrix}$.

3. a. Find all eigen values of a matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & -3 \\ 0 & 0 & -5 \end{bmatrix}$$

Also express each eigen space as a linear combination of fundamental eigen vectors.

b. Show that the set of vectors of the form $[2a - 3b, a - 5c, a, 4c - b, c]$ in $\mathbb{R}^5$ forms a subspace of $\mathbb{R}^5$ under usual operations.

c. Prove or disprove :

i) The set of polynomials of degree 5 is a vector space under the operations of usual addition and scalar multiplication of polynomials.

ii) The set of singular 2×2 matrices is a vector space under the operations of usual addition and scalar multiplication of matrices.

4. a. Use simplified span method to find a simplified general form for all vectors in $\text{span}(S)$, where

$$S = \left\{ \begin{bmatrix} -1 & 3 \\ -2 & 1 \end{bmatrix}, \begin{bmatrix} -2 & -5 \\ 3 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 4 \\ -3 & 4 \end{bmatrix} \right\}$$

is a subset of M_{22} .

b. Use independence test method to determine whether the set

$$T = \{[2, -1, 3], [4, -1, 6], [-2, 0, 2]\}$$

is linearly independent or a linearly dependent subset of $\mathbb{R}^3$?

c. Determine whether the subset

$$S = \{x^2 + x + 1, x^2 - 1, x^2 + 1\}$$

of P_2 forms a basis of P_2 , the set of all polynomials of degree ≤ 2 .

5. a. State True or False for the following statements. Give justification for your choice:

(i) A linear transformation T from vector space V to W , maps linearly independent subset S of V to a linearly independent subset of W . (3.5)

(ii) The function $L: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by $L(x, y, z) = (x + 1, y - 2, z + 3)$ is a linear transformation. (4)

b. Prove that the inverse of a linear transformation L , (if it exists), is also a linear transformation.

c. Find the matrix for the linear transformation $T: P_3(\mathbb{R}) \rightarrow \mathbb{R}^3$ defined by

$$T(ax^3 + bx^2 + cx + d) = (d + c, 2b, a - d)$$

with respect to the standard ordered bases for vector spaces $P_3(\mathbb{R})$ and $\mathbb{R}^3(\mathbb{R})$.

6. a. Find the kernel and range of the transformation $T: P_3(\mathbb{R}) \rightarrow \mathbb{R}^3$ defined by

$$T(p(x)) = (p(-1), p(0), p(1))$$

Also, verify the Dimension Theorem for T .

b. Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation that performs a counterclockwise rotation through an angle of 30° . Find the matrix representation of T with respect to ordered basis $B = \{[1, 1], [2, 3]\}$ for $\mathbb{R}^2$.

c. Let $T: P_3(\mathbb{R}) \rightarrow P_2(\mathbb{R})$ be a linear transformation given by

$$T(p(x)) = p'(x)$$

Check whether T is an invertible linear transformation or not?

[This question paper contains 8 printed pages.]

Your Roll No.....

L

Sr. No. of Question Paper : 2699

Unique Paper Code : 2342200010

Name of the Paper : Data Analysis and Visualization using Python

Name of the Course : B.A. (P) NEP

Semester : II

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **Section A** is compulsory.
3. Attempt any **four** questions from **Section B**.
4. Parts of a question must be answered together

Assume numpy has been imported as np and pandas has been imported as pd.

**Section- A
(Compulsory)**

1. a) Consider the following Pandas Series: (2)
`s = pd.Series([10, 20, 30], index=['a', 'b', 'c'])`
Write Python code to
i. Set the name of the Series as "Marks".
ii. Set the name of the index as "Subjects".

b) Consider a Matplotlib figure 'fig'. Write the command to save the figure as a file named (2)
'graph.png' at 300 dots-per-inch resolution.

c) Differentiate between descriptive and inferential statistics. (2)

d) Consider the following NumPy code: (2)

```
arr = np.array([[10, 20],  
                [30, 40],  
                [50, 60],  
                [70, 80]])  
  
print(arr[[0, 2]])
```


What will be the output?

e) Consider the following Python code:

```
df = pd.DataFrame({
    "key": ["A", "A", "B", "B"],
    "value": [2, 4, 6, 8]
})
result = df.groupby("key")["value"].agg(["sum", "mean"])
print(result)
```

(2)

What will be the output of the above code?

f) Given the following data:

2, 3, 1, 4, 5, 3, 7, 8

Calculate the mean, mode, and median of this data.

(3)

g) Consider the following Python code:

```
arr = np.array([10, 15, 20, 25])
```

- Write Python code to extract elements from index 1 to 3.
- What will be the output of `arr[::-1]`?
- Write Python code to extract every alternate element.

(3)

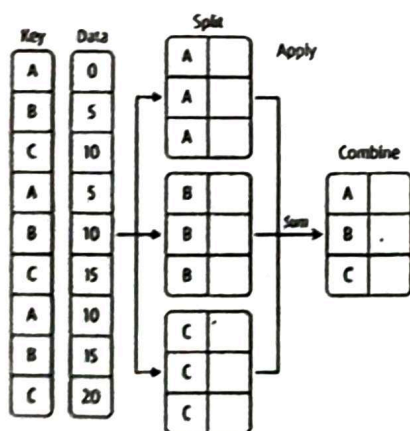
h) In a matplotlib plot, which functions are used for each of the following tasks:

(3)

- To display the legend at the best possible location.
- To set the limits of the x-axis.
- To set the limits of the y-axis.

j) Based on the concept of split-apply-combine, fill in the blank boxes in the given illustration:

(3)



- k) Write the Python code to read a file named "data.csv" into a DataFrame df. Also, write the code to display the first 5 rows of the DataFrame df. Write the Python code to save the DataFrame df into a CSV file named "output.csv" using a semicolon (;) as the separator. (4)

- l) Consider the following Python code: (4)

```
labels = pd.Index([0, 1, 2])
obj = pd.Series([10, 20, 30], index=labels)
print(obj.index is labels)
labels[1] = 5
```

- What will be the output of `print(obj.index is labels)`?
- Can a pandas Index object contain duplicate labels?
- What happens when `labels[1] = 5` is executed and why?

Section B

- 2 a) Write NumPy commands to perform the following operations: (5)
- Generate a 1D array of random numbers between 0 and 1 of size 7.
 - Create an array from the given list `l1=[5, 7, 9, 11]`.
 - Generate a 1D array of 5 random integers between 10 and 20.
 - Create a 1D array of size 10 having values ranging from 1 to 10 using `arange()` function.
 - Reshape the array created in part (iv) to (2, 5).
- b) Write Python code using Matplotlib to draw the following shapes in a single plot: (5)
- Draw a circle centered at (0, 0) with radius 1.
 - Draw a triangle with vertices at (0, 0), (1, 2), and (2, 0).
 - Draw a rectangle with bottom-left corner at (1, 1), width 2 and height 1.

c) Consider the following DataFrames:

(5)

```
df1 = pd.DataFrame({  
    'ID': [1, 2, 3], 'Name': ['A', 'B', 'C']})  
df2 = pd.DataFrame({  
    'ID': [2, 3, 4], 'Marks': [80, 90, 70]})
```

Write the output of the following: '

- i. `pd.merge(df1, df2, on='ID', how='left')`
- ii. `pd.merge(df1, df2, on='ID', how='right')`

3 a) A dataset contains the following variables:

(5)

- i. Gender
- ii. Temperature (in Celsius)
- iii. Income (in ₹)
- iv. Education level (Primary, Secondary, Higher)
- v. Number of siblings

Categorize each variable into Nominal, Ordinal, Interval, or Ratio.

b) Consider the following data:

(5)

```
scores = [35, 42, 48, 50, 55, 60, 65, 70, 75, 80, 85, 90]  
bins = [30, 50, 70, 90, 100]
```

- i. Write the Python code to divide the scores into bins.
- ii. What is the difference between `right=True` and `right=False` in `pd.cut()`?
- iii. Name the function used to create bins such that each bin contains approximately equal number of observations.

(5)

c) Consider the following DataFrame df:

Product	Category	Units_Sold	Price
Laptop	Electronics	5	50000
Mobile	Electronics	10	20000
Chair	Furniture	8	3000
Table	Furniture	6	7000
Tablet	Electronics	7	15000

- Write Python code to find the total units sold for each category.
- Write Python code to calculate the average price of products in each category.
- What is the purpose of the `groupby()` function in pandas?

4 a) Consider the following NumPy array:

(5)

```
arr = np.array([[1, 2, 3], [4, 5, 6]])
```

Write Python code to print the following:

- Shape of the array `arr`.
- Dimension of the array `arr`.
- Data type of the array `arr`.
- Print the values in `arr` that are greater than 2.

b) Consider the following Pandas DataFrame:

(5)

```
df = pd.DataFrame({
    'A': [10, 20, 30, 40],
    'B': [1, 3, 5, 7],
    'C': [2, 4, 6, 8]
}, index=['a', 'b', 'c', 'd'])
```

Write the output of the following:

- `df.count()`
- `df.sum(axis=1)`
- `df.prod()`

c) Differentiate between a pivot table and cross tabulation. Write Python functions that can be used to create each for a DataFrame. What is the use of index and columns in each function? (5)

5 a) Consider the following NumPy array:

(5)

```
arr = np.array([10, 20, 30, 40, 50])
```

- i. Write code to increase all elements of the array by 5 using array operations.
- ii. Write code to extract a slice containing elements from index 1 to 3.
- iii. Modify the slice obtained in part (ii) by setting all its values to 0.
- iv. After performing part (iii), what will be the final value of the original array `arr`?

b) Consider the following Python code:

(5)

```
x = [1, 2, 3, 4]
y = [10, 20, 15, 25]
```

Write Python code using the Matplotlib library to create the following plots:

- i. A scatter plot of `x` vs `y`. Label the axis and give title "Scatter" to chart.
- ii. A histogram of `y`. Label the axis and give title "Histogram" to chart.

c) Consider the following DataFrame:

(5)

```
data = pd.DataFrame({
    "city": ["Delhi", "Mumbai", "Chennai", "Kolkata", "Delhi", "Chennai"],
    "temperature": [35, 32, 30, 31, 36, 29]
})
```

- i. Write the Python code to create a dictionary `city_to_region` that maps between the cities and regions as:
"Delhi" -> "North"
"Mumbai" -> "West"
"Chennai" -> "South"
"Kolkata" -> "East"
- ii. Write the Python command to create a new column "Region" by mapping values from the "city" column using the dictionary `city_to_region`.

6 a) Briefly describe the following steps during the workflow of data analysis:

(5)

- i. Data collection
- ii. Data wrangling

b) Consider the following Python code:

(5)

```
import matplotlib.pyplot as plt
fig, ax = plt.subplots()
ax.plot(np.random.standard_normal(100).cumsum())
```

Complete the code to:

- i. Set the x-axis ticks at positions [0, 25, 50, 75, 100].
- ii. Set the tick labels as "one", "two", "three", "four", "five".
- iii. Rotate the tick labels by 45 degrees.
- iv. Set the font size of tick labels to 9.
- v. Set the xlabel as "Stages".

c) Consider the following Pandas DataFrame:

(5)

```
df = pd.DataFrame({
    'A': [1, np.nan, 3, np.nan],
    'B': [np.nan, np.nan, 6, 7],
    'C': [9, np.nan, np.nan, np.nan]
})
```

Write Python code to:

- i. Drop all rows containing at least one missing value.
- ii. Drop rows that contain at least two missing values
- iii. Drop only those columns where all values are missing.

7 a) Write the output of the following Python code:

(5)

```
a = np.zeros((2, 3))
b = np.ones((2, 2))
c = np.identity(3)
d = np.ones_like(a)
e = np.full_like(d, 4)
print(a)
print(b)
print(c)
print(d)
print(e)
```

- b) Differentiate between histogram and density plot. Given a series: (5)

```
s = pd.Series([12, 15, 14, 10, 18, 20, 22, 19, 17])
```

Write Python code using the Seaborn library to plot a histogram along with a density (KDE) curve for the given data, using 100 bins and black color.

- c) Consider the following Python DataFrame: (5)

```
df = pd.DataFrame(  
    [[1, 2, 3],  
     [4, 5, 6],  
     [7, 8, 9],  
     [10, 11, 12]],  
    index=[['A', 'A', 'B', 'B'], ['one', 'two', 'one', 'two']],  
    columns=[['X', 'X', 'Y'], ['x1', 'x2', 'y1']]  
)
```

Write the output of the following:

- i. `df.stack(level=0)`
- ii. `df.stack(level=1)`

[This question paper contains 2 printed pages.]

Your Roll No.....

L

Sr. No. of Question Paper : 2710

Unique Paper Code : 2352201202

Name of the Paper : DSC: Analytic Geometry

Name of the Course : Bachelor of Arts (Programme) with Mathematics as Major

Semester : II

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Attempt **two** parts from each question.
4. Each question carries equal marks.
5. Use of Calculator not allowed.

1. (a) Sketch the graph of the parabola: $y = 4x^2 + 8x + 5$, showing its vertex, focus and directrix.
(b) Identity and sketch the curve $x^2 + 4xy - 2y^2 - 6 = 0$.
(c) Find the equation of the ellipse that satisfies the given conditions: foci $(2, 1)$ and $(2, -3)$; major axis of length 6.
2. (a) Find an equation of the hyperbola that passes through the origin and has asymptotes $y = x - 2$, $y = -x + 4$.
(b) Sketch the following ellipse and label its foci, vertices, and ends of minor axis $9x^2 + 2y^2 = 9$.
(c) State reflection property of the parabola. Find an equation of the parabola that satisfies the given conditions: focus $(1, 1)$ and directrices $y = -2$.
3. (a) Find the angle that the vector $v = \sqrt{3}i + j$ makes with the positive x -axis. Also, find the vector whose length is 2 and same direction as v .
(b) A vector w is a linear combination of the vectors v_1, v_2 and v_3 if w can be expressed as $w = c_1v_1 + c_2v_2 + c_3v_3$, where c_1, c_2 and c_3 are scalars. Show that the vector $2i + j - k$ cannot be expressed as a linear combination of $v_1 = i - j$, $v_2 = 3i + k$ and $v_3 = 4i - j + k$.

- (c) Express the vector v as the sum of a vector parallel to b and a vector orthogonal to b , where $v = 4i - j + 7k$, $b = 2i + 3j - 6k$.
4. (a) Find the parametric equation of the line L passing through the points $P_1 = (2, 4, -1)$ and $P_2 = (5, 0, 7)$. Where does the line intersect the xy -plane?
- (b) Determine whether the line $L: x = t, y = 2t, z = 3t$ and the plane $x - y + 2z - 5 = 0$ are parallel, perpendicular or neither.
- (c) Find the magnitude and the equations of the line of shortest distance between the lines:

$$\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7},$$

$$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}.$$

5. (a) Find the equation of the sphere on the diameter with endpoints $(1, -2, 4)$ and $(3, 4, -12)$. Also, determine the center and the radius of the sphere.
- (b) Find the enveloping cone of the sphere $x^2 + y^2 + z^2 - 2x + 4z - 1 = 0$ and vertex $(1, 1, 1)$.
- (c) Find the equation of a right circular cylinder which envelopes a sphere with centre (a, b, c) and radius r and has the generators parallel to direction cosines (l, m, n) .
6. (a) Find the equation of the sphere that touches the sphere $x^2 + y^2 + z^2 - x + 3y + 2z - 3 = 0$ at the point $(1, 1, -1)$ and passes through the origin.
- (b) Find the equation of the right circular cone with vertex at $(1, -2, -1)$ and axis is the line $\frac{x-1}{3} = \frac{y+2}{4} = \frac{z+1}{5}$ and semi vertical angle is 60° .
- (c) Find the equation of the right circular cylinder of radius 2 whose axis passes through the point $(1, 2, 3)$ and has the direction cosines proportional to $(2, 3, -6)$.

This question paper contains 5 printed pages]

Roll No.

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S. No. of Question Paper : **2713**

Unique Paper Code : **2362201202**

Name of the Paper : **DSC-Statistics**

Name of the Course : **B.Sc. (Programme) Mathematical Science/
B.A. (Programme) NEP-UGCF-2022**

Semester : **II**

Duration : **3 Hours**

Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *five* questions.

Each question carries equal marks.

1. (a) Distinguish between Independent Events and Mutually Exclusive Events.
What is the probability that a randomly chosen card from a deck of cards will be either a king or a heart. 9
- (b) In a class of 100 students : 9
 - (i) 60 students like Mathematics (Event A)
 - (ii) 45 students like Statistics (Event B)
 - (iii) 25 students like both Mathematics and Statistics.

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Find out the following :

- (i) $P(A \cup B)$
- (ii) $P(A)$, $P(B)$, and $P(A \cap B)$
- (iii) Probability that a student likes neither subject.

2. (a) A factory produces items using two production units, Unit A and Unit B. 65% of the total output comes from Unit A and 35% from Unit B. The probability that an item produced by Unit A is defective is 1.5%, whereas the probability of a defective item produced by Unit B is 3.5%. If an item selected at random is found to be defective, find the probability that the item was produced by Unit B. 9
- (b) Explain the meaning of Probability and describe Conditional Probability with appropriate notation and formula. 9

3. (a) A random variable X has the following probability distribution : 9

(X)	0	2	4	6
P(X)	0.2	0.3	0.1	0.4

Find out the following of a random variable X :

- (i) Expectation $E[X]$
- (ii) Variance $V[X]$.

- (b) Define a probability density function (PDF). A continuous random variable X has the probability density function :

$$f(x) = \begin{cases} kx, & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the value of k .

(ii) Find $P(X > 1)$.

4. (a) The probability distribution of two discrete random variables X and Y is given below :

$(X \backslash Y)$	0	1	2
0	0.1	0.2	0.1
1	0.1	0.2	0.3

(i) Compute $P(X = 1, Y = 2)$

(ii) Find $P(X + Y \leq 2)$

(iii) Check whether X and Y are independent.

- (b) Let the joint probability density function of two continuous random variables X and Y be :

$$f(x, y) = \begin{cases} k(x + y), & 0 \leq x \leq 1; \quad 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the value of k .

5. (a) Define the Binomial distribution and solve the problem. A fair coin is tossed 6 times. Let X be the number of heads obtained. Find the probability of getting exactly 4 heads using the Binomial distribution. 9
- (b) Explain the properties of the Continuous Uniform Distribution. A continuous random variable X follows a uniform distribution with mean $\mu = 6$ and variance $\sigma^2 = \frac{16}{3}$. Find the probability that $X < 8$. 9
6. (a) Define a Poisson distribution. Write its probability mass function and explain the role of its parameter. Let X is a Poisson variate such that : $P(X = 2) = P(X = 3)$. Find the mean of X . 9
- (b) Explain the role of sample size in the Central Limit Theorem. The mean weight of a population is 70 kg with a standard deviation of 10 kg. A random sample of size 64 is selected. Using the Central Limit Theorem, find the approximate probability that the sample mean weight is less than 72 kg. 9

7. (a) Define the correlation coefficient. State its formula and explain its properties. The following data gives the values of two variables X and Y :

(X)	1	2	3	4	5
(Y)	2	4	5	4	5

Calculate the correlation coefficient between X and Y.

9

- (b) Explain the concept of regression lines. The following data gives the values of two variables X and Y :

(X)	2	4	6	8	10
(Y)	3	5	7	9	11

(i) Find the regression line of Y on X

(ii) Estimate the value of Y when X = 12.

