(1)

[This question paper contains 2 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5753 **K**

Unique Paper Code : 2352011101

Name of the Paper : DSC-1 Algebra

Name of the Course : **B.Sc. (H) Mathematics, UGCF-2022**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory and carry equal marks.
3. Attempt **two** parts from each question.

1. (a) (i) Solve the equation

$$x^3 - 14x^2 - 84x + 216 = 0,$$

given that the roots are in geometrical progression.

(ii) Find an upper limit to the roots of the equation $x^4 - 5x^3 + 7x^2 - 8x + 1 = 0$. (4.5+3)

(b) Find all the integral roots of $x^3 - 5x^2 - 2x + 24 = 0$. (7.5)

(c) Find all the rational roots of $32y^3 - 6y - 1 = 0$. (7.5)

2. (a) Find the value of z, where

$$z = \frac{(1-i)^{10}\left(\sqrt{3}+i\right)^5}{\left(-1-i\sqrt{3}\right)^{10}}$$ (7.5)

(b) Find the fourth roots of $z = \sqrt{3} + i$ and represent them geometrically to show that they lie on a circle of radius $2^{1/4}$. (7.5)

(c) Solve the equation

$$z^{10} + (-2 + i)z^5 - 2i = 0.$$ (7.5)

3. (a) Solve

$$y^3 - 15y - 126 = 0$$

using Cardan's method. (7.5)

(b) Prove that $n^2 - 2$ is never divisible by 4. (7.5)

(c) Let $a = qb + r$ where $a, b, q, r \in \mathbb{Z}$. Show that $\gcd(a,b) = \gcd(b, r)$. Find the gcd of 143 and 481. (7.5)

4. (a) (i) Let $a, b, c \in \mathbb{Z}$ be such that $a|bc$. If a, b are relatively prime, show that $a|c$.

(ii) Find the gcd of $a = 1575$, $b = 231$ and represent it in the form $ma + nb$ for suitable integers m and n. (4+3.5)

(b) Solve the following pair of congruences. If no solution exists, explain why.

$$x + 5y \equiv 3 \pmod 9$$
$$4x + 5y \equiv 1 \pmod 9$$ (7.5)

(c) Let $a \equiv x \pmod n$ and $b \equiv y \pmod n$. Then, show that

(i) $a + b = (x + y) \pmod n$

(ii) $a - b = (x - y) \pmod n$

(iii) $ab = (xy) \pmod n$ (2.5+2.5+2.5)

5. (a) Define a group. Is Z_5 under addition module 5, a group? Justify your answer. (2+5.5)

(b) Let $GL(2, \mathbb{R}) = \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} : a,b,c,d \in \mathbb{R}, |A| \neq 0\right\}$ be the set of all 2×2 matrices over $\mathbb{R}$ with nonzero determinant. Is $GL(2, \mathbb{R})$ a group with respect to matrix multiplication? Is it Abelian? Justify your answer. (5+1+1.5)

(c) Define the Centre Z(G) of a group G. Is Z(G) a subgroup of G? Is it Abelian? Give reasons to support your answer. (5+2.5)

6. (a) Let $G = \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} : a,b,c,d \in \mathbb{Z}\right\}$ be a group under addition and $H \subset G$ be such that $H = \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in G : a+b+c+d = 0\right\}$. Prove that H is a subgroup of G. What if 0 is replaced by 1? (5+2.5)

(b) Define inverse of an element in a group. If G is a group where every element is its own inverse, show that G is commutative. (2+5.5)

(c) Define a cyclic group. List down all subgroups of Z_{10}. Are these subgroups cyclic? What are their generators? What is the order of each subgroup? (1+2+1+2+1.5)

[This question paper contains 4 printed pages.]

Your Roll No.....

K

Sr. No. of Question Paper : 5987

Unique Paper Code : 2352011103

Name of the Paper : DSC-3: Probability and Statistics

Name of the Course : **B.Sc. (H) Mathematics**

Semester : I

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.
4. **All** questions carry equal marks.
5. Use of non-programmable scientific calculator and statistical tables are permitted.

1. (a) The following table represents the running times (in minutes) of 26 randomly selected recent American movies. Construct a stem-and-leaf plot and comment on interesting features of the display.

94	90	95	93	128	95	125	91	104	116	162	102	90
110	92	113	116	90	97	103	95	120	109	91	138	94

(b) The following data represents the actual blood pressure values for nine randomly selected individuals. Compute the median, upper fourth, and lower fourth.

118.6	127.4	138.4	130.0	113.7	122.0	108.3	131.5	133.2

(c) Following data consists of value of sodium content in one serving of cereal for a sample of cereals manufactured by certain company.

211	408	171	178	359	249	205	203	201

(i) Calculate sample mean, sample variance, and sample standard deviation.

(ii) If 50 is added to each observation of the sample to obtain the sample of transformed observations. Compute the sample variance and sample standard deviation of transformed observations without reperforming the calculations.

P.T.O.

2. (a) The route used by a certain motorist in commuting to work contains two intersections with traffic signals. The probability that he must stop at the first signal is 0.4, and the analogous probability for the second signal is 0.5, and the probability that he must stop at at least one of the two signals is 0.7. What is the probability that he must stop

(i) At both signals?

(ii) At the first signal but not at the second one?

(iii) At exactly one signal?

(b) State Baye's theorem. The reviews editor for a certain scientific journal decides whether the review for any particular book should be short (1-2 pages), medium (3-4 pages), or long (5-6 pages). Data on recent reviews indicates that 60% of them are short, 30% are medium, and the other 10% are long. Reviews are submitted in either Word or LaTex. For short reviews, 80% are in word, whereas 50% of medium reviews are in Word and 30% of long reviews are in Word. Suppose a recent review is randomly selected.

(i) What is the probability that the selected review was submitted in Word format?

(ii) If the selected review was submitted in Word format, what is the posterior probability of it being short?

(c) An oil exploration company currently has two active projects, one in Asia and the other in Europe. Let A be the event that the Asian project is successful and B be the event that the European project is successful. Suppose that A and B are independent events with $P(A) = 0.4$, and $P(B) = 0.7$.

(i) What is the probability that at least one of the two projects will be successful?

(ii) Given that at least one of the two projects is successful, what is the probability that only the Asian project is successful?

3. (a) A certain brand of upright freezer is available in three different rated capacities: 450 L, 500 L and 550 L. Let X is the rated capacity of a freezer of this brand sold at a certain store. Suppose that X has probability mass function *(pmf)* $p(x)$ with $p(450) = 0.2$, $p(500) = 0.5$ and $p(550) = 0.3$. If the price of a freezer having capacity X is $2.5X - 650$, what is the expected price and variance price paid by the next customer to buy a freezer?

(b) Twenty percent of all telephones of a certain type are submitted for service while under warranty. Of these, 60% can be repaired, whereas the other 40% must be replaced with new units. If a company purchases ten of these telephones, what is the probability that exactly two will end up being replaced under warranty?

(c) Let X be a Poisson random variable with parameter λ. Find the mean and variance of X.

4. (a) The current in a certain circuit as measured by an ammeter is a continuous random variable X with probability density function:

$$f(x) = \begin{cases} 0.075\,x + 0.2, & 3 \le x \le 5 \\ 0, & otherwise. \end{cases}$$

(i) Verify that the total area under the density curve is 1.

(ii) Calculate $P(X \le 4)$ and $P(3.5 \le X \le 4.5)$.

(b) Let X denote the amount of time a book on two-hour reserve is checked out and suppose the cumulative distribution function is

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x^2}{4}, & 0 \le x < 2 \\ 1, & 2 \le x. \end{cases}$$

(i) Calculate $P(X > 1.5)$.

(ii) Obtain the probability density function $f(x)$.

(iii) Calculate $E(X)$ and $V(X)$.

(c) In a road-paving process, asphalt mix is delivered to the hopper of the paver by trucks that haul the material from the batching plant. The random variable X = truck haul time is normally distributed with mean value 8.46 min and standard deviation 0.913 min.

(i) What is the probability that haul time will be at least 10 min?

(ii) What is the probability that haul time will exceed 15 min?

5. (a) It has been reported, the Lognormal distribution as the best option for describing the distribution of maximum pit depth data from cast iron pipes in soil. This distribution with $\mu = 0.353$ and $\sigma = 0.754$ is appropriate for maximum pit depth (mm) of buried pipelines.

(i) What are the mean value and variance of pit depth?

(ii) What value c is such that only 1% of all specimens have a maximum pit depth exceeding c?

(b) Given a standard normal distribution, find the area under the z-curve which lies

(i) between $z = -1.50$ and $z = 2.00$;

(ii) to the right of $z = 1.75$;

(iii) to the left of $z = -1.39$.

(c) The probability that a patient recovers from a rare blood disease is 0.4. If 100 people are known to have contracted this disease, use the normal approximation to binomial distribution to determine the probability that less than 30 survive.

6. (a) State Central Limit Theorem. A soft-drink vending machine is set so that the amount of drink dispensed is a random variable X with a mean of 200 ml and a standard deviation of 15 ml. What is the probability that the mean amount dispensed $\bar{X}$ in a random sample of size 36 is at least 204 ml?

(b) Consider the following data on the number of hours that 10 persons studied for a French test and their scores on the test:

Hours studied (x)	4	9	10	14	4	7	12	22	1	17
Test score (y)	31	58	65	73	37	44	60	91	21	84

(i) Find the equation of the least squares line that approximates the regression of the test scores on the number of hours studied.