9

[This question paper contains 2 printed pages.]

Your Roll No.....

L

Sr. No. of Question Paper : 3079

Unique Paper Code : 2352571201

Name of the Paper : Elementary Linear Algebra

Name of the Course : **B.A. / B.Sc. (Programme) with Mathematics as Non-Major / Minor**

Semester : II – DSC

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **any 5** parts from Question 1 and any **2** parts from Question 2-6.
- Each Question carries **15** marks.
- Use of calculator is **NOT** allowed.

1. Attempt any 5 parts: : (All parts carry 3 marks)

- i). Are the vectors $x = [3, -5, 2]$ and $y = [6, -10, 5]$ parallel? Justify.
- ii). Is $[-1, 3, 2]$ a linear combination of $[1, -2, 5]$ and $[0, 3, 1]$? Justify.
- iii). Prove that if $(a + b) \cdot (a - b) = 0$, then $\|a\| = \|b\|$.
- iv). Let x, y, z be vectors in $\mathbb{R}^n$. Is it true that $x \cdot y + x \cdot z = x \cdot (y + z)$? Justify.
- v). Find a unit vector in the direction of vector $[6, 2, 0, -3]$.
- vi). $(2, 5, -3), (-1, 3, -2), (-3, -2, -1)$ are vertices of an isosceles triangle. True or False? Justify.
- vii). Let x, y be vectors in $\mathbb{R}^n$. Is it true that $x \cdot y = 0$, but neither $x = 0$ nor $y = 0$? Justify.

2. a. Find the quadratic equation $y = ax^2 + bx + c$ that passes through the points $(3, 25), (-1, 1)$ and $(-2, 0)$.

- b. Use Gauss Jordan method to find complete solution set for the following homogeneous system of equations. Also give one particular solution.

$$\begin{aligned} 2x_1 + 6x_2 + 13x_3 + x_4 &= 0 \\ x_1 + 4x_2 + 10x_3 + x_4 &= 0 \\ 2x_1 + 8x_2 + 20x_3 + x_4 &= 0 \\ 3x_1 + 10x_2 + 21x_3 + 2x_4 &= 0 \end{aligned}$$

- c. Find the rank of the matrix $Y = \begin{bmatrix} 3 & 5 & 2 \\ 4 & 2 & 3 \\ -1 & 2 & 4 \end{bmatrix}$.

3. a. Find the characteristic polynomial and eigen values of the matrix

$$A = \begin{bmatrix} 4 & 0 & -2 \\ 6 & 2 & -6 \\ 2 & 0 & 1 \end{bmatrix}$$

Also find the eigen spaces corresponding to each eigen value of A.

- b. Check whether $\mathbb{R}$, the set of real numbers is a

i) vector space with ordinary addition but with scalar multiplication replaced by

$$a \odot x = 0 \quad \text{for every real number } a.$$

ii) vector space with usual scalar multiplication but with vector addition replaced by

$$x \oplus y = 2(x + y) \quad \text{for all } x, y \in \mathbb{R}.$$

c. Prove or disprove that the subset

$S = \{p(x) \in P_5 \mid p'(4) = 0, \text{ where } p'(x) \text{ is the derivative of } p(x)\}$ of P_5 is a subspace of P_5 under usual operations, where P_5 is a set of all polynomials of degree ≤ 5 .

4. a. Let S be a non empty subset of a vector space V . Then show that

i) $\text{Span}(S)$ is a subspace of V .

ii) If W is a subspace of V with $S \subseteq W$, then $\text{Span}(S) \subseteq W$.

b. Show that $S = \{[2, -1, 0, 5], [1, -1, 2, 0], [-1, 0, 1, 1]\}$ is a linearly independent subset of $\mathbb{R}^4$.

c. Let W be the solution set to the matrix equation $AX = 0$, where

$$A = \begin{bmatrix} -1 & -2 & -4 & 8 \\ 3 & 4 & 6 & -14 \\ -2 & -1 & 1 & 7 \\ 1 & 0 & -2 & 0 \\ 2 & 3 & 5 & -12 \end{bmatrix}$$

Show that $\dim W + \text{rank}(A) = 4$.

5. a. Check if the following mappings are linear transformation or not. Prove or disprove it with a suitable justification:

(i) $T_1: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T_1(x, y, z) = (x + y, y - z, 1 + z)$

(ii) $T_2: P_3(\mathbb{R}) \rightarrow \mathbb{R}$ defined by $T_2(ax^3 + bx^2 + cx + d) = a + b + c + d$

b. Find the matrix of the linear transformation: $P_3(\mathbb{R}) \rightarrow M_{2 \times 2}(\mathbb{R})$ defined by

$$L(ax^3 + bx^2 + cx + d) = \begin{bmatrix} 3a - 2c & -b + 4d \\ 4b - c + 3d & -6a - b + 2d \end{bmatrix}$$

with respect to standard ordered bases of $P_3(\mathbb{R})$ and $M_{2 \times 2}(\mathbb{R})$, respectively.

c. Does there exist a linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ such that $T(1, 0, 3) = (1, 1)$ and $T(-2, 0, -6) = (2, 1)$. Justify.

6. a. Let $T: P_2(\mathbb{R}) \rightarrow P_3(\mathbb{R})$ be the linear transformation defined by

$$T(f(x)) = 2f'(x) + \int_0^x 3f(t)dt$$

Find dimension of range space of T and kernel of T .

b. Consider the linear operator $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by

$$T(x, y) = (x \cos \alpha - y \sin \alpha, x \sin \alpha + y \cos \alpha)$$

where α is a fixed angle. Then check that T is an isomorphism or not.

c. Let $T: V \rightarrow W$ be a linear transformation, where V and W are finite dimensional vector spaces. Prove that T is onto if and only if $\dim(\text{Range}(T)) = \dim(W)$.

This question paper contains 4 printed pages]

Your Roll No. :
Sl. No. of Q. Paper : **6549** **I**
Unique Paper Code : 2354001202
Name of the Paper : Introduction to linear Algebra
Name of the Course : **Common Prog Group**
Semester : II
Time : 3 Hours **Maximum Marks : 90**

Instructions for Candidates :

- (a) Write your Roll No. on the top immediately on receipt of this question paper.
 - (b) Attempt **all** question by selecting **two** parts from each question.
 - (c) **All** questions carry equal marks.
1. (a) If a and b are unit vectors in $\mathbb{R}^n$. Then prove that $-1 \leq a \cdot b \leq 1$.
- (b) Solve the following system of linear equations using Gaussian-Elimination method. Examine the consistency of the system and if consistent, find the complete solution set.
- $$2x - y - z = 2$$
- $$x + 2y + z = 2$$
- $$4x - 7y - 5z = 2$$
- (c) Solve the following system of linear equations using Gauss-Jordan method:
- $$x - y - 2z = 0$$
- $$3x + y + 3z = 0$$
- $$2x + 3y - z = 0$$

2. (a) Define rank of a matrix. Find the rank of following matrix by reducing into its reduced

row echelon form: $A = \begin{bmatrix} 1 & 3 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{bmatrix}$.

- (b) Find the characteristic polynomial of the

matrix: $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. Also, find all the eigenvalues and eigenspaces for A.

- (c) Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$, Show that matrix A is not diagonalizable.

3. (a) Let V be a vector space over $\mathbb{R}$. Show that:

(i) $a0_v = 0_v$

(ii) If $av = 0_v$ then either $a = 0$ or $v = 0_v$

where $a \in \mathbb{R}, v \in V$ and 0_v denotes the zero vector of V.

- (b) Define a subspace. Show that the set $S = \{p \in p_3 \mid p(3) = 0\}$ is a subspace of p_3 under the usual operations.

- (c) Examine whether the set $S = \{[1, 2, 1], [2, 3, 1], [-1, 2, -3]\}$ forms a basis for $\mathbb{R}^3$ or not.

4. (a) Define linear independence and linear dependence for a finite set of vectors. Determine whether the set $S = \{[1, 9, -2], [3, 4, 5], [-2, 5, -7]\}$ is linearly independent or not.
- (b) Find a basis and dimension of the subspace W of $\mathbb{R}^3$, where $W = \{X \in \mathbb{R}^3 \mid AX = 0\}$ and

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 0 & 1 & -2 \\ 0 & -2 & 1 \end{bmatrix}.$$

- (c) Use the simplified span method to check whether the set $\{[1, 3, -1], [2, 7, -3], [4, 8, -7]\}$ spans $\mathbb{R}^3$ or not.
5. (a) Let $T: V \rightarrow W$ be a linear transformation from a vector space V to a vector space W . Define kernel of T and prove that kernel of T is a subspace of V .
- (b) Determine which of the following functions are linear transformations:
- (i) $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by
- $$T([x_1, x_2, x_3]) = [x_1 + 1, x_2 - 2, x_3]$$
- (ii) $f: M_{m \times n}(\mathbb{R}) \rightarrow M_{n \times m}(\mathbb{R})$ given by $f(A) = A^T$ for all $A \in M_{m \times n}(\mathbb{R})$.
- (c) Find the matrix for the linear transformation $L: p_3 \rightarrow M_{2 \times 2}(\mathbb{R})$ given by

$$L(ax^3 + bx^2 + cx + d) = \begin{bmatrix} -3a - 2c & -b + 4d \\ 4b - c + 3d & -6a - b + 2d \end{bmatrix}$$

with respect to standard ordered bases of P_3 and $M_{2 \times 2}(\mathbb{R})$.

Also compute $L(5x^3 - x^2 + 3x + 2)$ using this matrix.

6. (a) Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be the linear transformation given by

$$T([x_1, x_2, x_3]) = [-2x_1 + 3x_3, x_1 + 2x_2 - x_3]$$

Find the matrix of T with respect to the ordered bases $B = \{[1, -3, 2], [-4, 13, -3], [2, -3, 20]\}$ for $\mathbb{R}^3$ and $C = \{[-2, -1], [5, 3]\}$ for $\mathbb{R}^2$.

- (b) Consider the linear transformation

$T: P_2 \rightarrow P_3$ given by

$$T(ax^2 + bx + c) = cx^3 + bx^2 + ax$$

Find a basis for $\ker(T)$ and $\text{range}(T)$. Also, verify the Dimension theorem.

- (c) Consider the linear transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} -3 & 4 \\ -6 & 9 \\ 7 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Show that L is one-to-one but not onto.

[This question paper contains 2 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 2404

L

Unique Paper Code : 2352572401

Name of the Paper : Abstract Algebra

Name of the Course : B.A. / B.Sc. (Programme) with Mathematics as
Non-Program : Major/Minor – DSC

Semester : IV

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. The use of a calculator is not allowed.

1. (a) Define a group and show that the set of integers under multiplication is not a group. (7.5)
(b) Show that $G = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} : a \neq 0, a \in \mathbb{R} \right\}$ is an abelian group under multiplication. (7.5)
(c) Find the inverse of $\begin{bmatrix} 7 & 4 \\ 1 & 5 \end{bmatrix} \in GL(2, \mathbb{Z}_{13})$. (7.5)
2. (a) Define the order of an element "a" in G. Also find the orders of every element of $U(15)$. (7.5)
(b) Define center of a group and centralizer of $a \in G$. Prove that $C(a) = C(a^{-1})$. (7.5)
(c) Prove that in any group an element and its inverse will have the same order. (7.5)
3. (a) Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 & 1 & 7 & 8 & 6 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{bmatrix}$. (7.5)
Write α, β and $\alpha\beta$ as products of disjoint cycles and as products of 2-cycles.
(b) List the elements of the subgroups $\langle 3 \rangle$ and $\langle 7 \rangle$ in $U(20)$. (7.5)
(c) Let $H = \{0, \pm n, \pm 2n, \pm 3n, \dots\}$. Find all left cosets of H in $\mathbb{Z}$. (7.5)
4. (a) Let H be a normal subgroup of a group G and K be any subgroup of G , then prove that $HK = \{hk \mid h \in H, k \in K\}$ is a subgroup of G . (7.5)

- (b) Write down a complete Cayley table for D_3 , the group of symmetries of a triangle. Is D_3 Abelian? (7.5)
- (c) Let R^* be a group of nonzero real numbers under multiplication. Show that $\phi = A \rightarrow \det A$ is a homomorphism from $GL(2, R)$ to R^* . What is the Kernel. (7.5)
5. (a) State the subring test. Prove or disprove that the set (7.5)
- $$R = \left\{ \begin{bmatrix} a & a+b \\ a+b & b \end{bmatrix} \mid a, b \in \mathbb{Z} \right\}$$
- is a subring of the ring of all 2×2 matrices over $\mathbb{Z}$.
- (b) Define characteristic of a ring. Let R be a ring with unity 1 . Prove that if 1 has infinite order under addition then $\text{char} R = 0$. If 1 has order n under addition, then $\text{char} R = n$. (7.5)
- (c) Prove that a finite Integral Domain is a field. Hence, show that $\mathbb{Z}_p$ is a field, where p is a prime number. (7.5)
6. (a) Let $f: R \rightarrow S$ be a ring homomorphism. Prove that (7.5)
- (i) $\text{Ker} f$ is an ideal of R .
- Every ideal A of a ring R is the kernel of a ring homomorphism of R .
- (b) Prove that if an ideal I of a ring R contains a unit, then $I = R$. Hence, prove that the only ideals of a field F are $\{0\}$ and F itself. (7.5)
- (c) State and prove the first Isomorphism Theorem for Rings. (7.5)

[This question paper contains 2 printed pages.]

Your Roll No.....
L

Sr. No. of Question Paper : 2448

Unique Paper Code : 2352572401

Name of the Paper : Abstract Algebra

Name of the Course : **B.A./B.Sc. (Programme) with Mathematics as Non-Major/ Minor – DSC**

Semester : IV

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. The use of a calculator is not allowed.

1. (a) Define an abelian group. Show that the set of all 2×2 matrices with real entries is not a group under multiplication. (7.5)
(b) Let $G = \left\{ \begin{pmatrix} a & a \\ a & a \end{pmatrix} : a \neq 0, a \in \mathbb{R} \right\}$. Prove that G is a group under matrix multiplication. (7.5)
(c) Find the inverse of $\begin{bmatrix} 3 & 4 \\ 4 & 4 \end{bmatrix} \in \text{SL}(2, \mathbb{Z}_5)$. (7.5)
2. (a) What is a subgroup? Let G be an abelian group with identity e . Then show that $H = \{x \in G \mid x^2 = e\}$ is a subgroup of G . (7.5)
(b) Let G be an abelian group and H and K be subgroups of G . Then show that $HK = \{hk \mid h \in H, k \in K\}$ is a subgroup of G . (7.5)
(c) In the group $\mathbb{Z}_{12}$, find $|a|$, $|b|$ and $|a + b|$ when $a = 5$, $b = 4$. (7.5)
3. (a) Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 6 & 2 & 5 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 6 & 3 & 5 & 1 & 2 \end{bmatrix}$. Compute α^{-1} , β^{-1} , $O(\alpha)$, $O(\beta)$ and $\beta\alpha$. (7.5)
(b) List the elements of the subgroups $\langle 3 \rangle$ and $\langle 15 \rangle$ in $\mathbb{Z}_{18}$. Let a be a group element of order 18. List the elements of the subgroups $\langle a^3 \rangle$, and $\langle a^{15} \rangle$. (7.5)
(c) State Lagrange's Theorem. Suppose that K is a proper subgroup of H and H is a proper subgroup of G . If $|K| = 42$ and $|G| = 420$, what are the possible orders of H . (7.5)

4. (a) Prove that the group $SL(2, \mathbb{R})$ of 2×2 matrices with determinant 1 is a normal subgroup of $GL(2, \mathbb{R})$, the group of 2×2 matrices with nonzero determinant. (7.5)
- (b) Define factor group. What is the order of the factor group $\mathbb{Z}_{60}/\langle 15 \rangle$. (7.5)
- (c) Let ϕ be a homomorphism from a group G to a group $\overline{G}$ and let H be a subgroup of G . Then show that if H is cyclic then $\phi(H)$ is also cyclic. Check whether the mapping $\phi(x) = x^2$ from $\mathbb{R}$, the real numbers under addition, to itself is a homomorphism? (7.5)
5. (a) (i) Prove that the set $A = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \mid a, b \in \mathbb{Z} \right\}$ is a subring of the ring of all 2×2 matrices over $\mathbb{Z}$. (7.5)
- (ii) Prove that a unit of a ring divides every element of the ring.
- (b) Prove that the ring $\mathbb{Z}_p$ of integers modulo p where p is a prime is an integral domain. Is $\mathbb{Z} \oplus \mathbb{Z}$ an Integral Domain? Justify. (7.5)
- (c) Prove that characteristic of an Integral Domain is 0 or a prime number. (7.5)
6. (a) Let $f: R \rightarrow S$ be a ring homomorphism. Prove that (7.5)
- (i) $f(A)$ is a subring of S where A is a subring of R .
- (ii) if A is an ideal and f is onto S , then $f(A)$ is an ideal of S .
- (b) (i) Prove that every ring homomorphism f from $\mathbb{Z}_n$ to itself has the form $f(x) = ax$, where $a^2 = a$. (7.5)
- (ii) Prove or disprove that the correspondence $x \rightarrow 5x$ from $\mathbb{Z}_5$ to $\mathbb{Z}_{10}$ is a ring homomorphism.
- (c) Let $R[x]$ denote the set of all polynomials with real coefficients and let A denote the subset of all polynomials with constant term 0. Prove that A is an ideal of $R[x]$ and $A = \langle x \rangle$. (7.5)

This question paper contains 8 printed pages]

Your Roll No. :

Sl. No. of Q. Paper : **2636** **I**

Unique Paper Code : 2352202402

Name of the Paper : Introduction to Graph Theory

Name of the Course : **Bachelor of Arts
(Programme) with
Mathematics as Major
(DSC-2)**

Semester : IV

Time : 3 Hours

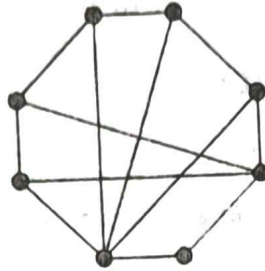
Maximum Marks : 90

Instructions for Candidates :

- (a) Write your Roll No. on the top immediately on receipt of this question paper.
 - (b) Attempt all the given **six** questions, selecting any **two** parts out of the three **parts (a), (b) and (c)** for each of the six questions.
 - (c) **All** the questions carry equal marks.
 - (d) Use of Calculator is not allowed.
- 1. (a)** Draw the following graphs and determine how many edges each has:
- (i) K_5
 - (ii) $K_{2,4}$
 - (iii) $K_{2,5}$

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- (b) Given the following degree sequences, either construct a graph with such a degree sequence, or explain why this would be impossible.
- (i) 1, 1, 1, 1, 1, 1
 (ii) 5, 4, 3, 2, 1
 (iii) 6, 6, 4, 2, 2, 2, 2, 1, 1.
- (c) Explain the difference between an Eulerian circuit and a Hamiltonian cycle. Is the following graph Eulerian? Does there exist an Eulerian trail?