(ii) Predict the average test score of a person who studied 14 hours for the test.

(c) Suppose that we want to determine on the basis of the following data whether there is a relationship between the time, in minutes, it takes a secretary to complete a certain form in the morning and in the late afternoon:

Morning (x)	8.2	9.6	7.0	9.4	10.9	7.1	9.0	6.6	8.4	10.5
Afternoon (y)	8.7	9.6	6.9	8.5	11.3	7.6	9.2	6.3	8.4	12.3

Compute and interpret the sample correlation coefficient.

(3)

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5811 **K**

Unique Paper Code : 2352012302

Name of the Paper : Riemann Integration

Name of the Course : **B.Sc. (H) Mathematics**

Semester : III (DSC)

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions, selecting any **three** parts from each question.
3. **All** questions carry equal marks.
4. Use of calculator is NOT allowed.

P.T.O.

1. (a) Calculate the upper and lower Darboux integrals for the function $f(x) = x^3$ on the interval [0, b] and show that its integral value is $\frac{b^4}{4}$.

(b) Let f be a bounded function on [a, b]. If P is a partition of [a, b] then

$$m(b-a) \leq L(f,P) \leq U(f,P) \leq M(b-a)$$

where $M = \sup\{f(x): x \in [a, b]\}$ and $m = \inf\{f(x): x \in b]\}$. Verify the inequality for the function $f(x) = 3x + 1$ defined on [0,3] and the partition $P = \left\{0, \frac{1}{2}, 1, 2, 3\right\}$ of [0,3].

(c) Let f be a bounded function on [a, b]. Show that f is integrable over [a,b] if and only if for each $\epsilon > 0$. there exists a partition P of [a,b] such that $U(f, P) - L(f,P) < \epsilon$.

(d) Define Riemann sum and Riemann integrability of a bounded function f on [a, b]. Let f be a bounded integrable function on [a, b] and $\langle S_n \rangle$ be a sequence of Riemann sums with corresponding partitions P_n satisfying $\lim_{n\to\infty} \text{mesh}(P_n) = 0$. Show that the sequence $\langle S_n \rangle$ converges to $\int_a^b f$.

2. (a) Show that if a function f is integrable on interval [a, b], then f is integrable on every interval $[c, d] \subseteq [a, b]$.

(b) Let f be a bounded function on [a, b]. If Q is a refinement of a partition P of [a, b], then show that $U(f,Q) - L(f,Q) \leq U(f, P) - L(f, P)$.

(c) Prove that a bounded function fon [a, b] is Riemann integrable if it is Darboux integrable. Also compare their corresponding values of the integrals.

(d) Let f and F be functions on [0, 1]. given by $f(x) = \cos(x)$ for all $x \in [0, 1]$ and

$$F(x) = \begin{cases} 0, & \text{for } 0 \le x < \frac{\pi}{6} \\ 1, & \text{for } \frac{\pi}{6} \le x \le 1 \end{cases}$$

Show that f is F-integrable and $\int_0^1 f \, dF = \frac{\sqrt{3}}{2}$.

3. (a) Let f be an integrable function on [a,b] and there exists $\lambda > 0$ such that $|f(x)| \ge \lambda \ \forall \ x \in [a, b]$. Show that 1/f If is integrable over [a, b].

(b) Let f be a continuous function on [a, b] such that $f(x) \ge 0$ for all $x \in [a, b]$. Show that fa $\int_a^b f = 0$ if and only if $f(x) = 0$ for all $x \in [a, b]$.

(c) Let f be a function defined on [a, b] and $c \in (a, b)$ be any point. Show that if f is integrable on [a, c] and [c, b], then f is integrable on [a, b] and

$$\int_a^b f = \int_a^c f + \int_c^b f.$$

(d) State and prove intermediate value theorem for integrals. Suppose f and g are continuous functions on [a, b] such that $\int_a^b f = \int_a^b g$. Prove there exists x in (a, b) such that $f(x) = g(x)$.

4. (a) Define piecewise continuous function. Show that every piecewise continuous function on [a, b] is integrable.

(b) State and prove Fundamental Theorem of Calculus I. Using the same, evaluate

$$\int_0^{\pi} x \cos x \, dx.$$

(c) Show that if f and g are integrable functions ove [a, b] and if $f(x) \leq g(x)$ for $x \in [a, b]$, then $\int_a^b f \leq \int_a^b g$. Hence deduce that

$$\int_{-2\pi}^{2\pi} \left| x^2 \sin^8 \left(e^x \right) \right| dx \leq \frac{16\pi^3}{3}.$$

(d) Show that if f is a continuous real-valued function on [a, b] satisfying $\int_a^b f(x)g(x)\,dx = 0$ for every continuous function g on [a, b], then $f(x) = 0$ for all x in [a, b].

5. (a) Use disk and washer method to find the volume of the solid generated when the region enclosed by $y = \sqrt{x+1}$, $y = \sqrt{2x}$ and $y = 0$ is revolved about x-axis.

(b) Use cylindrical shell method to find the volume of the solid generated when the region enclosed by $y = x^3$, $y = 1$, $x = 0$ is revolved about the line $y = 1$.

(c) Find the exact arc length of the curve $x = \frac{y^4}{8} + \frac{1}{4y^2}$ from $y = 1$ to $y = 4$.

(d) Find the area of surface generated by revolving the curve

$$x = t, \quad y = 2t^2, \quad 0 \le t \le 1$$

about y-axis.

6. (a) Discuss the convergence or divergence of the following integrals.

(i) $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\, dx$

(ii) $\int_{1}^{\infty} \frac{1}{x^3}\, dx$

(b) Test for convergence

$$\int_0^{\infty} \frac{1}{\sqrt{x}e^x} dx$$

(c) Suppose $\int_a^b f$ is an improper integral of type I.

Show that if $\int_a^b |f|$ converges, then so does $\int_a^b f$.

(d) Show that $\int_0^1 t^{p-1}(1-t)^{q-1} dt$ converges for positive values of p and q.

(4)

This question paper contains 5 printed pages]

Roll No. | | | | | | | | | | |

S. No. of Question Paper : **3575**

Unique Paper Code : **32357507**

Name of the Paper : **DSE-2 : Probability Theory and Statistics**

Name of the Course : **B.Sc. (Hons.) Mathematics (CBCS-LOCF)**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

There are six questions in all.

Attempt *all* questions selecting any *two* parts from each question.

Use of scientific calculator is allowed.

1. (*i*) If a random variable X has the probability distribution $f(x) = \frac{1}{8}\binom{n}{x}$, for $x = 0, 1, 2, 3$. Find the moment generating function of this random variable and use it to determine μ_1' and μ_2'. 6

   (*ii*) Find the mean and variance of the Binomial distribution. 6

   (*iii*) Let X be a continuous random variable with pdf $f(x) = ke^{-2x}, 0 \le x < \infty$. Find E(X), $E(X^2)$ and Var(X). 6

2. (*i*) If the probability is 0.40 that a child exposed to a certain contagious disease will catch it, what is the probability that the tenth child exposed to the disease will be the third to catch it ? 6

P.T.O.

(*ii*) Three distinct integers are chosen at random from the first 20 positive integers. Compute the probability that :

(*a*) their sum is even

(*b*) their product is even. 6

(*iii*) Let X be a discrete random variable with the probability mass function

$$p(x) = \frac{1}{2^x}, \; x = 1,2,3,...$$

Find :

(*a*) the cumulative distribution function of X,

(*b*) $P(2 < X \leq 5)$ and

(*c*) $P(X > 4)$. 6

3. (*i*) Show that if X is a random variable having the Poisson Distribution with the parameter λ, then the moment-generating function of

$$Z = \frac{X - \lambda}{\sqrt{\lambda}}$$

approaches the moment-generating function of the standard normal distribution as $\lambda \to \infty$. 6

(*ii*) Find the mean and variance of exponential distribution. 6

(*iii*) If X is a continuous uniformly distributed random variable with mean 1 and variance 4/3, find :

(*a*) $P(X < 0)$

(*b*) $P(X = 0)$

(*c*) $P(X > 0)$. 6

4. (*i*) Let X and Y have joint pdf

$$f(x, y) = \begin{cases} 12xy(1 - y), & 0 < x < 1, \quad 0 < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Compute $E(X^2)$ and $E(XY)$. Also find $E(3Y - X^2)$. 6.5

(*ii*) Show that two random variables X and Y are independent random variables if and only if

$$P(a < X < b, c < Y < d) = P(a < X < b)\, P(c < Y < d).$$ 6.5

(*iii*) Let 13 cards be taken, at random and without replacement, from an ordinary deck of playing cards. If X is the number of spades in these 13 cards and in addition, Y is the number of hearts in these 13 cards, find the probability $P(X = 2, Y = 5)$. Find the joint and marginal pmf of X and Y. 6.5

5. (*i*) Let X and Y have the joint pmf described by the following table :

(x, y)	(0,0)	(0,1)	(0,2)	(1,1)	(1,2)	(2,2)
$p(x, y)$	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{1}{12}$	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{1}{12}$