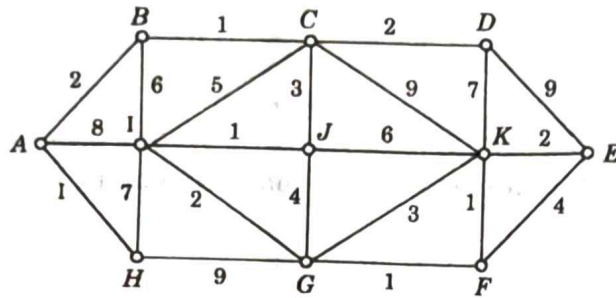


2. (a) Draw the pseudograph corresponding to each adjacency matrix:

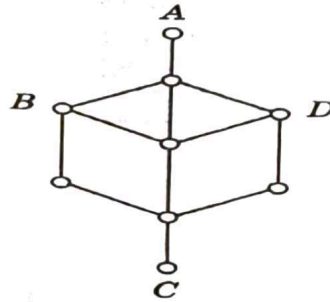
$$\begin{array}{l}
 \text{(i)} \quad \begin{array}{c} \begin{array}{ccc} & A & B & C \\ A & 0 & 0 & 2 \\ B & 0 & 0 & 1 \\ C & 2 & 1 & 0 \end{array} \end{array}
 \end{array}
 \quad
 \begin{array}{l}
 \text{(ii)} \quad \begin{array}{c} \begin{array}{cccc} & A & B & C & D \\ A & 0 & 0 & 1 & 1 \\ B & 0 & 1 & 1 & 0 \\ C & 1 & 1 & 0 & 1 \\ D & 1 & 0 & 1 & 0 \end{array} \end{array}
 \end{array}$$

- (b) Show that every graph is isomorphic to a subgraph of some complete graph. Prove that if two graphs are isomorphic, they must contain the same number of triangles.

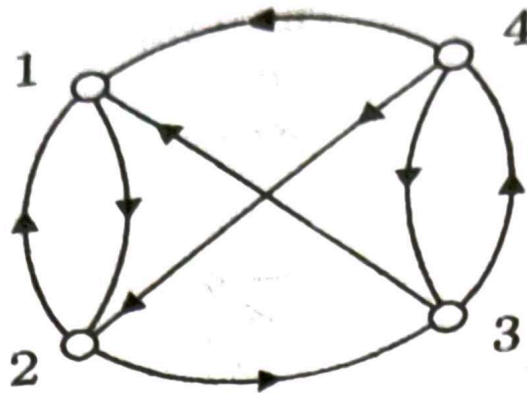
- (c) Apply the first form of Dijkstra's algorithm to find a shortest path from A to E in the graph shown. Label all the vertices.



3. (a) Solve the Chinese postman problem for the following graph, where the edges of the graph represent the streets and all streets have the same unit length (starting with vertex A).



- (b) Find the adjacency matrix A of the following graph and also determine the (3,3) and (1,4) entries of A^2 without calculating the matrix A^2 and explain your reasoning.

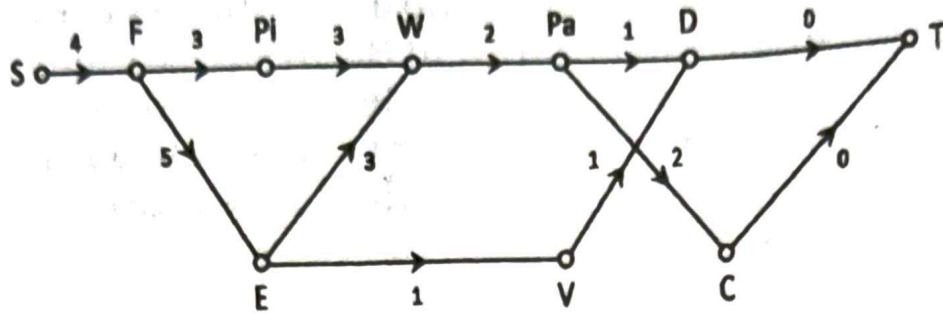


- (c) Find a permutation matrix P such that $A_2 = PA_1P^T$.

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

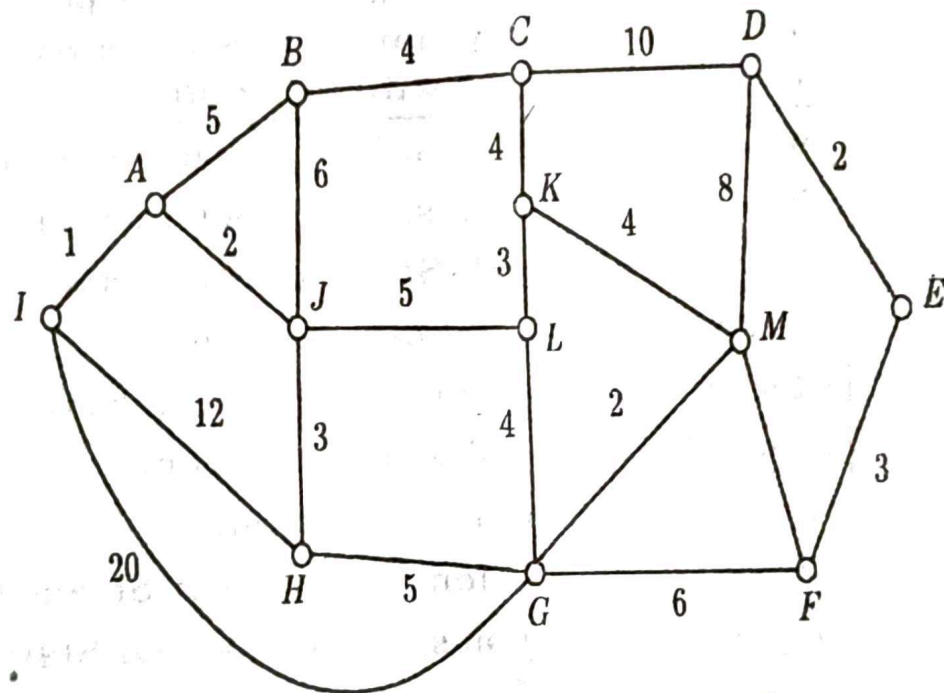
4. (a) To finish a basement, a contractor must arrange for certain task to be completed. As shown in below table, brief codes for reference and the time required for each activity is given.

Task	Code	Time
Floor installation	F	4
Plumbing	Pl	3
Electrical work	E	5
Wallboard	W	3
Varnish doors and moldings	V	1
Paint	Pa	2
Install doors and moldings	D	1
Lay Carpet	C	2



Referring to the above graph, find the minimum time to complete the task.

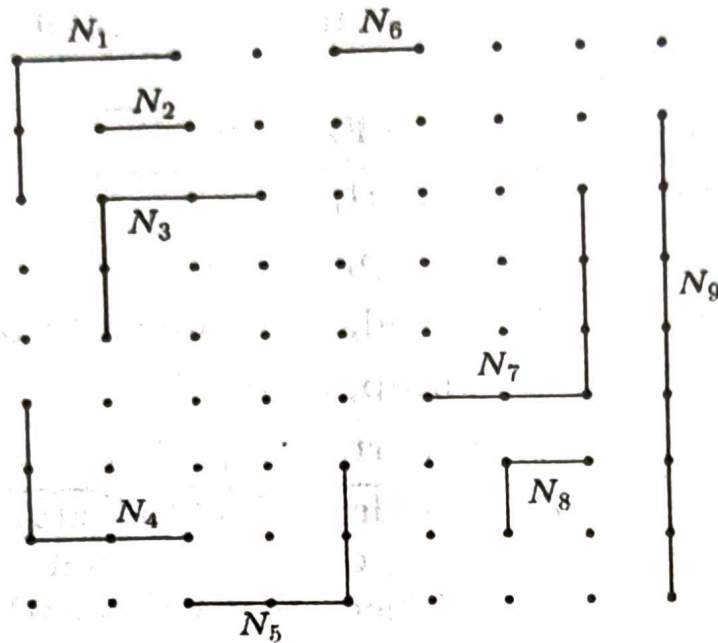
- (b) Use Prim's algorithm to find the minimum spanning tree for the following graph (starting with the vertex I)



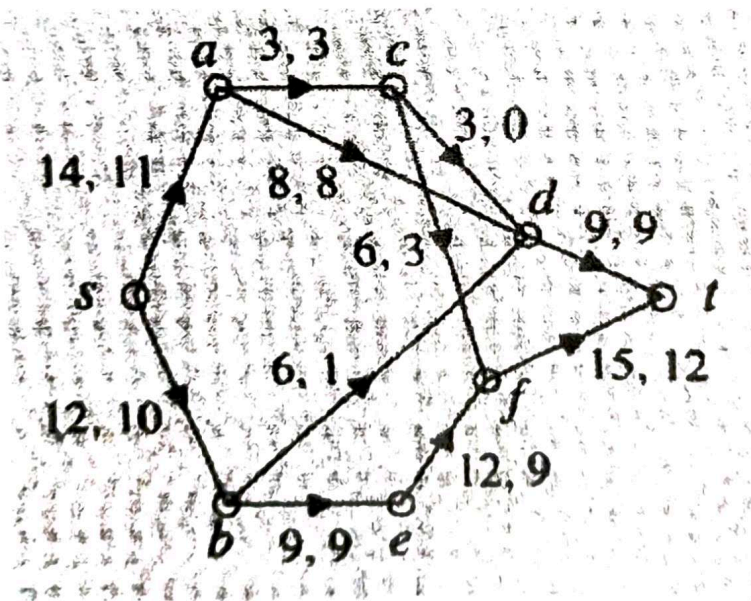
- (c) It's the weekend and Harry has several ways to spend his Friday and Saturday evenings. He can study, read, watch television or go out with the boys. Draw a tree that shows all possible ways Harry can spend his Friday and Saturday evenings.
5. (a) Define a planar graph and a plane graph. Show that the complete bipartite graph $K_{2,5}$ is planar by drawing an isomorphic plane graph with straight edges. Also, label the region defined by your plane graph and list the edges that form the boundary of each region.
- (b) There are ten students who, in the coming semester, will be taking the courses shown in the following table. How many time periods must be allowed in order for these students to take the courses they want without conflicts?

Arnold	Physics, Mathematics, English
Bill	Physics, Earth Science, Economics
Carol	Earth Science, Business
Calvin	Statistics, Economics
Eleanor	Mathematics, Business
Frederick	Physics, Earth Science
George	Business, Statistics
Huber	Mathematics, Earth Science
Ingrid	Physics, Water Skiing, Statistics
Jacquie	Physics, Economics, Water Skiing

- (c) Draw the line of sight graph G associated with the given net pattern. Is this graph planar. Explain your answer.



6. (a) Define capacity of the (s,t) -cut. Find the capacity of the (s,t) -cut defined by $S = \{s, a, b, d\}$ and $T = \{c, e, f, t\}$ for the given network.



- (b) State and prove Max Flow-Min Cut theorem.
- (c) Five ladies have men friends as shown in the following table.

Lady	Men Friends
Karen	Stuart, Paul
Mary	Philip, Stuart
Aurie	David, Paul, Roger, Stuart
Pamela	Philip, Paul
Lynne	Stuart, Philip, Paul

Draw a bipartite graph depicting this situation. Find a way in which each lady can marry a man she knows, or use Hall's Marriage Theorem to explain why no such matching exists.

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 2978

L

Unique Paper Code : 2353572006

Name of the Paper : DSE-2(iii) Linear Programming

Name of the Course : B.Sc. (Prog.) Physical Sciences / Mathematical Sciences

Semester : IV

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting any **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of ordinary calculator is allowed.
5. Graph paper will be provided.

Q1 (a) Solve the following linear programming problem graphically.

$$\text{Maximize } z = 3x_1 + 2x_2$$

Subject to

$$x_1 + x_2 \geq 1$$

$$x_1 + x_2 \leq 7$$

$$x_1 + 2x_2 \geq 10$$

$$x_2 \leq 3$$

$$x_1, x_2 \geq 0.$$

(7.5)

(b) Find all basic feasible solutions of the following equations.

$$2x_1 + x_2 + x_3 = 6$$

$$x_1 + 2x_2 + x_3 + x_4 = 12.$$

(7.5)

(c) Test the convexity of the set $S = \{(x, y) \in R^2 : 3x^2 + 2y^2 \leq 6\}$.

(7.5)

Q2(a) Use Simplex Method to solve the following Linear Programming Problem.

$$\text{Minimize } z = -x_1 + 2x_2$$

Subject to the constraints

$$-x_1 + 3x_2 \leq 10$$

$$x_1 + x_2 \leq 6$$

$$x_1 - x_2 \leq 2$$

$$x_1, x_2 \geq 0.$$

(7.5)

(b) Use Simplex Method to solve the following Linear Programming Problem.

$$\text{Maximize } z = 3x_1 + 2x_2 + x_3$$

Subject to

$$2x_1 - 3x_2 + 2x_3 \leq 3$$

$$x_1 - x_2 - x_3 \geq -5$$

$$x_1, x_2, x_3 \geq 0.$$

(7.5)

(c) Using Artificial variable technique Method, solve the following Linear Programming problem.

$$\text{Maximize } z = x_1 - 4x_2 + 3x_3$$

Subject to

$$2x_1 - x_2 + 5x_3 = 40$$

$$x_1 + 2x_2 - 3x_3 \geq 22$$

$$3x_1 + x_2 + 2x_3 = 30$$

$$x_1, x_2, x_3 \geq 0.$$

(7.5)

Q3 (a) Define a convex set.

Examine the convexity of the set $S = \{(x, y) \in R^2 : xy \leq 1, x \geq 0, y \geq 0\}$. (7.5)

(b) Formulate the dual of the following Linear programming problem.

$$\text{Minimize } z = x_1 + x_2 + 3x_3$$

Subject to

$$4x_1 + 8x_2 \geq 3$$

$$7x_2 + 4x_3 \leq 6$$

$$3x_1 - 2x_2 + 5x_3 = 7$$

$$x_1 \leq 0, x_2 \geq 0, x_3 \text{ is unrestricted.}$$

(7.5)

(c) Determine the values of m and n that will make (A_2, B_2) a saddle point from the payoff matrix of player A.

	B ₁	B ₂	B ₃
A ₁	1	n	6
A ₂	m	5	10
A ₃	6	2	3

(7.5)

Q4(a) Minimize cost in the following assignment problem.

	I	II	III	IV	V
A	8	4	2	6	1
B	0	9	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	9	5

(7.5)

(b) Find initial basic feasible solution by using North-West corner rule, Least Cost method and Vogel's approximation method. Compare the three solutions.

	A	B	C	D	Supply
I	19	14	23	11	11
II	15	16	12	21	13
III	30	25	16	39	19
Demand	6	10	12	15	

(7.5)

(c) Solve the following cost minimizing assignment problem:

	A	B	C	D	E	F
1	14	9	16	18	11	13
2	10	12	14	16	9	11
3	13	11	9	12	14	10
4	15	14	12	10	11	9
5	9	13	15	11	10	12
6	11	10	13	14	12	8

(7.5)

Q5(a) Starting with Initial Basic feasible solution obtained using Least Cost Method, find the optimum solution to following transportation problem:

Sources	Destinations					Supply
	D1	D2	D3	D4	D5	
S1	12	18	25	14	20	15
S2	16	22	18	20	15	20
S3	20	10	22	16	18	25
Demand	10	10	15	10	15	

(7.5)

- (b) Find all basic feasible solutions of the following equations.

$$5x_1 + 4x_2 + x_3 + x_4 + 2x_5 = 8$$

$$x_1 + x_2 + 3x_3 + 2x_4 + 6x_5 = 12 \quad (7.5)$$

- (c) Solve the following Linear programming problem by using artificial variables.

$$\text{Maximize } z = x_1 + 5x_2$$

Subject to

$$3x_1 + 4x_2 \geq 6$$

$$x_1 + 3x_2 \leq 2$$

$$x_1, x_2 \geq 0. \quad (7.5)$$

- Q6(a) Use principle of dominance to solve the following game:

$$\begin{bmatrix} 5 & -7 & -2 & 2 \\ -2 & 2 & -5 & 5 \\ -2 & 5 & -2 & 7 \end{bmatrix} \quad (7.5)$$

- (b) Convert the following game problem, involving "two-person zero-sum game" into a linear programming problem for player A and player B:

Player B

$$\text{Player A} \begin{bmatrix} 1 & -1 & -1 \\ -1 & -1 & 3 \\ -1 & 2 & -1 \end{bmatrix} \quad (7.5)$$

- (c) (i) Define the following terms in "Two person Zero-Sum" game:

(i) Maximin and minimax value for a fair game

(ii) Mixed strategy

- (ii) Determine the saddle point(s) in the pay-off matrix given below.

$$\begin{bmatrix} 4 & 4 & 4 & 4 \\ 4 & 4 & 6 & 7 \end{bmatrix} \quad (4+3.5)$$

[This question paper contains 2 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 3002

L

Unique Paper Code : 2352572401

Name of the Paper : Abstract Algebra

Name of the Course : B.A./B.Sc. (Programme) with Mathematics as Non-Major/ Minor DSC

Semester : IV

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. The use of a calculator is not allowed.

1. (a) For each element a in a group, show that there is a unique element b in G such that $ab = ba = e$. Also find the inverse of 13 in $U(14)$. (7.5)
- (b) Show that $G = \{1, -1, i, -i\}$ forms a group under multiplication. (7.5)
- (c) Find the inverse of $\begin{bmatrix} 2 & 6 \\ 3 & 5 \end{bmatrix} \in GL(2, Z_{11})$. (7.5)
2. (a) Show that $\{1, 2, 3\}$ under multiplication modulo 4 is not a group, but that $\{1, 2, 3, 4\}$ under multiplication modulo 5 is a group. (7.5)
- (b) State and prove the one-step subgroup test. (7.5)
- (c) Find the order of the group and the order of each element in the group when $G = U(10)$ and Z_{12} . (7.5)
3. (a) Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{bmatrix}$. (7.5)
Compute $\alpha\beta, \beta\alpha$ and β^{-1} . Check whether α and β are even or odd permutations.
- (b) List the elements of the subgroups $\langle 20 \rangle$ and $\langle 10 \rangle$ in Z_{30} . Let a be a group element of order 30. List the elements of the subgroups $\langle a^{20} \rangle$, and $\langle a^{10} \rangle$. (7.5)
- (c) Find all of the left cosets of $\{1, 11\}$ in $U(30)$. (7.5)

4. (a) Define normal subgroup. Show that the alternating group A_n of even permutations is a normal subgroup of S_n . (7.5)
- (b) Construct a complete Cayley table for D_4 , the group of symmetries of a square. Find the inverse of each of the element of D_4 . (7.5)
- (c) Define group homomorphism. Show that the Kernel of homomorphism $\phi: \mathbb{R}^* \rightarrow \mathbb{R}^*$ given by $\phi(x) = |x|$ is a cyclic group. (7.5)
5. (a) Prove that intersection of any collection of subrings of a ring R is a subring of R . Does the result hold for the union of subrings? Justify. (7.5)
- (b) Prove that finite Integral Domains are fields. Can the condition of finiteness be dropped? Justify. (7.5)
- (c) (i) State and prove the subring test. (7.5)
(ii) Prove that the ring $\mathbb{Z}_p$ of integers modulo p where p is a prime, is an integral domain.
6. (a) Determine all the ring homomorphisms from $\mathbb{Z}_{12}$ to $\mathbb{Z}_{30}$. (7.5)
- (b) (i) Prove that the map $f(x) = x \bmod n$ from $\mathbb{Z}$ to $\mathbb{Z}_n$ is a ring homomorphism. (7.5)
(ii) Prove that if A be an ideal of a ring R and $1 \in A$, then $A = R$.
- (c) Let R be the ring of all real-valued functions of a real variable. Prove that the subset S of all differentiable functions is a subring of R but not an ideal of R . (7.5)

(32)
[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 2249

L

Unique Paper Code : 2352203601

Name of the Paper : DSC – Probability and Statistics

Name of the Course : Bachelor of Arts / Bachelor of Science (Programme)
with Mathematics as non-major/minor

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Attempt any two parts from each question.
4. All questions carry equal marks.
5. Use of non-programmable scientific calculator and statistical tables is permitted.