Find the correlation coefficient of X and Y. 6.5

(*ii*) Let X_1 and X_2 be two random variables having the joint pdf :

$$f(x_1, x_2) = \begin{cases} 6x_2, & 0 < x_2 < x_1 < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Find the conditional mean of X_2 given $X_1 = x_1 \in (0, 1)$. 6.5

(*iii*) If X and Y have a bivariate normal distribution, then find the marginal density function of Y. 6.5

6. (*i*) Let $X_1, X_2, \ldots X_n$ constitute a random sample from an infinite population with mean μ and variance σ^2 and moment generating function $M_X(t)$. Show that the limiting distribution of $Z = \frac{X-\mu}{\sigma}$ as $n \to \infty$ is the standard normal distribution. 6.5

(*ii*) If the probability density of X is given by

$$f(x) = \begin{cases} 630\, x^4 (1-x)^4, & 0 < x < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Find the probability that it will take on a value within two standard deviations of the mean and compare this probability with the lower bound provided by Chebyshev's theorem. 6.5

(*iii*) Given the joint density

$$f(x, y) = \begin{cases} \frac{2}{5}(2x + 3y), & 0 < x < 1, 0 < y < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Find the regression equation of Y on X. 6.5

(5)

[This question paper contains 4 printed pages.]

Your Roll No.....

K

Sr. No. of Question Paper : 5714

Unique Paper Code : 2352013501

Name of the Paper : Metric Spaces

Name of the Course : **B.Sc. (Hons) Mathematics (DSC-13)**

Semester : V

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting **two** parts from each question.
3. Part of the questions to be attempted together.
4. **All** questions carry equal marks.
5. Use of Calculator is not allowed.

1. (a) (i) Let $X = \mathbb{R}$ and for $x, y \in \mathbb{R}$, define $d(x, y)$ by

$$d(x,y)=\begin{cases}|x-y|+1, & \text{if exactly one among x and y is strictly positive}\\ |x-y|, & \text{otherwise}\end{cases}$$

Prove that (X, d) is a metric space. (4)

(ii) Let X denote the set of all Riemann integrable functions defined on [a, b]. For $f, g \in X$, define $d(f, g) = \int_a^b |f(x)-g(x)|\,dx$. Show that (X, d) is not a metric space. (3.5)

(b) Let X be the set of all continuous functions defined on [0, 2] equipped with metrics $d_\infty(f, g) = \sup_{x\in[0,2]}|f(x) - g(x)|$ and $d_1(f, g) = \int_0^2 |f(x)-g(x)|\,dx$, $f, g \in X$. Compute the distance $d_\infty(f, g)$ and $d_1(f, g)$ where

P.T.O.

$$f(x)=\begin{cases}\sin x, & 0\le x<\frac{\pi}{4}\\ \frac{1}{\sqrt{2}}, & \frac{\pi}{4}\le x\le 2\end{cases} \text{ and } g(x)=\begin{cases}\cos x, & 0\le x<\frac{\pi}{4}\\ \frac{1}{\sqrt{2}}, & \frac{\pi}{2}\le x\le 2\end{cases}.$$

(7.5)

(c) (i) Define complete metric space. Let X = $\mathbb{N}$ the set of natural numbers. Define $d(m,n)=\left|\frac{1}{m}-\frac{1}{n}\right|$; m, n $\in$ X. Show that (X, d) is an incomplete metric space. (4)

(ii) Is the metric space (X, d) of the set X of rational numbers with usual metric d a complete metric space? Justify your answer. (3.5)

2. (a) (i) Define an open set in a metric space (X, d). Show that a subset G in a metric space (X, d) is open if and only if it is the union of all open balls contained in G. (5)

(ii) Let S(x, r) be an open ball in a metric space (X, d). Let A be a subset of X such that diameter of A, d(A) < r and $S(x, r) \cap A \neq \phi$. Show that $A \subseteq S(x, 2r)$. (2.5)

(b) (i) Let (X, d) be a metric space. Prove that the closed ball $\overline{S}(x,r)$, where x $\in$ X and r > 0, is a closed subset of X. (3.5)

(ii) Let X = $\mathbb{R}$ with usual metric. Let F = {1, 1/2, 1/3,...}. Find the derived set of F. (4)

(c) Let (X, d) be a metric space and A_1 and A_2 be subsets of X. Prove that

(i) $\overline{(A_1 \cup A_2)} = \overline{A_1} \cap \overline{A_2}$.

(ii) $\overline{(A_1 \cap A_2)} \subseteq \overline{A_1} \cap \overline{A_2}$.