1. (a) Every corporation has a governing board of directors. The number of individuals on a board varies from one corporation to another. Consider the accompanying data on the number of directors on each board in a random sample of 204 corporations.

No. of directors	4	5	6	7	8	9	10	11	12	13	14	15	16	17	21	24	32
Frequency	3	12	13	25	24	42	23	19	16	11	5	4	1	3	1	1	1

Construct a histogram of the data based on relative frequency and comment on any interesting features.

- (b) Calculate the mean and variance of the following data.

116.4	115.9	114.6	115.2	115.8
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Subtract 100 from each observation to obtain a sample of transformed values. Now calculate the sample variance of these transformed values and compare it the variance of original data.

- (c) The accompanying data consists of observations on the time until failure (1000s of hours) for a sample of turbochargers from one type of engine:

1.6	2.0	2.6	3.0	3.5	3.9	4.5	4.6	4.8	5.0
5.1	5.3	5.4	5.6	5.8	6.0	6.0	6.1	6.3	6.5

Compute the median, upper fourth and lower fourth.

2. (a) Consider randomly selecting a student at a large university, and let A be the event that the selected student has a Visa card and B be the analogous event for MasterCard. Suppose that

$$P(A) = 0.6, P(B) = 0.4 \text{ and } P(A \cap B) = 0.3.$$

- (i) Describe, in terms of A and B , the event that the selected student has a Visa card but not a MasterCard, and then calculate the probability of this event.
 - (ii) Calculate the probability that the selected student has exactly one of the two types of cards.
- (b) State Bayes's theorem.

At a certain gas station, 40% of the customers use regular gas (A_1), 35% use plus gas (A_2) and 25% use premium (A_3). Of those customers using regular gas, only 30% fill their tanks (event B). Of those customers using plus, 60% fill their tanks, whereas of those using premium, 50% fill their tanks. If the next customer fills the tank, what is the probability that regular gas is requested?

- (c) A university warehouse has received a shipment of 25 printers, of which 10 are laser printers and 15 are inkjet models. If 6 of these 25 are selected at random to be checked by a particular technician, what is the probability that exactly 3 of those selected are laser printers.

3. (a) A store carries flash drives with either 1 GB, 2 GB, 4 GB, 8 GB, or 16 GB of memory. The accompanying table gives the distribution of Y = the amount of memory in a purchased drive :

y	1	2	4	8	16
$P(y)$	.05	.10	.35	.40	.10

- (i) Find the cdf of Y .
- (ii) Plot the graph of cdf.

- (b) Suppose $E(X) = 5$ and $E[X(X - 1)] = 27.5$. Find
- $E(X^2)$
 - $V(X)$.
- (c) A company that produces fine crystal knows from experience that 10% of its goblets have cosmetic flaws and must be classified as "seconds."
- Among six randomly selected goblets, how likely is it that only one is a second?
 - Among six randomly selected goblets, what is the probability that at least two are seconds?
4. (a) A pediatrician wishes to recruit 5 couples, each of whom is expecting their first child, to participate in a new natural childbirth regimen. Let $p = P$ (a randomly selected couple agrees to participate). If $p = .2$, what is the probability that 15 couples must be asked before 5 are found who agree to participate? That is, with $S = \{\text{agrees to participate}\}$ and $F = \{\text{doesn't agree to participate}\}$, what is the probability that 10 F 's occur before the fifth S ?
- (b) Let X denote the number of traps (defects of a certain kind) in a particular type of metal oxide semiconductor transistor, and suppose it has a Poisson distribution with $\mu = 2$.
- Find the probability that there are exactly three traps.
 - Find the probability that there are at most three traps.
- (c) Let X denote the amount of time a book on two-hour reserve is actually checked out, and suppose the cdf is
- $$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{4} & 0 \leq x < 2 \\ 1 & 2 \leq x \end{cases}$$
- Obtain the probability density function $f(x)$.
 - Calculate $P(X \leq 1)$.
5. (a) Suppose the pdf of the magnitude X of a dynamic load on a bridge (in newtons) is given by

$$f(x) = \begin{cases} \frac{1}{8} + \frac{3}{8}x & 0 \leq x \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$

Find the probability that the magnitude of a dynamic load is between 0.5 to 1.5 newton. Also, find the probability that the load exceeds 1 newton.

- (b) The authors used data collected from an experiment with a specified initial crack length and number of loading cycles to propose a normal distribution with mean value 5.496 mm and standard deviation 0.067 mm for the random variable X = final crack depth. For this model, what value of final crack depth would be exceeded by only 0.5% of all cracks under these circumstances?
- (c) The breakdown voltage of a randomly chosen diode of a particular type is known to be normally distributed. What is the probability that a diode's breakdown voltage is within 1 standard deviation of its mean value? And, what is the probability that a diode's breakdown voltage is within 2 standard deviation of its mean value?
6. (a) Consider the following data on y = porosity (%) is related to x = unit weight (pcf) in concrete specimens.

x	99	101	102	103	105	107	108	110
y	29	28	27	25	23	22	21	20

Find the equation of regression line. And estimate the porosity (y) when unit weight (x) is 100.

- (b) The amount of a particular impurity in a batch of a certain chemical product is a random variable with mean value 4.0 g and standard deviation 1.5 g. If 50 batches are independently prepared, what is the (approximate) probability that the sample mean amount of impurity $\bar{X}$ is between 3.5 and 3.8 g?
- (c) Consider the following data of on x = rainfall volume (m^3) and y = runoff volume (m^3) for a particular location.

x	0.5	1.2	1.4	1.7	2.3	3.0	4.0	4.7	5.5	6.7
y	0.4	1.0	1.3	1.5	1.5	2.5	2.7	4.6	3.8	4.6

Compute the coefficient of correlation between x and y .

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[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 2285

L

Unique Paper Code : 2352203601

Name of the Paper : DSC – Probability and Statistics

Name of the Course : Bachelor of Arts / Bachelor of Science (Programme)
with Mathematics as non- major / minor

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Attempt any two parts from each question.
4. All questions carry equal marks.
5. Use of non-programmable scientific calculator and statistical tables is permitted.

1. (a) Construct stem and leaf display of the following data and comment on any interesting features.

136	140	155	150	144	147	169	152	161	139
142	145	149	155	158	160	150	155	160	

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 2285

L

Unique Paper Code : 2352203601

Name of the Paper : DSC – Probability and Statistics

Name of the Course : Bachelor of Arts / Bachelor of Science (Programme)
with Mathematics as non- major / minor

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Attempt any two parts from each question.
4. All questions carry equal marks.
5. Use of non-programmable scientific calculator and statistical tables is permitted.

1. (a) Construct stem and leaf display of the following data and comment on any interesting features.

136	140	155	150	144	147	169	152	161	139
142	145	149	155	158	160	150	155	160	

- (b) The May 1, 2009, issue of The Mont Clarian reported the following home sale amounts for a sample of homes in Alameda, C A that were sold the previous month (1000s of \$) :

590	815	575	608	350	1285	408	540	555	679
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- (i) Calculate and interpret the sample mean and median.
- (ii) Suppose the 6th observation had been 985 rather than 1285. How would the mean and median change?
- (c) An article reported the following data on oxidation-induction time (min) for various commercial oils :

87	103	130	160	180	195	132	145	211	105
145	153	152	138	87	99	93	119	129	

Calculate the sample variance and standard deviation. Answer without reperforming the calculations, what will be the new sample variance and deviation if each observation is multiplied by 4.

2. (a) An individual has three different email accounts. Most of her messages, in fact 70%, come into account #1, whereas 20% come into account #2 and the remaining 10% into accounts #3. Of the messages into account #1, only 1% are spam, whereas the corresponding percentage for accounts #2 and #3 are 2% and 5%, respectively. What is the probability that a randomly selected message is spam?

- (b) A new magazine publishes three columns entitled “Art” (A), “Books” (B) and “Cinema” (C). Reading habits of a randomly selected readers with respect to these columns are as follows:

Reads Regularly	A	B	C	$A \cap B$	$A \cap C$	$B \cap C$	$A \cap B \cap C$
Probability	0.14	0.23	0.37	0.08	0.09	0.13	0.05

Then determine the following probabilities $P(A|B)$, $P(A|B \cup C)$ and $P(A \cup B|C)$.

- (c) It is known that 30% of a certain company’s washing machines require service while under warranty, whereas only 10% of its dryers need such service. If someone purchases both a washer and a dryer made by this company, then
- What is the probability that both machines will need warranty service?
 - What is the probability that neither machine needs service?
3. (a) In a group of five potential blood donors—a, b, c, d, and e—only a and b have type O- positive blood. Five blood samples, one from each individual, will be typed in random order until an O- positive individual is identified. Let the random variable Y = the number of typing necessary to identify an O- positive individual.
- Find the probability mass function (pmf) of Y .
 - Draw the line graph of pmf.

- (b) A library has an upper limit of 6 on the number of DVDs that can be checked out to an individual at one time. Consider only those who currently have DVDs checked out, and let X denote the number of DVDs checked out to a randomly selected individual. The pmf of X is as follows :

x	1	2	3	4	5	6
$P(x)$	.30	.25	.15	.05	.10	.15

- (i) Find the variance of X .
- (ii) Find the standard deviation of X .
- (c) Eighteen individuals are scheduled to take a driving test at a particular DMV office on a certain day, eight of whom will be taking the test for the first time. Suppose that six of these individuals are randomly assigned to a particular examiner, and let X be the number among the six who are taking the test for the first time.
- (i) What kind of a distribution X does have (name and values of all parameters)?
- (ii) Compute $P(X = 2)$, $P(X \leq 2)$, and $P(X \geq 2)$.
- (iii) Calculate the mean value and standard deviation of X .
4. (a) A mail-order computer business has six telephone lines. Let A denote the number of lines in use at a specified time. Suppose the pmf of A is as given in the accompanying table :

x	0	1	2	3	4	5	6
P(x)	.10	.15	.20	.25	.20	.06	.04

Calculate the probability of each of the following events:

- (i) {at most three lines are in use}
 - (ii) {fewer than three lines are in use}
 - (iii) {at least three lines are in use}
 - (iv) {between two and five lines, inclusive, are in use.}
- (b) Let A denote the amount of space occupied by an article placed in a 1-ft³ packing container. The pdf of A is

$$f(x) = \begin{cases} 90x^8(1-x) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

- (i) Find cdf of X.
 - (ii) What is $P(X \leq 5)$.
- (c) The article "Expectation Analysis of the Probability of Failure for Water Supply Pipes" (J. of Pipeline Systems Engr. and Practice, May 2012: 36-46) proposed using the Poisson distribution to model the number of failures in pipelines of various types. Suppose that for cast-iron pipe of a particular length, the expected number of failures is 1 (very close to one of the cases

considered in the article). Then A , the number of failures, has a Poisson distribution with $\mu = 1$.

(i) Obtain $P(X \leq 5)$.

(ii) Determine $P(2 \leq X \leq 4)$.

(iii) What is the probability that X exceeds its mean value by more than one standard deviation.

5. (a) "Time headway" in traffic flow is the elapsed time between the time that one car finishes passing a fixed point and the instant that the next car begins to pass that point. Let X = the time headway for two randomly chosen consecutive cars on a freeway during a period of heavy flow. The following pdf of X is essentially the one suggested in "The Statistical Properties of Freeway Traffic" (Transp. Res., vol. 11: 221-228):

$$f(x) = \begin{cases} 0.15e^{-0.15(x-0.5)} & x \geq 0.5 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability that headway time is at most 5 sec.

- (b) Let X , the thickness of a certain metal sheet, have a uniform distribution on $[A, B]$. Write the probability density function (pdf) of X and find the cumulative distribution function (cdf) of X .
- (c) The amount of distilled water dispensed by a certain machine is normally distributed with mean value 64 oz and standard deviation 0.78 oz. What container size c will ensure that overflow occurs only 0.5% of the time?

6. (a) The distribution of egg weights (g) of a certain type is normal with mean value 53 and standard deviation 0.3. Let $X_1, X_2, \dots, X_{12}$ denote the weights of a dozen randomly selected eggs; these X_i 's constitute a random sample of size 12 from the specified normal distribution. Find the mean and standard deviation of the sample mean weight $\bar{X}$. Also, find the probability that the sample mean weight exceeds 53.5 g.
- (b) Medical researchers have noted that adolescent females are much more likely to deliver low-birth-weight babies than are adult females. Because such babies have higher mortality rates, numerous investigations have focused on the relationship between mother's age and birth weight. For the following data on x = maternal age (year) and y = baby's birth weight (kg).

x	15	17	18	15	16	19	17	16	18	19
y	2.3	3.4	3.3	2.6	2.9	3.3	2.9	2.5	3.1	3.6

Compute the coefficient of correlation between x and y . And, how would you describe the nature of relationship between the two variables.

- (c) Consider the following data on fracture strength (x , as a percentage of ultimate tensile strength) and attenuation (y , in neper/cm, the decrease in amplitude of the stress wave) in fiberglass-reinforced polyester composites.

x	12	30	36	40	45	57	62	67	71	78
y	3.3	3.2	3.4	3.0	2.8	2.9	2.7	2.6	2.5	2.6

Find the equation of regression line. And estimate the attenuation (y) when fracture strength (x) is 50.

[This question paper contains 2 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 2318

L

Unique Paper Code : 2352203602

Name of the Paper : DSC- Elementary Mathematical Analysis

Name of the Course : NEP-UGCF B.A. (Prog) with Mathematics as Major

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any two parts from each question.
3. All questions are compulsory.

1. (a) Define the limit of a function in terms of ϵ and δ . Use $\epsilon - \delta$ definition, to show that $\lim_{x \rightarrow 2} (4x - 5) = 3$. (7.5)

- (b) State the sequential criterion for limits. Use it to prove that $\lim_{x \rightarrow x_0} f(x)$ does not

exist, where $f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ -1, & \text{if } x \text{ is irrational.} \end{cases}$ (7.5)

- (c) Let $f: A \rightarrow \mathbb{R}$ where $A \subseteq \mathbb{R}$ and let x_0 be a cluster point of A . Then $\lim_{x \rightarrow x_0} f(x) = L$ if and only if for each sequence $\langle x_n \rangle$ in $A \setminus \{x_0\}$ that converges to x_0 , the sequence $\langle f(x_n) \rangle$ converges to L . (7.5)

2. (a) Define the continuity of a function at a point in terms of $\epsilon - \delta$. Use $\epsilon - \delta$ definition, to show that $f(x) = 4x^2 - 5x - 3$ is continuous at $x_0 = 2$. (7.5)

- (b) State the Intermediate Value Theorem. Also, prove that $\cos x = x$ for some x in $(0, \frac{\pi}{2})$. (7.5)

- (c) Define uniform continuity of a function f on interval I . Use it to prove that the function $f(x) = 3x^2 - 2x - 1$ is uniformly continuous on the interval $[-1, 5]$. (7.5)

3. If $f: [a, b] \rightarrow \mathbb{R}$ is a bounded function and $\mathcal{P}$ and $\mathcal{Q}$ are any partitions of $[a, b]$, then prove that $\underline{S}(f, \mathcal{P}) \leq \bar{S}(f, \mathcal{Q})$.

- (b) Prove that $f(x) = 7$ is integrable on $[-1, 1]$. Also, find $\int_{-1}^1 f$. (7.5)
- (c) Let $f(x) = 5 - 3x$ be defined on $[0, 2]$. For the partition $\mathcal{P} = \{0, \frac{1}{2}, 1, \frac{3}{2}, 2\}$ of $[0, 2]$, find $\underline{S}(f, \mathcal{P})$ and $\bar{S}(f, \mathcal{P})$. (7.5)
4. (a) Use sequential criteria for integrability to show that the function $f(x) = x^2$ is integrable on $[0, 1]$ and find $\int_0^1 f$. (7.5)
- (b) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = \begin{cases} 1, & \text{if } 1 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$. Then prove that f is integrable on $[0, 5]$ and find $\int_0^5 f$. (7.5)
- (c) Let $f, g: [a, b] \rightarrow \mathbb{R}$ be integrable functions on $[a, b]$.
- (i) If $f(x) \geq 0$, for all $x \in [a, b]$, then prove that $0 \leq \int_a^b f$.
- (ii) If $f(x) \leq g(x)$, for all $x \in [a, b]$, then prove that $\int_a^b f \leq \int_a^b g$. (7.5)
5. (a) Discuss the pointwise convergence of the following sequences of functions:
- (i) $\langle f_n \rangle$ where $f_n(x) = \frac{\sin(nx + n)}{n}$, $x \in \mathbb{R}$
- (ii) $\langle g_n \rangle$ where $g_n(x) = \frac{x^2 + nx}{n}$, $x \in [0, 1]$. (7.5)
- (b) Define uniform convergence for a sequence of functions.
- Let $f_n(x) = x^n$, $x \in [0, 1]$, then show that $\langle f_n \rangle$ is uniformly convergent on $[0, c]$ where $c < 1$ but it is not uniformly convergent on $[0, 1]$. (7.5)
- (c) State the Cauchy Criterion for uniform convergence of a sequence of functions.
- Test the sequence $\langle f_n \rangle$ for uniform convergence on $[0, 1]$ where
- $$f_n(x) = \frac{x}{n}, \quad \forall n \in \mathbb{N}, x \in [0, 1]$$
6. (a) State Weierstrass M-Test for uniform convergence of series of functions.
- Test the following series for uniform convergence on $\mathbb{R}$, (7.5)
- (i) $\sum_{k=1}^{\infty} \frac{\sin kx}{k^2}$, (ii) $\sum_{k=1}^{\infty} r^k \cos kx$, $0 < r < 1$.
- (b) Find the interval of convergence and the radius of convergence for the following power series, (7.5)
- (i) $\sum_{k=1}^{\infty} \frac{x^k}{k^k}$ (ii) $\sum_{k=1}^{\infty} \left(\frac{k+1}{k}\right)^k (x+4)^k$
- (c) Show that $\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$, $-1 \leq x \leq 1$.
- And use Abel's theorem to deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$ (7.5)

[This question paper contains 4 printed pages]

Your Roll No. :

Sl. No. of Q. Paper : **2490 I**

Unique Paper Code : 2343010020

Name of the Paper : Research Methodology

Name of the Course : **B.A(Prog)/B.Sc(Prog)**

Semester : VI

Time : 3 Hours **Maximum Marks : 90**

Instructions for Candidates :

- Write your Roll No. on the top immediately on receipt of this question paper.
- All** parts of Question **1 (Section-A)** are compulsory.
- Attempt any **four** questions from **Section-B**.
- Parts of a question must be answered together.
- Use of scientific calculator is allowed.