Is the closure of the union of an arbitrary family of the subsets of X equal to the union of the closures of the members of the family? Justify your answer. (7.5)

3. (a) Let (X, d) be a metric space and $Z \subseteq Y \subseteq X$. If $cl_X Z$ and $cl_Y Z$ denotes the closures of Z in the metric spaces X and Y respectively, then show that $cl_Y Z = Y \cap cl_X Z$. (7.5)

(b) Let (X, d_X) and (Y, d_Y) be metric spaces and let $f: X \to Y$ be a function. Prove that the following statements are equivalent :

(i) f is continuous on X;

(ii) $\overline{f^{-1}(B)} \subseteq f^{-1}(\overline{B})$ for all $B \subseteq Y$. (7.5)

(c) Let (X, d_X) and (Y, d_Y) be metric spaces. When will a function $f: X \to Y$ is said to be uniformly continuous? For any metric space (X, d) and $A \subseteq X$, let $g: X \to \mathbb{R}$ be defined as $g(x) = \inf\{d(x,y) : y \in A\}$. Prove that g is uniformly continuous mapping using ε-δ method. (7.5)

4. (a) Define equivalent metrics. Show that the metrics d_1, d_2, d_∞ defined on $\mathbb{R}^n$ by

$$d_1(x,y) = \sum_{i=1}^{n} |x_i - y_i|, \quad d_2(x,y) = \left(\sum_{i=1}^{n} (x_i - y_i)^2 \right)^{\frac{1}{2}},$$

$d_\infty(x, y) = \max\{|x_i - y_i| : 1 \le i \le n\}$ are equivalent metrics. (7.5)

(b) Define fixed point of the mapping T. Show that if $T: X \to X$ is a mapping and T^n has a fixed point $x_0 \in X$, then T also has a fixed point. Is the converse true? Justify your answer. (7.5)

(c) Prove that every isometry between metric spaces (X, d_X) and (Y, d_Y) is also a homeomorphism between (X, d_X) and (Y, d_Y). Let X be a metric space and $x_0 \in X$, C(X) denotes the space of all real valued, continuous bounded functions on X with uniform metric $d(f, g) = \sup\{|f(x) - g(x)| : x \in X\}$. Prove that the mapping $f_x(y) = d(y, x) - d(y, x_0)$ for $x, y \in X$ defines an isometry of X into C(X). (7.5)

5. (a) Let (X, d_X) be a connected metric space and $f: (X, d_X) \to (Y, d_Y)$ be a continuous mapping. Show that the space $f(X)$ with the metric induced from Y is connected. (7.5)

(b) Let (X, dx) be a metric space. If every continuous function $f: (X, d_X) \to (\mathbb{R}, d)$ has the intermediate value property, prove that (X, d_X) is a connected metric space. (7.5)

(c) If every two points in a metric space X are contained in some connected subset of X, prove that X is connected. (7.5)

6. (a) Let Y be a subset of metric space (X, d). If (X, d_Y) is compact, prove that Y is a closed subset of (X, d). (7.5)

(b) If f is a continuous real-valued function on a compact metric space (X, d_X), show that f is bounded and attains its bounds, i.e., if $M = \sup f(X)$, $m = \inf f(X)$, there exist x and y in X such that $f(x) = M$ and $f(y) = m$. (7.5)

(c) Let A be a compact subset of a metric space (X, d) and B be a close subset of X such that $A \cap B = \phi$. Show that $d(A, B) > 0$. (7.5)

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5849 **K**

Unique Paper Code : 2352013503

Name of the Paper : Partial Differential Equations

Name of the Course : **B.Sc. (H) Mathematics – DSC-15**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory and are of **15** marks each.
3. Attempt any **two** parts from each Questions. Each part is of **7.5** marks.

1. (a) Define characteristic curves of a first order partial differential equation. Find the partial differential equation arising from the two-parameter family of curves :

$$u - ax - by - ab = 0.$$

(b) Determine the integral surfaces of the equation:

$x(y^2 + u)u_x - y(x^2 + u)u_y = (x^2 - y^2)u$; with the data $x + y = 0$ and $u = 1$.

P.T.O.

(c) Find the solution of the initial value system:

$$u_t - 2uu_x = v - x,$$

$$v_t + cv_x = 0,$$

$$u(x, 0) = x, \; v(x, 0) = x.$$

2. (a) Reduce the given equation into canonical form and find the general solution x :

$$u_x + xu_y = y.$$

(b) Using $v = \ln u$ and $v(x,y) = f(x) + g(y)$, solve the equation:

$$x^2u_x^2 + y^2u_y^2 = u^2.$$

(c) Find a complete integral of the following equation by using Charpit's method :

$$z^2 = pqxy.$$

3. (a) Derive the three-dimensional Laplace equation:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{d^2 V}{\partial z^2} = 0,$$

where V is Gravitational potential at any point.

(b) Determine the region of the following equation, and transform it to canonical form:

$$x^2u_{xx} - 2xyu_{xy} + y^2u_{yy} = e^x.$$

(c) Derive the equation of traffic flow :

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = 0,$$

where $\rho(x, t)$ is traffic density and $q(x, t)$ is traffic flow at any time t.

4. (a) Find the general solution of the equation:

$yu_{xx} + 3yu_{xy} + 3u_x = 0,\ y \neq 0.$

(b) Reduce the following equation into canonical form :

$u_{xx} - 2\cos x\, u_{xy} + (1 + \cos^2 x)u_{yy} + u = 0.$

(c) Find the general solution of the equation:

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial^2 z}{\partial y^2} = e^{x+y}.$$

5. (a) Determine the solution of the initial-value problem for an infinite string with initial condition:

$$u_{tt} - u_{xx} = 0, \quad x \in \mathbb{R}, \quad t > 0,$$
$$u(x, 0) = sin3x, \ x \in \mathbb{R},$$
$$u_t(x, 0) = cos3x, \ x \in \mathbb{R}.$$

(b) Find the solution of the initial-value problem for a semi-infinite string with fixed end:

$$u_{tt} - 16u_{xx} = 0, \quad 0 < x < \infty, \ t > 0,$$
$$u(x, 0) = \sin x, \quad 0 \leq x < \infty,$$
$$u_t(x, 0) = x^2, \quad 0 \leq x < \infty,$$
$$u(0, t) = 0, \quad 0 \leq t < \infty.$$

P.T.O.

(c) Determine the solution of wave equation of semi-infinite string with free end for non- homogeneous boundary value problem:

$$u_{tt} - 16u_{xx} = 0, \quad x > 0, \quad t > 0,$$
$$u(x,0) = \cos x, u_t(x,0) = 2, \quad x \geq 0,$$
$$u_x(0,t) = 1 - t, \; t \geq 0.$$

6. (a) Obtain the solution for a semi-infinite string with free end:

$$u_{tt} = c^2 u_{xx}, \quad 0 < x < \infty, \quad t > 0,$$
$$u(x,0) = f(x), \quad 0 \leq x < \infty,$$
$$u_t(x,0) = g(x), \quad 0 \leq x < \infty,$$
$$u_x(0,t) = 0, \quad 0 \leq t < \infty.$$

(b) Determine the solution of wave equation for non-homogeneous boundary value problem of semi-infinite string with fixed end:

$$u_{tt} - 4u_{xx} = 0, \quad x > 0, \quad t > 0,$$
$$u(x,0) = \log(1+x), \; x \geq 0$$
$$u_t(x,0) = 1, \; x \geq 0,$$
$$u(0,t) = t, \quad t \geq 0.$$

(c) Determine the solution of Cauchy problem for non-homogeneous wave equation:

$$u_{tt} = u_{xx} - 2, \quad x \in \mathbb{R}, \quad t > 0,$$

with initial condition $u(x, 0) = \sin x$, $u_t(x, 0) = x$, $x \in \mathbb{R}$.

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5917 **K**

Unique Paper Code : 2353010008

Name of the Paper : DSE – Linear Programming and Applications

Name of the Course : **NEP-UGCF 2022 – B.Sc. (H) Mathematics**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions carry equal marks.

P.T.O.

1. (a) Solve the following Linear Programming Problem by Graphical Method:

Minimize $Z = -x_1 - 3x_2$

subject to $x_1 + x_2 \leq 6$

$-x_1 + 2x_2 \leq 8$

$2x_1 + x_2 \geq 1$

$x_1, x_2 \geq 0.$

(b) Find all the Basic Feasible Solutions for the following system of equations

$$x_1 + x_2 + 2x_3 - x_4 = 6$$

$$x_1 + 2x_2 - 2x_4 = 3.$$

(c) Define a convex set and use its mathematical definition to prove that the following set S is a convex set :

$$S = \{(x_1, x_2) : 3x_1^2 + 2x_2^2 \leq 6\}.$$

2. (a) Consider the following problem:

Minimize $z = cx$

subject to $Ax = b,$

$x \geq 0,$

Given a Basic Feasible Solution $x_B = B^{-1}b$, if for any column a_k of A not in B, $z_k - c_k > 0$ and $y_{ik} \leq 0$, i = 1,2,... ... ..., m; then prove that the problem will have an unbounded solution.

(b) Solve the following Linear Programming Problem by Simplex Method :

Maximize $Z = 5x_1 + 4x_2$

subject to $x_1 + 2x_2 \leq 6$

$-2x_1 + x_2 \leq 4$

$5x_1 + 3x_2 \leq 15$

$x_1, x_2 \geq 0$

(c) Solve the following Linear Programming Problem by Big-M method

Minimize $Z = -x_1 - x_2$

subject to $x_1 - x_2 - x_3 = 1$

$-x_1 + x_2 + 2x_3 - x_4 = 1$

$x_1, x_2, x_3, x_4 \geq 0$.