Section - A

- Why are objectivity and systematic approach important characteristics of research? 3
 - What do you mean by problem identification in research? 3

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- (c) What is the purpose of conducting a literature review? 3
- (d) Explain the three measures of central tendency. 3
- (e) Why is interpretation of data important in research? 3
- (f) How are conclusions drawn from research findings? 3
- (g) Explain the role of descriptive statistics in data analysis. 3
- (h) What do you mean by peer review in academic publishing? Describe the role of reviewers in the peer review process? 3
- (i) Define citation index and its importance. 3
- (j) What is a conflict of interest in research? How can a personal conflict of interest affect ethical publication practices? 3

Section - B

- 2. (a) Differentiate between the following types of research with suitable examples 7.5
 - (i) Experimental vs Non-Experimental Research
 - (ii) Prospective vs Retrospective Research

- (b) A Student has scored the following marks in different tests 10 subjects. 7.5
78 89 67 76 57 88 92 83 85 72
Calculate the mean, median, mode and inter quartile range of the student's marks.
3. (a) A journal had published 120 articles in 2019, 130 articles in 2020 and 150 articles in 2021. What will be the cite score and impact factor if journal receive 2200 citations in 2022? 7.5
- (b) Discuss the complete research process in detail, explaining each step and its importance. 7.5
4. (a) Researcher has published 8 papers and citations count for each paper are: 7.5
20, 16, 15, 14, 12, 19, 17, 18
Calculate the h-index, g-index, and i10-index for this researcher.
- (b) Explain standard error of mean and estimated standard error of mean. A factory produces metal rods. A quality control team is analysing the length of rods, which is known to have a population standard deviation of 0.05 cm. They take a random sample of 36 rods to estimate the mean length. Calculate the standard error of mean for this sample. 7.5

5. (a) Describe the steps involved in hypothesis testing with the help of an example. 7.5
 (b) What is plagiarism? Mention any two plagiarism checker tools and explain their purpose. 7.5
6. (a) Explain fraud and misconduct in research. Explain its types, effects on society, and ways to prevent it. 7.5
 (b) Explain copyright and royalty. What type of works are protected under copyright? What happens when a copyright expires? 7.5
7. Explain the placebo effect with suitable examples. How is it used in experimental research? 7.5
 (b) Explain any three intellectual property rights (IPR) in research. How do they protect the work of researchers? 7.5

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 2556

L

Unique Paper Code : 2353010015

Name of the Paper : Research Methodology

Name of the Course : **Bachelor of Science (Honours Course) / Bachelor of Arts / Bachelor of Science (Programme) – DSE**

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
 2. Attempt all the **six** questions, selecting any **two** parts from each question.
 3. Parts of the question are to be attempted together.
 4. All the **six** questions carry equal marks.
-
1. (a) Complete the following sentences, that describe the rules that must be observed for avoiding errors and for the betterment of a mathematical writing.
 - (i) Begin sentences with words, not _____.
 - (ii) Adopt a _____ notation.
 - (iii) Use a balance of _____ and _____ while writing mathematical sentences or statements.
 - (iv) Write words fully, without _____.

- (v) Write sentences in _____ style, not _____ style.
- (vi) Use math symbols within math expressions, not as _____ between math expressions and words.
- (b) List out the basic techniques that successful mathematicians have found useful in discovering mathematics and that are central part of the mathematical thought process.
- (c) What is the general sequence of components of a research/review paper. Explain each component in detail.
2. (a) Write short notes on the following terms, with reference to mathematical writing, giving an example in each case:
- (i) Parallelism
 - (ii) Otiose Symbols
 - (iii) Different kinds of ellipsis
- (b) What points should be kept in mind, while writing the 'Introduction' section of a research paper?
- (c) Explain the key factors that a researcher should consider, to choose an appropriate journal for publication of his/her research paper?
3. (a) Describe how mathematical expressions are written in LaTeX, explaining the LaTeX environments for producing, an in-text formula and a displayed formula. Give one example each, for a fraction, an integral, and matrix, to show how each of these is written using LaTeX commands.

- (b) Write a code in LaTeX software to obtain the following output :

1 Introduction

1.1 Importance of LaTeX

LaTeX is widely used for professional documents.

1. High-quality typesetting
2. Easy equations
3. Auto-numbering

$$x + y = 10 \quad (1)$$

$$x - y = 4 \quad (2)$$

- (c) What is cross-referencing in LaTeX? Explain the steps to add a figure, give it a label, and refer to it in the text
4. (a) What is the advantage of giving a presentation of the research work? Write a short note on poster presentation. Give the general format of a poster.
- (b) Make a Beamer presentation on the topic 'The Mathematical Research' with at least one bullet list and one equation.
- (c) Explain the use of the PSTricks commands `\begin{pspicture}`, `\pscircle`, and `\psline`, and describe how coordinates are used in PSTricks.
5. (a) Describe the following web resources for mathematics, with reference to research in Mathematics:

(i) MAA

(ii) MathsSciNet

(iii) ArXiv

- (b) What is Impact Factor of a journal? Who introduced Impact Factor and when? Which database can be used to find the impact factor of a journal?
 - (c) Explain in detail, the journal metrics SJR and MCQ.
6. (a) Describe research ethics concerning :
- (i) the participants
 - (ii) the researchers.
- (b) Explain the key features of the databases 'MathSciNet and 'Scopus'. Write the official websites for accessing these databases.
- (c) What do you mean by plagiarism with reference to a mathematical writing? Name any plagiarism checker software. What can be the possible penalties, that a researcher might face in case, the publisher finds out the researcher's work to be plagiarized?

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 2558

L

Unique Paper Code : 2353200011

Name of the Paper : Introduction to Mathematical Modeling

Name of the Course : B.A. (P) / B.Sc. (Physical Science & Mathematical Science) – DSE

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of Scientific Calculator is allowed.

1. (a) The following differential equation describes the level of pollution in the lake

$$\frac{dC}{dt} = \frac{F}{V} C_{in} - C$$

Where V is the volume F is the flow (in and out), C is the concentration of pollution at time t and C_{in} is the concentration of pollution entering lake. Let $V = 28 \times 10^6 m^3$, $F = 4 \times 10^6 m^3/\text{month}$. If only freshwater enters the lake.

- (i). How long would it take for the lake with pollution concentration 10^7 parts/ m^3 to draw below the safety threshold 4×10^6 parts/ m^3 ?
- (ii). How long will it take to reduce the pollution level to 5% of its current level? (7.5)

- (b) Solve the following potassium-argon decay model

$$\frac{dK(t)}{dt} = -(k_1 + k_2)K,$$

$$\frac{dA(t)}{dt} = k_1K,$$

$$\frac{dC(t)}{dt} = k_2K$$

Where $K(t)$, $A(t)$ and $C(t)$ represent the potassium, argon and calcium in the sample of rock respectively. Then

- (i). Solve the system to find $K(t)$, $A(t)$ and $C(t)$ in terms of k_1 , k_2 and $k_3 = k_1 + k_2$, using the initial conditions $K(0) = k_0$, $A(0) = C(0) = 0$. (7.5)
 - (ii). Show that $K(t) + A(t) + C(t) = k_0$ for all $t \geq 0$.
- (c) Write down the word equations which describe the movement of the drug between the two compartments of the body, the GI-Tract and the bloodstream when the patient takes the single pill.
- (i). From the word equation, develop the differential equation system which describes this process, defining all variables and parameters as required.
 - (ii). The constants of proportionality associated with the rate at which the drug diffuses from the GI-Tract into the blood stream, and then is removed from the blood stream are 0.72/hour and 0.15/hour respectively. Suppose initially the amount of the drug in the GI-Tract is 0.0001 mg and there is none in the blood stream. Determine the level of the drug in the blood stream after 6 hours. (7.5)

2. (a) A mouse population, initially consisting of 1,000 mice, has a per-capita birth rate of 8 mice per month (per mouse) and a per-capita death rate of 2 mice per month (per mouse). Also, 20 mouse traps are set each week and they are always filled. Draw a compartment diagram and write a word equation describing the rate of change in the number of mice and then formulate a differential equation for the population with an initial condition. (7.5)

- (b) Consider the fish populations as an alternative to the discrete logistic equation. The difference equation is

$$\frac{dN}{dt} = Ne^{r(1-N/K)}, \quad r > 0,$$

Where N is the population size in generation k , r is the intrinsic growth rate and K is the carrying capacity, which are positive constants. Find all the equilibrium solutions and determine their stability. (7.5)

- (c) Consider the discrete logistic equation (with $K = 1$)

$$\frac{dN_n}{dt} = N_n + rN_n(1 - N_n).$$

Show that every second term in the sequence $N_0, N_1, N_2, \dots$ satisfies the difference equation

$$N_{n+2} = (1 + 2r + r^2)N_n - (2r + 3r^2 + r^3)N_n^2 + (2r^2 + 2r^3)N_n^3 - r^3N_n^4$$

And for equilibrium solution, let $S = N_{n+2} = N_n$ and obtain a quartic equation. (7.5)

3. (a) Formulate the differential equations of a SIR-model. State the assumptions on which it is based. What are its limitations? (7.5)

- (b) Find the basic reproduction number r_0 for the SIR model. What is the condition for disease outbreak to become an endemic? If $r_0 = 15$, what proportion of susceptibles need to be vaccinated. (7.5)

- (c) Formulate the differential equations for the prey and predator model. Mention the assumptions taken to formulate the problem. (7.5)

4. (a) Consider the following model for two competing species, with densities, $X(t)$ and $Y(t)$, given by the differential equations

$$\frac{dX}{dt} = \beta_1 X - d_1 X^2 - c_1 XY$$

$$\frac{dY}{dt} = \beta_2 Y - d_2 Y^2 - c_2 XY$$

with parameter values $\beta_1 = 3$, $\beta_2 = 3$, $d_1 = 2$, $d_2 = 1$, $c = 2$ and $c_2 = 2.5$. What is the carrying capacity for each of the species, evaluated for the given parameter values? (7.5)

- (b) State Gause's Principle of Competitive Exclusion. Extend the competition model to account for logistic growth in both species, in the absence of the other species. (7.5)

- (c) Develop a model (a pair of differential equations) for a battle between two armies where both groups use aimed fire. Assume that the red army has a significant loss due to disease, where the associated death rate (from disease) is proportional to the number of soldiers in that army. (7.5)

5. (a) The following data shows the growth of a bacteria culture over time:

t (days)	2	4	6	8	10
P (number of bacteria)	5	15	45	105	175

Assume the model $P = ae^{bt}$. Transform the data using natural logarithms. Plot the transformed data on a semi-log graph. Estimate the values of a and b . (7.5)

- (b) For the following dataset:

x	1	2	3	4	5
y	2	5	12	20	40

Assume a linear model $y = ax + b$. Formulate the model that minimizes the maximum deviation between the data points and the line. Estimate the values of a and b . (7.5)

- (c) The following data is given:

x	1	2	3	4	5
y	3	12	18	27	42

Fit the data using the least squares method for $y = ax^2$. (7.5)

6. (a) Generate 10 random numbers using the seed $x_0 = 12467$. (7.5)
- (b) Use Monte-Carlo simulation to approximate the area under $f(x) = x^2$ over the interval $0 \leq x \leq 1$. (7.5)
- (c) Find the area enclosed between the curves $y = x^2$ and $y = 4 - x$ within the region bounded by $x = 0$ and $y = 0$. (7.5)

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 2925

L

Unique Paper Code : 2352203601

Name of the Paper : DSC – Probability and Statistics

Name of the Course : Bachelor of Arts / Bachelor of Science (Programme)
with Mathematics as non-major/minor

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Attempt any two parts from each question.
4. All questions carry equal marks.
5. Use of non-programmable scientific calculator and statistical tables is permitted.

1. (a) Construct a stem and leaf display of the following data:

5.9	7.2	7.3	6.3	8.1	6.8	7.0	7.6	6.8	7.7
6.5	7.0	6.3	7.9	9.0	8.2	8.7	7.8	9.7	7.8
7.4	7.7	9.7	11.8	10.7	11.6	11.3			

Also, comment on interesting features of the display.

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- (b) Consider the following data on endotoxin concentration in settled dust both for a sample of urban homes and for a sample of farm homes:

U	6.0	5.0	11.0	33.0	4.0	5.0	80.0	18.0	35.0	17.0	23.0
F	4.0	14.0	11.0	9.0	9.0	8.0	4.0	20.0	5.0	8.9	21.0
	9.2	3.0	2.0	0.3							

- (i) Determine the value of the sample standard deviation for each sample, interpret these values and then contrast variability in the two samples.
- (ii) Compute the fourth spread for each sample and compare.
- (c) Zinfandel is a popular red wine varietal produced almost exclusively in California. It is rather controversial among wine connoisseurs because its alcohol content varies quietly substantially from one producer to other. In May 2003, the author went to the website klwines.com, randomly selected 10 zinfandels from among the 325 available, and obtained the following values of alcohol content (%) :

14.8	14.5	16.1	14.2	15.9	13.7	16.2	14.6	13.8	15.0
------	------	------	------	------	------	------	------	------	------

Calculate mean, median and sample variance.

2. (a) Let X be a Bernoulli random variable with pmf $p(1) = p$ and $p(0) = 1 - p$.

- (i) Compute $E(X^2)$.

(ii) Show that $V(X) = p(1 - p)$.

(iii) Compute $E(X^{79})$.

(b) A friend of mine is giving a dinner party. His current wine supply includes 8 bottles of zinfandel, 10 of merlot, and 12 of cabernet (he only drinks red wine), all from different wineries.

(i) If 6 bottles of wines are to be randomly selected from the 30 for serving, how many ways are there to do this.

(ii) If 6 bottles are randomly selected, what is the probability that all of them are the same variety.

(c) For any three events A, B and C prove that

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

3. (a) Six boxes of components are ready to be shipped by a certain supplier. The number of defective components in each box is as follows :

Box	1	2	3	4	5	6
No. of Defectives	0	2	0	1	2	0

One of these boxes is to be randomly selected for shipment to a particular customer. Let X be the number of defectives in the selected box. Find pmf of X.

- (b) Let X be the number of students who show up for a professor's office hour on a particular day. Suppose that the pmf of X is $p(0) = .20$, $p(1) = .25$, $p(2) = .30$, $p(3) = .15$, and $p(4) = .10$.
- (i) Draw the corresponding probability histogram.
 - (ii) What is the probability that at least two students show up? More than two students show up?
 - (iii) What is the probability that between one and three students, inclusive, show up?
 - (iv) What is the probability that the professor shows up?
- (c) A certain brand of upright freezer is available in three different rated capacities: 16 ft^3 , 18 ft^3 , and 20 ft^3 . Let X = the rated capacity of a freezer of this brand sold at a certain store. Suppose that X has pmf

x	16	18	20
$p(x)$	.2	.5	.3

- (i) Compute $E(X)$, $E(X^2)$, and $V(X)$.
- (ii) If the price of a freezer having capacity X is $70X - 650$, what is the expected price paid by the next customer to buy a freezer?
- (iii) What is the variance of the price paid by the next customer?

4. (a) Each of six randomly selected cola drinkers is given a glass containing cola S and one containing cola F. The glasses are identical in appearance except for a code on the bottom to identify the cola. Suppose there is actually no tendency among cola drinkers to prefer one cola to the other. Then $p = P(\text{a selected individual prefers S}) = .5$, so with $X = \text{the number among the six who prefer S}$.

(i) What is the probability that exactly three prefer S?

(ii) What is the probability that at least three prefer S?

(iii) What is the probability that at most one prefers S?

- (b) Compute the following binomial probabilities directly from the formula $b(x; n, p)$.

(i) $b(3; 8, .35)$

(ii) $P(3 \leq X \leq 5)$ when $n = 7$ and $p = .6$

(iii) $P(1 \leq X)$ when $n = 9$ and $p = .1$

- (c) The current in a certain circuit as measured by an ammeter is a continuous random variable X with the following density function:

$$f(x) = \begin{cases} .075x + .2 & 3 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

(i) Verify that the total area under the density curve is indeed 1.

(ii) Calculate $P(X \leq 4)$.

(iii) Calculate $P(3.5 \leq X \leq 4.5)$.

5. (a) The Department of Statistics has a lab with six computers reserved for statistics majors. Let X denote the number of these computers that are in use at a particular time of day. Suppose that the probability distribution of X is as given in the following table; the first row of the table lists the possible X values and the second row gives the probability of each such value.

x	0	1	2	3	4	5	6
$p(x)$	0.05	0.10	0.15	0.25	0.20	0.15	0.10

Find the probability that (a) at most 2 computers are in use, (b) at least 3 computers are in use, (c) between 2 and 5 computers inclusive are in use, (d) the number of computers in use is strictly between 2 and 5.

- (b) Suppose pulses arrive at a counter at an average rate of six per minute. Find the probability that in a 0.5-min interval at least one pulse is received.

- (c) The weight distribution of parcels sent in a certain manner is normal with mean value 12 lb and standard deviation 3.5 lb. The parcel service wishes to establish a weight value c beyond which there will be a surcharge. What value of c is such that 99% of all parcels are under the surcharge weight?
6. (a) The distribution of egg weights (g) of a certain type is normal with mean value 53 and standard deviation 0.3. Let $X_1, X_2, \dots, X_{12}$ denote the weights of a dozen randomly selected eggs; these X_i 's constitute a random sample of size 12 from the specified normal distribution. Find the mean and standard deviation of the sample total weight T_0 . And, determine the probability that the sample total weight T_0 is between 635 and 640.
- (b) Consider the following data on $y =$ porosity (%) is related to $x =$ unit weight (pcf) in concrete specimens.

x	110	112	112	113	113	115	115	120
y	20	17	19	16	17	13	14	11

Find the equation of regression line. And estimate the porosity (y) when unit weight (x) is 100.

- (c) Consider the following data of on $x =$ rainfall volume (m^3) and $y =$ runoff volume (m^3) for a particular location.

x	5	12	14	17	23	30	40	47	55	67
y	4	10	13	15	15	25	27	46	38	46

Compute the coefficient of correlation between x and y. And, how would you describe the nature of relationship between the two variables.

[This question paper contains 2 printed pages.]

Your Roll No.....

L

Sr. No. of Question Paper : 5782

Unique Paper Code : 2354000009

Name of the Paper : Abstract Algebra

Name of the Course : **COMMON PROG GROUP**

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt all questions by selecting **two** parts from each question.
3. **All** parts of a question are to be attempted together.
4. Each part carries **7.5** marks.
5. Use of Calculator not allowed.

1. (a) Define the group $GL(2, F)$, where F is any field. In $GL(2, \mathbb{Z}_{13})$, find the inverse of $\begin{pmatrix} 7 & 4 \\ 1 & 5 \end{pmatrix}$.
(b) Prove that a group G is Abelian iff $(ab)^{-1} = a^{-1}b^{-1} \quad \forall a, b \in G$.
(c) Define the group $U(n)$, for $n \in \mathbb{N}$ Construct the Cayley table for $U(20)$. Find the inverse of each element of $U(20)$.
2. (a) Let G be an Abelian group under multiplication with identity e . Prove that the subset $H = \{x^2 \mid x \in G\}$ is a subgroup of G .
(b) Let G be a group. Define the Centralizer, $C(a)$, of an element a in G . Prove that $C(a) = C(a^{-1}) \quad \forall a \in G$.
(c) Define the order of an element in a group G . Find the order of each element in the group $U(15)$.

3. (a) Define a cyclic group. Show that $U(14) = \langle 3 \rangle = \langle 5 \rangle$. Is $U(14) = \langle 11 \rangle$?
- (b) Define the order of a permutation. Determine whether the following permutations are even or odd :
- (i) $(1243)(3521)$
- (ii) $(12)(134)(152)$
- (c) State & prove Lagrange's theorem for groups.

4. (a) Prove that a quotient group of a cyclic group is cyclic.
- (b) Define a normal subgroup of a group. Let

$$H = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in \mathbb{R}, ad \neq 0 \right\}$$

Is H a normal subgroup of the group $GL(2, \mathbb{R})$?

- (c) Show that map $\phi: \mathbb{R}^* \rightarrow \mathbb{R}^*$ defined by $\phi(x) = |x|$ is a group homomorphism. Also show that $\text{Ker } \phi$ is a cyclic group.
5. (a) (i) Prove that the set $A = \left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} : a, b \in \mathbb{Z} \right\}$ is a subring of the ring R of 2×2 matrices over $\mathbb{Z}$.
- (ii) Prove that the ring $\mathbb{Z}_p$ of integers modulo p , where p is a prime number, is an integral domain.
- (b) Prove that intersection of any collection of subrings of a ring R is a subring of R . Does the result hold for union of subrings ? Justify.
- (c) Show that the set $\mathbb{Q}[\sqrt{2}] = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$ is a commutative ring with unity.
6. (a) Define an ideal of a ring. State the ideal test. Hence prove that $n\mathbb{Z}$ is an ideal of $\mathbb{Z}$, where $n \in \mathbb{Z}$.
- (b) Let $f: R \rightarrow S$ be a ring homomorphisms. Prove that
- (i) $f(A)$ is a subring S , where A is a subring of R
- (ii) $f(B) = \{r \in R : f(r) \in B\}$ is an ideal of R , where B is an ideal of S .
- (c) Prove that the only ideals of a field F are $\{0\}$ and F itself.

[This question paper contains 2 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 101

L

Unique Paper Code : 2352204801

Name of the Paper : Topics in Multivariate Calculus

Name of the Course : B.A./B.Sc.(Programme)

Semester : VIII

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carries equal marks.
4. Use of Calculator is not allowed.

Q.1.(a) Let f be the function defined by $f(x, y) = \begin{cases} \frac{2xy}{x^2+y^2} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$. Is f continuous at $(0, 0)$? Explain.

(b) Let $z = 4x - y^2$, where $x = uv^2$ and $y = u^3v$. Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$.

(c) Find all second order partial derivatives for $f(x, y) = 5x^2 - 2xy + 3y^3$.

Q.2.(a) Find an equation for the tangent plane to the surface $z = \tan^{-1}\left(\frac{y}{x}\right)$ at the point $P(1, \sqrt{3}, \pi/3)$.

(b) Find all relative extrema and saddle points of the function

$$f(x, y) = 2x^2 + 2xy + y^2 - 2x - 2y + 5.$$

(c) Use the method of Lagrange multipliers to find the extrema of $f(x, y) = 1 - x^2 - y^2$ subject to the constraint $x + y = 1$ with $x \geq 0$ and $y \geq 0$.

Q.3. (a) Find the volume of the solid bounded below by the rectangle R in the x - y -plane and above by the graph of $z = 2x + 3y$; $R: 0 \leq x \leq 1, 0 \leq y \leq 2$.

(b) Sketch the region of integration and evaluate $\int_0^1 \int_x^1 e^{y^2} dy dx$ by reversing the order of integration.

(c) Evaluate $\iint_D \frac{1}{x} dA$ where D is the region that lies inside the circle $r = 3 \cos \theta$ and outside the cardioid $r = 1 + \cos \theta$.

Q.4. (a) Find the volume of the tetrahedron T bounded by the plane $2x + y + 3z = 6$ and the coordinate planes $x = 0, y = 0$ and $z = 0$

(b) Evaluate the integral $I = \int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{x^2+y^2}^{\sqrt{2-x^2-y^2}} z \, dz \, dy \, dx$ using the given rectangular form, or by transforming to either cylindrical or spherical coordinates.

(c) Evaluate $\iint_D (x-y)^5 (x+y)^3 \, dy \, dx$ where D is the region in x - y plane that is bounded by the coordinate axes and the line $x + y = 1$.

Q.5 (a) Evaluate the line integral $\int_C (y \, dx - z \, dy + x \, dz)$ where C is the curve with parametric equations: $x = t^2, y = e^{-t}, z = e^t$ for $0 \leq t \leq 1$.

(b) Verify the vector field: $F(x, y) = (y - x^2) \hat{i} + (x + y^2) \hat{j}$ is conservative and find a scalar potential f for F . Then evaluate the line integral: $\int_C F \cdot dR$, where C is any smooth path connecting $A(0, 0)$ to $B(1, 1)$.

(c) Verify Green's theorem for the line integral: $\oint_C (x^2 - y^2) \, dx + 2xy \, dy$, where C is the boundary of rectangle: $0 \leq x \leq 1, 0 \leq y \leq 1$.

Q.6 (a) Evaluate: $\iint_S (x^2 + y^2) \, dS$, where S is the surface of the hemisphere: $z = \sqrt{1 - x^2 - y^2}$.

(b) Use Stokes' theorem to evaluate: $\oint_C (y \, dx - 2x \, dy + z \, dz)$, where C is the intersection of the surface: $z = x^2 + y^2$ and the plane: $x + y + z = 1$, oriented counter-clockwise when viewed from the origin.

(c) Use the divergence theorem to evaluate: $\iiint_S F \cdot N \, dS$, where $F = xyz \hat{i} + xyz \hat{j} + 3x \hat{k}$ and S is the surface of the box: $0 \leq x \leq 1, 0 \leq y \leq 2, 0 \leq z \leq 3$ with outward unit normal vector N .

[This question paper contains 2 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5150

L

Unique Paper Code : 2354000017

Name of the Paper : Elements of Partial Differential Equations

Name of the Course : Common Prog Group

Semester : VIII

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the **six** questions are compulsory.
3. Attempt any **two** parts from each question.
4. Each part carries **7.5** marks.
5. Use of Calculator not allowed.

- 1.(a) (i) Eliminate the constants a and b from the equation

$$2z = (ax + y)^2 + b. \quad 2.5$$

- (ii) Eliminate the arbitrary function f from the equation

$$z = xy + f(x^2 + y^2). \quad 5$$

- (b) (i) Find the general integral of the partial differential equation $p + q = pq$. 3

- (ii) Show that the equations $xp = yq$ and $z(xp + yq) = 2xy$ are compatible and solve them. 4.5

- (c) Find the equation of the integral surface of the differential equation

$$2y(z - 3)p + (2x - z)q = y(2x - 3)$$

which passes through the circle $z = 0, x^2 + y^2 = 2x$. 7.5

- 2.(a) Find the equation of the system of surfaces which cut orthogonally the cones of the system $x^2 + y^2 + z^2 = cxy$. 7.5

- (b) Find the integral surface of the differential equation

$$(y + zq)^2 = z^2(1 + p^2 + q^2)$$

circumscribed about the surface $x^2 - z^2 = 2y$. 7.5

- (c) Prove that an equation of the "Clairaut" form

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = f\left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}\right)$$

is always solvable by Jacobi's method. Hence solve the equation

$$\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\right) \left(x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}\right) = 1. \quad 7.5$$

- 3.(a) Solve

$$(D^2 - 2DD' + D'^2)z = 12xy \quad 7.5$$

- (b) Find the particular integral of the partial differential equation

$$(D^2 - D')z = e^{2x+y} \quad 7.5$$

P.T.O.

- (c) Find the general solution of the partial differential equation

$$(D^2 - DD' - 2D)z = \sin(3x + 4y) \quad 7.5$$

4. (a) Solve

$$(x^2 D^2 - y^2 D'^2 + xD - yD')z = \log x \quad 7.5$$

- (b) Solve by Monge's Method

$$q^2 r - 2pqs + p^2 t = 0 \quad 7.5$$

- (c) Solve using Monge's Method

$$y^2 r - 2ys + t = p + 6y. \quad 7.5$$

5. (a) Using separation of variables, show that the one-dimensional wave equation

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 z}{\partial t^2}, \quad 0 < x < a, \quad t > 0$$

$$z(0, t) = 0, \quad z(a, t) = 0 \text{ for all } t.$$

has solution

$$z(x, t) = \sum_r \left\{ A_r \cos \frac{r\pi ct}{a} + B_r \sin \frac{r\pi ct}{a} \right\} \sin \frac{r\pi x}{a}$$

Where A_r 's and B_r 's are constants.

7.5

- (b) The faces
- $x = 0, x = a$
- of an infinite slab are maintained at zero temperature.

The initial distribution of temperature in the slab is described by the equation

$$\theta = f(x) (0 \leq x \leq a). \text{ Determine the temperature at a subsequent time } t. \quad 7.5$$

- (c) Find the D'Alembert solution of the Cauchy problem for one dimensional wave equation given by

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 y}{\partial t^2}, \quad x \in R, t > 0$$

$$y = \eta(x), \quad \frac{\partial y}{\partial t} = v(x) \text{ at } t = 0, x \in R \quad 7.5$$

6. (a) Determine the solution of the initial boundary value problem

$$\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \infty, t > 0$$

$$u(x, 0) = x^4, \quad 0 \leq x < \infty,$$

$$u_t(x, 0) = 0, \quad 0 \leq x < \infty,$$

$$u(0, t) = 0, \quad t \geq 0 \quad 7.5$$

- (b) Determine the solution of the vibrating string problem

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0, \quad 0 < x < l, t > 0$$

$$u(x, 0) = f(x), \quad 0 \leq x \leq l,$$

$$u_t(x, 0) = g(x), \quad 0 \leq x \leq l,$$

$$u(0, t) = 0, \quad t \geq 0$$

$$u(l, t) = 0, \quad t \geq 0 \quad 7.5$$

- (c) Derive the equation of traffic flow

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = 0,$$

Where $\rho(x, t)$ is traffic density and $q(x, t)$ is traffic flow at any time t .

7.5

(700)

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5152

L

Unique Paper Code : 2354000019

Name of the Paper : Optimization Techniques

Name of the Course : Common Prog Group-NEP-UGCF 2022

Semester : VIII

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory and carry equal marks.
3. Attempt any **one** part from each question.

1. (a) A firm having two sources S_1 and S_2 wishes to ship its products to two destinations D_1 and D_2 . The number of units available at S_1 and S_2 are 5 and 25 respectively and the product demands at D_1 and D_2 are 20 and 10 respectively. The firm, instead of shipping directly from sources to destinations, decides to investigate the possibility of transshipment. The unit transportation costs (in rupees) are given in the following table:

		Sources		Destinations		
		S_1	S_2	D_1	D_2	Available
Sources	S_1	0	2	3	4	5
	S_2	2	0	2	4	25
Destinations	D_1	3	2	0	1	----
	D_2	4	4	1	0	----
Demand		----	----	20	10	

- (i) Convert the given problem into transportation problem. Also, make a node diagram for the given problem.

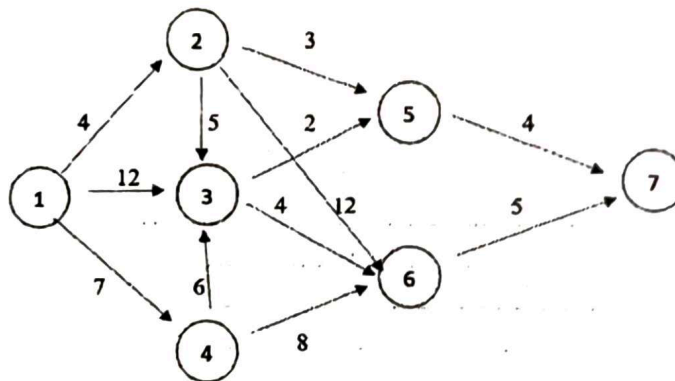
(ii) Find the initial basic feasible solution by North West Corner Rule.

- (b) In a cargo loading problem, there are 4 items of different weights/unit and different value/unit which are given as below:

Item (i)	w_i (kg/unit)	p_i (Rs/ unit)
1	1	1
2	3	5
3	4	7
4	6	11

The maximum cargo load is restricted to 17. How many units of each item be loaded to maximize the value?

2. (a) Suppose we want to select the shortest highway route between two cities. The network in given figure provides the possible routes between the starting city node '1' and the destination city at node '7'. Solve by the method of backward recursive equation and use it to find the optimal solution for the following data:



- (b) Consider the following transshipment problem with two sources and two destinations. The costs for shipment in rupees are given in the following table. Determine an optimum shipping schedule.

		Source		Destination		Supply
		S1	S2	D1	D2	
Source	S1	0	1	6	2	4
	S2	1	0	8	2	5
Destination	D1	0	7	0	3	-
	D2	1	0	3	0	-
Demand		-	-	3	6	

3. (a) Solve the following ILPP by using Gomory's Cutting Plane Method

$$\begin{aligned} \text{Max } Z &= x_1 + x_2 \\ \text{Subject to } 3x_1 + 2x_2 &\leq 12 \\ x_2 &\leq 2 \\ x_1, x_2 &\geq 0 \text{ and both are integers.} \end{aligned}$$

(b) Consider the following ILPP

$$\begin{aligned} \text{Max } Z &= 4x_1 + 5x_2 \\ \text{Subject to } x_1 + 4x_2 &\leq 10 \\ 53x_1 - 4x_2 &\leq 6 \\ x_1, x_2 &\geq 0 \text{ and both are integers} \end{aligned}$$

- (i) Convert the above LPP into the standard Linear programming problem.
(ii) Write the two LPP's by branching from x_2 and find the next branching variable, if the initial solution is $x_1 = 4$ and $x_2 = 1.5$, the maximum value of $Z = 23.5$, with the last simplex table as follows:

c_j			4	5	0	0
c_B	B	x_B	x_1	x_2	s_1	s_2
5	x_2	3/2	0	1	3/16	-1/16
4	x_1	4	1	0	1/4	1/4
$z_j - c_j$			0	0	-31/16	-11/16

4. (a) Consider the following ILPP

$$\begin{aligned} \text{Max } Z &= 4x_1 + 3x_2 \\ \text{Subject to } 3x_1 + 4x_2 &\leq 12 \\ 64x_1 + 2x_2 &\leq 9 \\ x_1, x_2 &\geq 0 \text{ and both are integers.} \end{aligned}$$

- (i) Find the optimal solution of the defined problem by using the simplex method.
(ii) Solve the given ILPP graphically.

- (b) Consider the following optimal simplex tableau of the associated Linear Programming Problem for a given AILP (with maximization form and x_3, x_4 as slack variables) with $\text{Max } Z = 53/10$. Generate the Gomory's cut through x_2 and hence solve the given AILP.

c_j			1	1	0	0
c_B	B	x_B	x_1	x_2	x_3	x_4
1	x_2	9/5	0	1	3/10	-1/10
1	x_1	7/2	1	0	-1/4	1/4
	$z_j - c_j$		0	0	1/20	3/20

5. (a) (i) Define a convex function on a convex set $S \subseteq R^n$.
Is the function $f(x) = -|x|, x \in R$, convex? Justify your answer using definition.
(ii) Suppose that $f(x)$ and $g(x)$ are concave functions defined on a convex set $S \subseteq R^n$.
Show that the sum of these two concave functions, $(f + g)(x)$ is also a concave function on $S \subseteq R^n$.
- (b) Examine the convexity/strict convexity for the following functions:
(i) $f(x_1, x_2) = 2x_1^2 + x_2^2 + 4x_1x_2$
(ii) $f(x_1, x_2) = 4x_1^2 + x_2^2 + 4x_1x_2$
6. (a) (i) Define a quasi-convex function on a convex set $S \subseteq R^n$. Give an example of a quasi-convex function which fails to be continuous in its domain.
(ii) Define a pseudolinear function on a convex set $S \subseteq R^n$. Give an example of a pseudolinear function.
- (b) (i) Show that sum of two pseudo-convex functions need not be a pseudo-convex function on R .
(ii) Solve the following linear fractional programming problem by the simplex algorithm:

$$\begin{aligned} \text{Max } Z &= \frac{x_1}{x_1 + x_2 + 1} \\ \text{subject to } & x_1 + x_2 \leq 1 \\ & x_1 + 2x_2 \leq 1 \\ & x_1, x_2 \geq 0 \end{aligned}$$

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5419

L

Unique Paper Code : 2354000019

Name of the Paper : Optimization Techniques

Name of the Course : Common Prog Group-NEP-UGCF 2022

Semester : VIII

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory and carry equal marks.
3. Attempt any **one** part from each question.

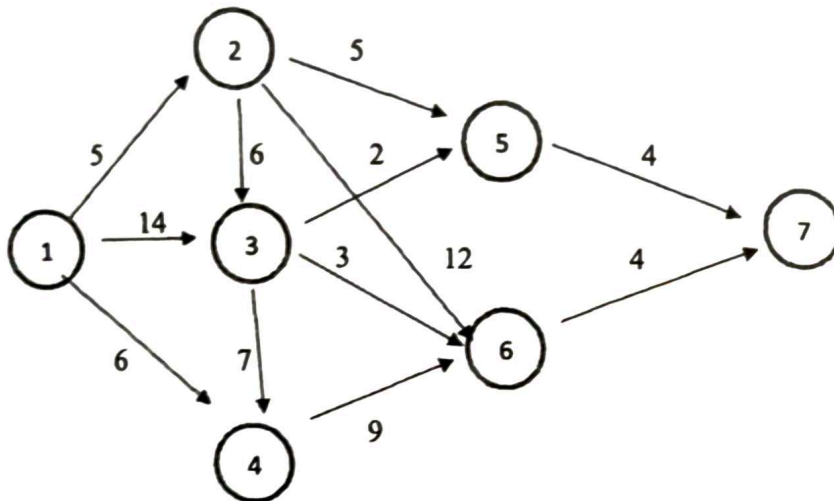
1.(a) A firm has two factories X and Y. It wishes to ship its products from its two factories to three retail stores A, B and C. The number of units available at factories X and Y are 200 and 300 respectively, while the demands at retail stores A, B and C are 100, 150 and 250 respectively. Rather than shipping directly from factories to retail stores, it is added to investigate possibility of transshipment. The unit transportation cost (in rupees) is given in the following table. Find the optimal shipping schedule.

		Source		Destination		
		X	Y	A	B	C
Source	X	0	8	7	8	9
	Y	6	0	5	4	3
Destination	A	7	2	0	5	1
	B	1	5	1	0	4
	C	8	9	7	8	0

(b) A 4-ton vessel can be loaded with one or more three items. The following table gives the unit weight w_i in tons and the unit revenue in thousands of dollars r_i , for item. How many units of each item be loaded to maximize the value?

Item (i)	Weight (w)	Value (r)
1	1	20
2	2	40
3	3	60

2. (a) Suppose we want to select the shortest highway route between two cities. The network in given figure provides the possible route between the starting city node '1' and the destination city at node '7'. Solve by the method of forward recursive equation and use it to find the optimal solution for the following data:



(b) In a cargo loading problem, there are 4 items of different weights/unit and different values/unit as given below:

Item (i)	Weight (w)	Value (v)
1	2	2
2	3	5
3	4	7
4	5	9

The maximum cargo load is restricted to 17. How many units of each item be loaded to maximize the value?

3. (a) Solve the following ILPP by using Gomory's Cutting Plane Method

$$\text{Max } Z = 2x_1 + x_2$$

$$\text{subject to } x_1 + x_2 \leq 5$$

$$36x_1 + 2x_2 \leq 21$$

$$x_1, x_2 \geq 0 \text{ and both are integers.}$$

(b) Consider the following ILLP

$$\begin{aligned} \text{Max } Z &= x_1 + 4x_2 \\ \text{subject to } 2x_1 + 4x_2 &\leq 7 \\ 5x_1 + 3x_2 &\leq 15 \\ x_1, x_2 &\geq 0 \text{ and both are integers} \end{aligned}$$

- (i) Convert the above defined problem into the standard Linear programming problem.
 (ii) Write the two LPP's by branching from x_2 and find the next branching variable, if the initial solution is $x_1 = 0$ and $x_2 = \frac{7}{4}$, the maximum value of $Z = 7$, with the last simplex table as follows:

c_j			1	4	0	0
c_B	B	x_B	x_1	x_2	s_1	s_2
4	x_2	7/4	1/2	1	1/4	0
0	s_2	39/4	7/2	0	-3/4	1
$z_j - c_j$			1	0	1	0

4. (a) Consider the following ILPP

$$\begin{aligned} \text{Max } Z &= x_1 - x_2 \\ \text{subject to } x_1 + 2x_2 &\leq 4 \\ 6x_1 + 2x_2 &\leq 9 \\ x_1, x_2 &\geq 0 \text{ and both are integers.} \end{aligned}$$

- (i) Find the optimal solution of the LPP by using the simplex method.
 (ii) Solve the given ILPP graphically.

(b) Consider the following optimal simplex tableau of the associated Linear Programming Problem for a given AILP (with maximization from and x_3, x_4 as slack variables) with $\text{Max } Z = 18.75$. Generate the Gomory's cut through Z and hence solve the given AILP.

c_j			5	2	0	0
c_B	B	x_B	x_1	x_2	x_3	x_4
2	x_2	1.25	0	1	0.75	-0.5
5	x_1	3.25	1	0	-0.25	0.5
$z_j - c_j$			0	0	0.25	1.5

5. (a) (i) Define a convex function on a convex set $S \subseteq R^n$. Is the function $f(x) = x^2, x \in R$, convex? Justify your answer using definition.
(ii) Suppose that $S \subseteq R^n$ be a convex set and $f: S \rightarrow R$. Prove that f is a convex function on S if and only if its epigraph E_f is a convex set.
- (b) (i) Is every convex function continuous? Explain with an example.
(ii) Using Charnes and Cooper algorithm, solve the following linear fractional programming problem:

$$\begin{aligned} \text{Max } Z &= \frac{x_1}{x_1 + x_2 + 1} \\ \text{subject to } & x_1 + x_2 \leq 1 \\ & x_1 + 2x_2 \leq 1 \\ & x_1, x_2 \geq 0 \end{aligned}$$

6. (a) (i) For the following problem, check whether it is a convex programming problem with justification:

$$\begin{aligned} \text{Max } & x_2 \\ \text{subject to } & x_1^2 + x_2^2 \leq 1 \\ & x_1^2 \geq x_2 \end{aligned}$$

- (ii) Define a quasilinear function on a convex set $S \subseteq R^n$. Give an example of a quasilinear function.
- (b) (i) Define a quasi-convex function on a convex set $S \subseteq R^n$.
Is the function $f(x) = [x], x \in R$, quasi-convex? Justify your answer. Give an example of a quasi-convex function on R which is not pseudo-convex.
- (ii) Define a pseudo-concave function on a convex set $S \subseteq R^n$. Give an example of a pseudo-concave function.

43.5
[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 7385

L

Unique Paper Code : 2353200017

Name of the Paper : Applied Algebra

Name of the Course : NEP UGCF-2022 BACHELOR OF ARTS/
BACHELOR OF SCIENCE (PROGRAMME)

Semester : VIII

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All Questions are compulsory.
3. Attempt any two parts from each question.

1. (a) An economy consists of two sectors: Goods and Services.
Each year:

Goods sector sells 80% of its output to Services and retains the rest
Services sector sells 70% of its output to Goods and retains the rest.

- (i) Construct the exchange table for this economy.
- (ii) Form a system of linear equations representing the condition that income equals expenditure for each sector.
- (iii) Solve the system to find the equilibrium prices of the outputs of Goods and Services.
- (iv) Interpret your solution economically.

- (b) Solve the Leontief Production equation for an economy with three sectors, given that:

$$\text{Consumption matrix } C = \begin{bmatrix} 0.2 & 0.2 & 0.0 \\ 0.3 & 0.1 & 0.3 \\ 0.1 & 0.0 & 0.2 \end{bmatrix} \text{ and final demand vector } d = \begin{bmatrix} 50 \\ 30 \\ 20 \end{bmatrix}$$

- (c) Use matrix multiplication to find the image of the triangle with data matrix

$$D = \begin{bmatrix} 5 & 2 & 4 \\ 0 & 2 & 3 \end{bmatrix}$$

under the transformation that reflects points through the y-axis. Sketch or give the vertices of both the original triangle and its image.

P.T.O.

2. (a) Solve the difference equation

$$y_{k+3} - 2y_{k+2} - 5y_{k+1} + 6y_k = 0$$

- (i) Find the auxiliary equation.
 (ii) Determine the general solution.
 (iii) State the dimension of the solution space.

- (b) Let $P = \begin{bmatrix} 0.6 & 0.3 \\ 0.4 & 0.7 \end{bmatrix}$ be the stochastic matrix.

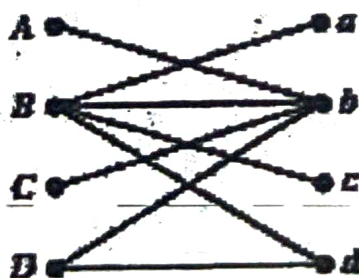
- (i) Find all vectors x such that $Px = x$.
 (ii) Determine a steady-state vector for P and interpret the result.

- (c) Find a least-square solution of the inconsistent system $Ax = b$ for

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$$

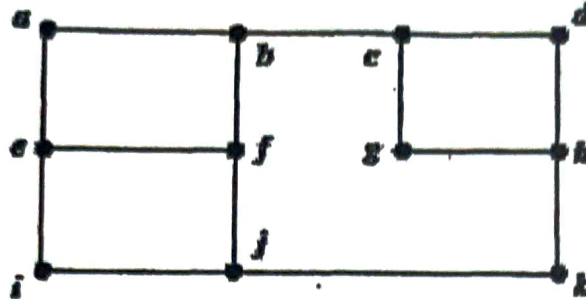
3. (a) A drug company wishes to test four different pain relievers A, B, C and D on four groups of objects each day from Monday to Thursday. Use Latin Square to design an experiment so as to reduce the effects due to difference between the objects and different days.

- (b) Find all possible matching jobs or explain why none exists:

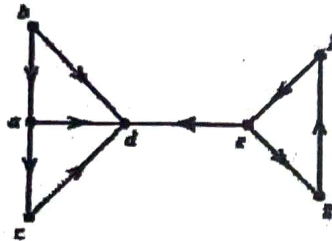


- (c) Three pitchers of sizes 10L, 4L and 7L are given. If initially 10L pitcher is full and other two empty, find a minimal sequence of pouring so as to have exactly 2L in either the 7L or the 4L pitcher.

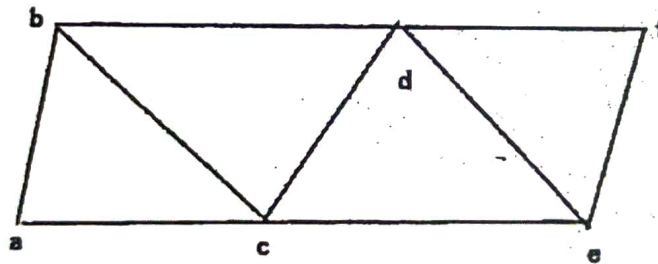
4. (a) The following figure represents a section of city's street map. We want to position police at corners (vertices) so that they can keep every block (edge) under surveillance i.e., every edge should have a policeman at least one of its corners. What is the smallest number of police that can do this job?



- (b) Find a vertex basis for the following graph. Justify your answer.



- (c) Find a minimal edge cover for the graph:



5. (a)

- (i) Suppose the prime factorization of an integer n contains the prime factors 2, 3 and 11. That is, $n = 2^r 3^s 11^t$ for positive integers $r \geq 1, s \geq 1$ & $t \geq 1$.

Show that (1) 6 divides n
 (2) 22 divides n
 (3) 7 does not divide n
 (4) 35 does not divide n

P.T.O.

- (ii) Calculate $(12.18) \pmod{6}$ and $33^7 \pmod{10}$
- (b) Explain how the check digit is computed in the US postal money order system. Verify whether 10962801047 is valid using the US postal money order check digit scheme.
- (c) The number 0 – 037 – 1222494315 is used to identify an Airplane ticket. Use the Airplane Ticket check digit scheme to assign a check digit to this number and write out the entire document number.
6. (a) Define the symmetry and rigid motion of an object. What are the different types of rigid motions. Explain with the help of examples.
- (b) Define a regular polygon. Examine the six symmetries of a triangle in detail.
- (c) Define the Verhoeff check digit scheme. Show that the identification number 3170092 is a valid number using the Verhoeff scheme.

[This question paper contains 2 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 7386

L

Unique Paper Code : 2353200018

Name of the Paper : Elements of Partial Differential Equations

Name of the Course : DSE-Bachelor of Arts / Bachelor of Science
(Programme)

Semester : VIII

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Answer any **two** parts from each question.
3. All questions carry equal marks.

Q.1 (a) Find the integral surface of the linear partial differential equation

$$x(y^2 + z) \frac{\partial z}{\partial x} - y(x^2 + z) \frac{\partial z}{\partial y} = (x^2 - y^2)z$$

(b) Find the solution to the equation $z = \frac{1}{2}(p^2 + q^2) + (xp = yq, y^2 = z(xp + yq) = 2xy$.

(c) Find the general solution to the differential equation

$$x(z + 2a) \frac{\partial z}{\partial x} + (xz + 2yz + 2ay) \frac{\partial z}{\partial y} = z(z + a).$$

Q.2 (a) Find the surface which intersects the surfaces of the system $z(x + y) = c(3z + 1)$.

Orthogonally and which passes through the circle $x^2 + y^2 = 1, z = 1$.

(b) Find the solution to the equation $(y + zx)u_x - (x + yz)u_y = x^2 - y^2$

(c) Find the solution to the equation $z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$. which passes through the x -axis.

Q.3 (a) Find the solution to the characteristic equations $pq = z$ and determine the integral surface which passes through the parabola $x = 0, y^2 = z$

(b) Show that the equations $xp = yq, z(xp + yq) = 2xy$ are compatible and solve them.

(c) If $z = f(x^2 - y) + g(x^2 + y)$ where the functions f, g are arbitrary, prove that

$$\frac{\partial^2 z}{\partial x^2} - \frac{1}{x} \frac{\partial z}{\partial x} = 4x^2 \frac{\partial^2 z}{\partial y^2}$$

Q.4 (a) Show that if f and g are arbitrary functions of a single variable, then

$u = f(x - vt + i\alpha y) + g(x - vt - i\alpha y)$ is a solution of the equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} \quad \text{provided that } \alpha^2 = 1 - \frac{v^2}{c^2}$$

(b) Find the solution of $(D - D' - 1)(D - D' - 2)z = e^{2x-y}$

(c) Solve using Monge's method

$$(q^2 - 1)zr - 2pqzs + (p^2 - 1)zt + z^2(rt - s^2) = p^2 + q^2 - 1$$

Q.5 (a) Find the solution of $4D^2 + 12DD' + 9D'^2)z = 0$

(b) Find the solution of $r + 3s + t + (rt - s^2) = 1$

(c) Determine the solution of wave equation with non-homogeneous boundary -value problem

$$u_{tt} - 4u_{xx} = 0, \quad x > 0, t > 0,$$

$$u(x, 0) = \log(1+x), \quad x \geq 0,$$

$$u_t(x, 0) = 1, \quad x \geq 0,$$

$$u(0, t) = t, \quad t \geq 0,$$

Q6 (a) Determine the solution of the initial-value problem for a semi-infinite string with fixed end

$$u_{tt} - 4u_{xx} = 0, \quad 0 < x < \infty, t > 0,$$

$$u(x, 0) = x^4, \quad 0 \leq x < \infty,$$

$$u_t(x, 0) = 0, \quad 0 \leq x < \infty,$$

$$u(0, t) = 0, \quad 0 \leq t < \infty.$$

(b) Determine the solution of Cauchy problem for non-homogeneous wave equation:

$$u_{tt} = c^2 u_{xx} + h(x, t), \quad x \in \mathbb{R}, t > 0$$

with initial condition $u(x, 0) = f(x), u_t(x, 0) = g(x), x \in \mathbb{R}$

(c) Find the solution of wave equation with non-homogeneous boundary conditions:

$$u_{tt} - c^2 u_{xx} = 0, \quad x > 0, t > 0,$$

$$u(x, 0) = f(x), u_t(x, 0) = g(x), \quad x \geq 0,$$

$$u(0, t) = p(t), \quad t \geq 0.$$

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 7388

L

Unique Paper Code : 2353200020

Name of the Paper : Optimization Techniques

Name of the Course : Bachelor of Arts/Bachelor of Science (Programme)

Semester : VIII

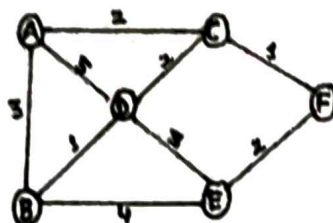
Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of calculator (not scientific) is allowed.

1(a)



To develop forward recursive equation and use it find the shortest path from A to E.

(7.5)

1(b) Solve the following Cost Minimizing Transportation problem.

	44	13	15
24	2	1	3
14	4	6	5
34	8	9	7

(7.5)

1(c) Find the optimal solution of the fractional knapsack problem with $M=15$, $n=7$

Object	1	2	3	4	5	6	7
Wright	1	3	5	4	1	3	2
Profit	5	10	15	7	8	9	4

(7.5)

2(a) Consider the following transshipment problem with two sources and destinations, the costs for shipment in rupees are given below. Determine an optimal shipment schedule:

		Source		Destination		Supply
		S_1	S_2	D_1	D_2	
Source	$\{S_1$	0	1	6	2	4
	$\{S_2$	1	0	8	2	5
Destination	$\{D_1$	0	7	0	3	—
	$\{D_2$	1	0	1	0	—
Demand		—	—	3	6	

(7.5)

2(b) In a cargo-loading problem, there are three items of different per unit weight and revenue as given below

Item i	$w_i(\text{ton})$	$r_i(\$)$
1	4	70
2	1	20
3	2	40

Maximum cargo loaded is restricted to 8 tons. The goal is to determine the number of units of each item that will maximize the total return. (7.5)

2(c) State the 'Principle of optimality' in dynamic programming and give a mathematical formulation of a dynamic programming problem. (7.5)

3 (a) Solve the integer linear programming problem graphically

$$\text{Max } z = x_1 - x_2$$

$$\text{subject to } x_1 + 2x_2 \leq 4 \quad ,$$

$$6x_1 + 2x_2 \leq 9 \quad ;$$

$$x_1, x_2 \geq 0, \quad x_1 \text{ and } x_2 \text{ integers} \quad (7.5)$$

3(b) Solve the integer linear programming problem using Gomory's cutting plane methods

$$\text{Max } z = x_1 + x_2$$

$$\text{subject to } x_1 + 2x_2 \leq 4 \quad ,$$

$$x_2 \leq 2 \quad ;$$

$$x_1, x_2 \geq 0, \quad x_1 \text{ and } x_2 \text{ integers} \quad (7.5)$$

3 (c) Solve the mixed integer linear programming problem

$$\begin{aligned}
 \text{Max } z &= 2x_1 + x_2 \\
 \text{subject to } x_1 + x_2 &\leq 5 \\
 6x_1 + 2x_2 &\leq 21 \\
 x_1, x_2 &\geq 0 \quad x_1 \text{ is an integer}
 \end{aligned} \tag{7.5}$$

4(a) Solve the mixed integer linear programming problem

$$\begin{aligned}
 \text{Min } z &= x_1 - 3x_2 \\
 \text{subject to } x_1 + x_2 &\leq 5 \\
 -2x_1 + 4x_2 &\leq 11 \\
 x_1, x_2 &\geq 0 \text{ and } x_2 \text{ is an integer}
 \end{aligned} \tag{7.5}$$

4(b) Use branch and bound method to solve the integer linear programming problem

$$\begin{aligned}
 \text{Max } z &= 3x_1 + 4x_2 \\
 \text{subject to } 7x_1 + 16x_2 &\leq 52, \\
 3x_1 - 2x_2 &\leq 9; \\
 x_1, x_2 &\geq 0, \quad x_1 \text{ and } x_2 \text{ integers}
 \end{aligned} \tag{7.5}$$

4(c) Use branch and bound method to solve the integer linear programming problem

$$\begin{aligned}
 \text{Max } z &= x_1 + 2x_2 \\
 \text{subject to } x_1 + 2x_2 &\leq 12, \\
 4x_1 + 3x_2 &\leq 14; \\
 x_1, x_2 &\geq 0, \quad x_1 \text{ and } x_2 \text{ integers.}
 \end{aligned} \tag{7.5}$$

5(a) Define concavity of a function. Let S be a non-empty convex set in $\mathbb{R}$. Examine the concavity/convexity of the function

$$f(x) = 6x - x^2 - 3 \text{ for } x \in \mathbb{R}. \tag{7.5}$$

5(b) Let S be a non-empty convex subset of $\mathbb{R}^n$ and let $f : S \rightarrow \mathbb{R}$ be a convex function

and $\alpha \geq 0$. Then, prove that $h(x) = \alpha f(x)$ is a convex function. (7.5)

5(c) Let $S \subseteq \mathbb{R}^3$ be a convex set and $f : S \rightarrow \mathbb{R}$ be given by

$$f(x_1, x_2, x_3) = -x_1^2 + 2x_1x_2 + 3x_1x_3 + 6x_2x_3, \quad (x_1, x_2, x_3) \in \mathbb{R}^3.$$

Examine the convexity/concavity of f . (7.5)

6(a) Is the following problem a convex programming problem? Give reasons for your answer.

$$\begin{aligned} \text{Max } & x_2 \\ \text{subject to } & x_1^2 + x_2^2 \leq 1 \\ & x_1^2 \geq x_2 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Also, show it geometrically. (7.5)

6(b) Solve the following linear fractional programming problem by the Charnes and Cooper Method

$$\begin{aligned} \text{Max } & z = \frac{2x_1 + x_2}{-x_1 - 2x_2} \\ \text{subject to } & x_1 + x_2 \leq 6 \\ & 2x_1 + x_2 \geq 4 \\ & x_1, x_2 \geq 0. \end{aligned} \quad (7.5)$$

6(c) Solve the following linear fractional programming problem by the simplex Algorithm:

$$\begin{aligned} \text{Max } & z = \frac{2x_1 + 3x_2}{x_1 + x_2 + 7} \\ \text{subject to } & 3x_1 + 5x_2 \leq 15 \\ & 4x_1 + 3x_2 \leq 12 \\ & x_1, x_2 \geq 0. \end{aligned} \quad (7.5)$$

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 8063

L

Unique Paper Code : 2353200017

Name of the Paper : Applied Algebra

Name of the Course : NEP UGCF-2022 BACHELOR OF ARTS/
BACHELOR OF SCIENCE (PROGRAMME)

Semester : VIII

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All Questions are compulsory.
3. Attempt any two parts from each question.

1. (a) An economy consists of three sectors: Manufacturing, Agriculture, and Services. The input requirements per unit of output of each sector are given below:

Purchased From/Produced By	Manufacturing	Agriculture	Services
Manufacturing	0.50	0.40	0.20
Agriculture	0.20	0.30	0.10
Services	0.10	0.10	0.30

Suppose the Manufacturing sector plans to produce 100 units of output.

- (i) Write the unit consumption vector for the Manufacturing sector.
- (ii) Determine the intermediate demand vector generated by the Manufacturing sector.
- (iii) Explain briefly how this model illustrates the structure of interdependence in an economy.

- (b) Consider an economy with two sectors having production model as

$$X = Cx + d$$

where C is the consumption matrix given by $\begin{bmatrix} 0 & 5 \\ 6 & 2 \end{bmatrix}$ and d is the final demand vector given by $\begin{bmatrix} 50 \\ 30 \end{bmatrix}$

Use an inverse matrix to determine the production level necessary to satisfy the final demand.

P.T.O.

- (c) Any linear transformation of $\mathbb{R}^2$ can be represented in homogeneous coordinates by a matrix of the form $\begin{bmatrix} A & 0 \\ 0 & 1 \end{bmatrix}$, where A is a 2×2 matrix.

Explain how homogeneous coordinates are used to represent linear transformations in $\mathbb{R}^2$ and write the homogeneous coordinate matrix for the transformation: Reflection about the line $y = x$.

2. (a) Consider the difference equation:

$$y_{k+3} - 2y_{k+2} - 5y_{k+1} + 6y_k = 0$$

Assume a solution of the form $y_k = r^k$ and derive the auxiliary equation.

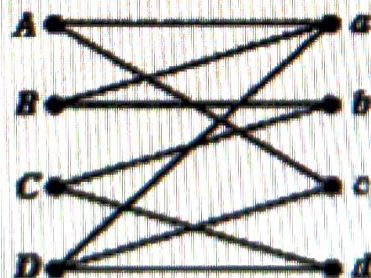
- (b) Consider a Markov Chain with transition matrix

$$P = \begin{bmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{bmatrix}$$

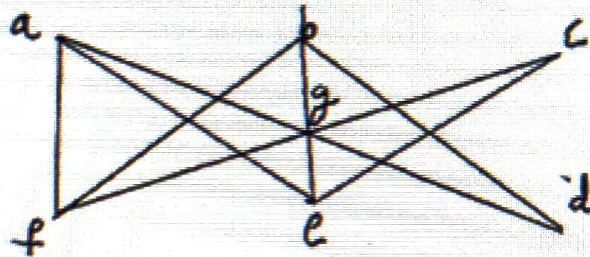
- (i) Compute P^2 and P^3 and describe the pattern observed in the powers of P .
 (ii) Based on the pattern, determine whether the Markov Chain appears to approach a steady-state matrix.
- (c) Determine the least-squares error in the least-squares solution of $Ax = b$ for

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$$

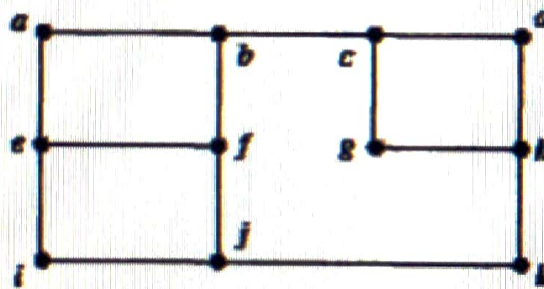
3. (a) Find all matchings of jobs for the following graph:



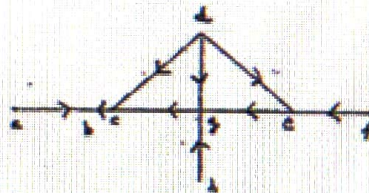
- (b) Prove that the table for any finite group $(G, +)$ of order n is a Latin square of order n based on G .
- (c) Find a minimal edge cover for the following graph. Give a detailed logical analysis.



4. (a) Find a largest independent set in the graph:

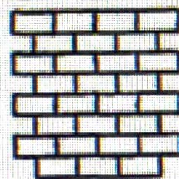


- (b) Find a vertex basis for the following graph. Justify your answer.



- (c) A supermarket wishes to test the effect of putting cereals on four shelves at differential heights, show how to design such an experiment lasting four weeks and using four brands of cereals.
5. (a) Define the 14 digit Airline Ticket check digit scheme. Explain the cases where the Airline Ticket check digit scheme fails to catch single-digit errors.
- (b) Using the ISBN check digit scheme, determine which of the following is a valid ISBN and which is invalid.
 $0 - 201 - 52032 - X$ and $3 - 357 - 02001 - 4$

- (c) Define the 12-character Washington State Driver's License number check digit scheme. Create a Washington State Driver's License number, complete with check digit, for Joseph Marshall Pollan born on October 19, 1970.
6. (a) Explain mathematically how the UPC check digit scheme catches all single-digit errors.
- (b) Define a regular polygon. Examine the twelve symmetries of a hexagon in detail.
- (c) Define Glide Reflective rigid motion. Explain the generation of the pattern below by using rigid motion:



[This question paper contains 2 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 8064

L

Unique Paper Code : 2353200018

Name of the Paper : Elements of Partial Differential Equations

Name of the Course : DSE-Bachelor of Arts / Bachelor of Science
(Programme)

Semester : VIII

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Answer any two parts from each question.
3. All questions carry equal marks.

Q.1 (a) Find the solution of the system of surface which cut orthogonally the cones of the system
 $x^2 + y^2 + z^2 = cxy$.

(b) Find the characteristics of the equation $pq = z$, and determine the integral surface which passes through the parabola $x = 0, y^2 = z$.

(c) Show that the equation $z = px + qy$ is compatible with any equation $f(x, y, z, p, q) = 0$ that is homogeneous in x, y and z . Solve completely the simultaneous equations
 $z = px + qy, 2xy(p^2 + q^2) = z(yq + xp)$

Q.2 (a) Find the complete integrals of the partial differential equations $z^2 = pqxy$.

(b) Find the solution to the equation $p^2q^2 + x^2y^2 = x^2q^2(x^2 + y^2)$.

(c) Find the integral surface of the differential equation $(y + zq)^2 = z^2(1 + p^2 + q^2)$ circumscribed about the surface $x^2 - z^2 = 2y$.

Q.3 (a) Find the solution of the equations $xp - yq = x, x^2p + q = xz$ is compatible and find their solution.

(b) Find the complete integrals of the partial differential equations $(p^2 + q^2)x = pz$ and deduce the solution which passes through the curve $x = 0, z^2 = 4y$.

(c) Show that a family of spheres $x^2 + y^2 + (z - c)^2 = r^2$, satisfies the partial differential equation $yp - xq = 0$

Q.4 (a) Find the solution of $r + 2s + t + 2p + 2q + z = 0$

(b) Solve using Monge's method $3s + (rt - s^2) = 2$

(c) Find the solution of $r - s + 2q - z = x^2 y^2$

Q.5 (a) Find the solution of $(D - D^2)z = \cos(x - 3y)$

(b) Find the solution of $y^2 r - 2ys + t = p + 6y$

(c) Solve the wave equation $u_{tt} - c^2 u_{xx} = 0$

with initial conditions $u(x, 0) = 0$, $u_t(x, 0) = 1$.

Q.6 (a) Derive the one-dimensional wave equation of a string

$$u_{tt} = c^2 u_{xx} + F,$$

where, $c^2 = T / \rho$, and T is the tension at the end points

$F = f / \rho$, and f be the external force, acting on the string per unit length.

(b) Determine the solution of wave equation with non-homogeneous boundary value problem:

$$u_{tt} - 9u_{xx} = 0, \quad x > 0, t > 0,$$

$$u(x, 0) = \log(1+x), u_t(x, 0) = 1, \quad x \geq 0,$$

$$u(0, t) = t, \quad t \geq 0.$$

(c) Determine the solution of Cauchy problem for non-homogeneous wave equation:

$$u_{tt} = u_{xx} - 2, \quad x \in \mathbb{R}, t > 0,$$

with initial condition $u(x, 0) = \sin x$, $u_t(x, 0) = x$, $x \in \mathbb{R}$.

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 8743

L

Unique Paper Code : 2353200020

Name of the Paper : Optimization Techniques

Name of the Course : Bachelor of Arts/ Bachelor of Science (Programme)

Semester : VIII

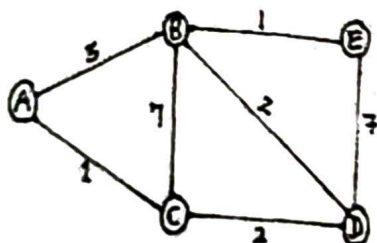
Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of calculator (not scientific) is allowed.

1(a) Find the shortest path in the weighted graph by Dijkstra's algorithm with A as the source node to node E.



(7.5)

1(b) Use North west Corner rule to find a starting solution to the following balanced transportation problem

	13	18	14
10	2	1	4
15	6	3	2
20	4	2	3

(7.5)

1(c) Solve the following 0-1 knapsack problem

Object	1	2	3	4
Wright	5	2	4	3
Profit	20	30	15	10

(7.5)

2(a) Consider the following transshipment problem with two source and destination, the cost for shipment in rupees are given below. Determine an optimal shipment schedule:

		Source		Destination		Supply
		S_1	S_2	D_1	D_2	
Source	$\{S_1$	0	1	6	2	4
	$\{S_2$	1	0	8	2	5
Destination	$\{D_1$	0	7	0	3	—
	$\{D_2$	1	0	1	0	—
Demand		—	—	3	6	

(7.5)

2(b) A wilderness hiker must pack three item: food, first-aid kits, and cloths. The backpack has a capacity of 3 ft^3 . Each unit food takes 1 ft^3 . A first-aid kit occupies $1/4 \text{ ft}^3$, and each piece of cloth takes about $1/2 \text{ ft}^3$. The hiker assigns the priority weights 3, 4, and 5 to food, first-aid, and clothes, respectively, which means that clothes are the most valuable of the three items. From experience, the hiker must take at least one unit of each and no more than two first-aid kits. How many of each item should the hiker take? (7.5)

2(c) Describe recursive equation approach to solve dynamic programming problem. (7.5)

3 (a) Solve the integer linear programming problem graphically

$$\text{Max } z = 15x_1 + 32x_2$$

$$\text{Subject to } 7x_1 + 16x_2 \leq 52 \quad ,$$

$$3x_1 - 2x_2 \leq 9$$

$$x_1, x_2 \geq 0 \quad x_1 \text{ and } x_2 \text{ are integers} \quad (7.5)$$

3(b) Solve the integer linear programming problem using Gomory's cutting plane method

$$\text{Max } z = 4x_1 + 3x_2$$

$$\text{Subject to } 3x_1 + 4x_2 \leq 12 \quad ,$$

$$4x_1 + 2x_2 \leq 9$$

$$x_1, x_2 \geq 0 \quad x_1 \text{ and } x_2 \text{ integers} \quad (7.5)$$

3 (c) Solve the integer linear programming problem using Gomory's cutting plane method

$$\text{Max } z = x_1 + x_2$$

$$\text{subject to } 6x_1 + 4x_2 \leq 24 \quad ,$$

$$x_1 + 2x_2 \leq 6$$

$$x_1, x_2 \geq 0 \text{ and integer} \quad (7.5)$$

4(a) Solve the mixed integer linear programming problem

$$\text{Min } z = x_1 + x_2$$

$$\text{subject to } 3x_1 + 2x_2 \leq 5$$

$$x_2 \leq 2$$

$$x_1, x_2 \geq 0 \text{ and } x_1 \text{ is an integer} \quad (7.5)$$

4(b) Use branch and bound method to solve the integer linear programming problem

$$\text{Max } z = x_1 + x_2$$

$$\text{subject to } 2x_1 + 5x_2 \leq 16$$

$$6x_1 + 5x_2 \leq 30$$

$$x_1, x_2 \geq 0, x_1 \text{ and } x_2 \text{ are integers} \quad (7.5)$$

4(c) Use branch and bound method to solve the integer linear programming problem

$$\text{Max } z = 3x_1 + 4x_2$$

$$\text{subject to } 7x_1 + 16x_2 \leq 52$$

$$3x_1 - 2x_2 \leq 9$$

$$x_1, x_2 \geq 0, x_1 \text{ and } x_2 \text{ are integers} \quad (7.5)$$

5(a) Define concavity of a function. Let S be a non-empty convex set in $\mathbb{R}$. Check whether the function $f(x) = e^{-x}$ for $x \in \mathbb{R}$ is concave or convex. (7.5)

5(b) Let $S \subseteq \mathbb{R}^n$ be a convex set and let $f : S \rightarrow \mathbb{R}$. Prove that f is convex if and only if its epigraph is a convex set. (7.5)

5(c) Let $S \subseteq \mathbb{R}^2$ be an open convex set and $f : S \rightarrow \mathbb{R}$ be given by

$$f(x_1, x_2) = x_1^2 + 2x_2^2 + 2x_1x_2 - 5x_1 + 4x_2, (x_1, x_2) \in \mathbb{R}^2. \text{ Examine the convexity/strict convexity of } f. \quad (7.5)$$

6(a) Is the following problem a convex programming problem? Give reasons for your answer.

$$\text{Min } x_1^2 + x_2$$

$$\text{subject to } |x_1| + |x_2| \leq 2$$

$$x_1^2 \geq x_2.$$

Also, show it geometrically. (7.5)

6(b) Solve the following linear fractional programming problem by the Charnes and Cooper Method

$$\text{Max } z = \frac{x_1}{x_1 + x_2 + 1}$$

$$\text{subject to } x_1 + x_2 \leq 1$$

$$x_1 + 2x_2 \leq 1$$

$$x_1, x_2 \geq 0.$$

(7.5)

6(c) Solve the following linear fractional programming problem by the simplex Algorithm:

$$\text{Max } z = \frac{x_2 - 5}{-x_1 - x_2 + 9}$$

$$\text{subject to } 2x_1 + 5x_2 \geq 10$$

$$4x_1 + 3x_2 \leq 20$$

$$-x_1 + x_2 \leq 2$$

$$x_1, x_2 \geq 0.$$

(7.5)

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 10039

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Unique Paper Code : 2353200017

Name of the Paper : Applied Algebra

Name of the Course : NEP UGCF-2022 BACHELOR OF ARTS/
BACHELOR OF SCIENCE (PROGRAMME)

Semester : VIII

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All Questions are compulsory.
3. Attempt any two parts from each question.

1. (a) An economy consists of three sectors: Coal, Electric (power), and Steel. The distribution of output of each sector among all the sectors is given below:

Purchased By/Produced By	Coal	Electric	Steel
Coal	0.0	0.4	0.6
Electric	0.6	0.1	0.2
Steel	0.4	0.5	0.2

Let P_C, P_E, P_S denote the prices of the total annual outputs of the Coal, Electric and Steel sectors respectively.

- (i) Form a system of linear equations representing the condition that income of each sector equals its expenditure.
- (ii) Solve the system using Row Reduction Method.
- (iii) Find the general equilibrium price vector.
- (iv) Interpret the economic meaning of your solution.

P.T.O.

- (b) Consider an economy consisting of three sectors: Manufacturing, Agriculture, and Services. The consumption matrix of the economy is given by

$$C = \begin{bmatrix} 0.50 & 0.40 & 0.20 \\ 0.20 & 0.30 & 0.10 \\ 0.10 & 0.10 & 0.30 \end{bmatrix}$$

Suppose the final demand vector is $d = \begin{bmatrix} 50 \\ 30 \\ 20 \end{bmatrix}$

- (i) Write the Leontief input-output model for this economy.
 - (ii) Form the system of linear equations corresponding to the model.
 - (iii) Solve for the production vector x using matrix method.
 - (iv) Interpret the economic meaning of the production vector obtained.
- (c) Define homogeneous coordinates and explain their importance in computer graphics. Show how a translation

$$(x, y) \rightarrow (x + h, y + k)$$

can be represented using matrix multiplication in homogeneous coordinates.

2. (a) Verify that $y_k = k^2$ is a solution of the difference equation

$$y_{k+2} - 4y_{k+1} + 3y_k = -4k, \forall k$$

Find the general solution of the equation.

- (b) Consider a system whose state is described by a Markov Chain:

$$x_{k+1} = Px_k, \quad k = 0, 1, 2, \dots$$

where the transition matrix is

$$P = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.3 & 0.8 & 0.3 \\ 0.2 & 0 & 0.4 \end{bmatrix}$$

and the initial state vector is

$$x_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Compute the state vectors x_1, x_2, x_3 and continue the process to obtain x_k for sufficiently large k (upto $k = 10$).

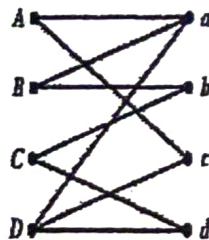
- (c) Find a least-square solution of the system $Ax = b$ for

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}, b = \begin{bmatrix} -3 \\ -1 \\ 0 \\ 2 \\ 5 \\ 1 \end{bmatrix}$$

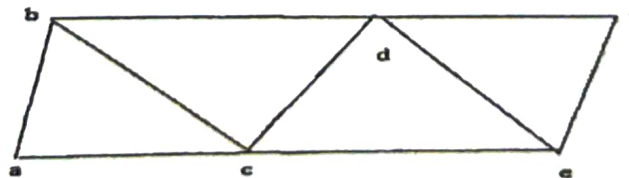
3. (a) Show that the Latin square illustrated in the table cannot be obtained from a group table.

A	B	C	D	E
B	A	E	C	D
C	D	A	E	B
D	E	B	A	C
E	C	D	B	A

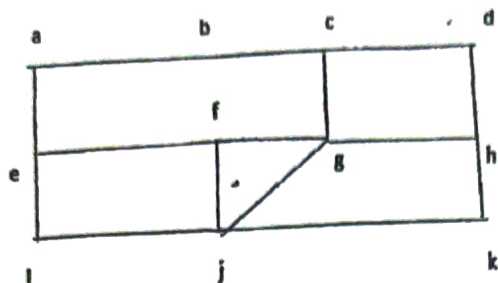
- (b) Find a matching or explain why none exists for the following graph:



- (c) Find a maximal independent set in the following graph:



4. (a) Find a minimal edge cover for the following graph. Give a detailed logical analysis.



- (b) Define a Latin square. Give examples. Construct a Latin square of order 7 on $\{a, b, c, d, e, f, g\}$.
- (c) Define an independent set and an edge cover in a graph. Give examples. State a relationship between an independent set and an edge cover in a graph.
5. (a) The US postal money order check digit scheme detects errors in identification Numbers. Discuss whether this scheme detects all single-digit errors. If not, identify the cases where it fails and explain the mathematical reason behind the failure.
- (b) Using the UPC check digit scheme, assign a check digit to the number 3 - 88207 - 29431. In which cases does the UPC check digit scheme fail to catch transposition-of-adjacent-digits errors? Illustrate with examples.
- (c) Define the ten-digit ISBN check digit scheme. Explain mathematically why this scheme catches all single-digit errors.
6. (a) Define a rigid motion. Show that translation is a rigid motion. Is every rigid motion a translation? Justify your answer with an example.
- (b) Define a regular polygon. Examine the ten symmetries of a pentagon in detail.
- (c) Define the Verhoeff check digit scheme. Show that the identification number 57014 is an invalid number using the Verhoeff scheme.