3. (a) Solve the following Linear Programming Problem by Two Phase Method:

Minimize $Z = 3x_1 - 3x_2 + x_3$

subject to $x_1 + 2x_2 - x_3 \geq 5$

$-3x_1 - x_2 + x_3 \leq 4$

$x_1, x_2, x_3 \geq 0.$

(b) Prove that to every extreme point of the feasible region, there corresponds a basic feasible solution of the Linear Programming Problem:

Minimize $z = cx$

subject to $Ax = b, x \geq 0.$

(c) Solve the following Linear Programming Problem by Simplex Method:

Maximize $Z = x_1 + 1.5x_2$

subject to $2x_1 + 3x_2 \leq 6$

$x_1 + 4x_2 \leq 4$

$x_1 \geq 0$, x_2 is unrestricted.

Find an alternate optimal solution, if it exits. How many optimal feasible solutions does the problem has?

4. (a) Write the Dual of the following Linear Programming Problem so that dual variables and the requirement vectors are non-negative :

Minimize $Z - x_1 - 3x_2 - 2x_3$

subject to

$$3x_1 - x_2 + 2x_3 \geq 10$$
$$x_1 - 4x_2 + 2x_3 \leq 8$$
$$3x_1 + 5x_2 - 3x_3 = 12$$
$$x_1 \leq 0,\ x_2 \geq 0,\ x_3 \text{ is unrestricted.}$$

(b) Prove that Dual of Dual is Primal. Verify the same for the following problem:

Maximize $Z = -x_1 + 3x_2$

subject to $2x_1 - 5x_2 \leq 8$

$$-x_1 + 4x_2 \geq 7$$
$$x_3 \geq 0,\ x_2 \leq 0.$$

(c) Using Complementary Slackness Theorem, find optimal solutions of the following Linear Programming Problem and its Dual.

Minimize $Z = 4x_1 + 6x_2 + 5x_3 + x_4$

subject to

$$x_1 + x_3 \geq 3$$
$$x_2 + x_3 + x_4 \geq -2$$
$$x_1, x_2, x_3, x_4 \geq 0.$$

5. (a) Find the optimal solution of the Assignment Problem with the following cost matrix. Also find an alternate optimal solution, if any

	I	II	III	IV	V	VI
A	2	6	7	3	8	7
B	6	1	3	9	7	3
C	3	6	5	7	3	5
D	2	2	7	8	4	8
E	4	9	6	8	7	6
F	7	5	5	7	7	5

(b) For the following cost minimization Transportation Problem, find initial basic feasible solution by using North-West comer rule, Least Cost method and Vogel's approximation method. Compare the three solutions (in terms of cost).

	I	II	III	IV	Supply
A	8	9	6	3	18
B	6	11	5	10	20
C	3	8	7	9	18
Demand	15	16	12	13	

(c) A company has four machines to be installed at five vacant places available. Each machine can be installed to one and only one place. Find the optimal solution of the Assignment Problem and the total minimum cost with the cost matrix given below. Also find which place remain vacant:

Place

Machines	A	B	C	D	E
I	13	15	19	14	15
II	16	13	13	14	13
III	14	15	18	15	11
IV	18	12	16	12	10

6. (a) Define a Saddle point. The pay-off matrix of a game is given below. Find the solution of the game of Player A and Player B. Is it a Fair Game?

Player B

Player A	I	II	III	IV	V
I	4	0	1	7	-1
II	0	-3	-5	-7	5
III	3	2	3	4	5
1V	-6	1	-1	0	5
V	0	0	6	0	0

(b) Convert the following Game problem into a pair of Linear Programming Problems for Player A and Player B.

	Player B			
Player A	-1	-2	3	2
	3	0	-3	-2

(c) Solve the following cost minimization Transportation Problem :

	I	II	III	IV	Supply
A	10	11	10	13	30
B	12	12	11	10	50
C	13	11	14	18	20
Demand	20	40	30	10	

(8)

[This question paper contains 12 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5918 **K**

Unique Paper Code : 2353010009

Name of the Paper : Mathematical Statistics

Name of the Course : **B.Sc. (H) Mathematics (NEP UGCF 2022) - DSE**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.
4. **All** questions carry equal marks.
5. Use of non-programmable scientific calculators and statistical tables is permitted.

P.T.O.

1. (a) A service station has both self-service and full-service islands. On each island, there is a single regular unleaded pump with two hoses. Let X denote the number of hoses being used on the self-service island at a particular time, and let X denote the number of hoses on the full-service island in use at that time. The joint pmf of X and Y appears in the accompanying tabulation.

		y		
x	p(x, y)	0	1	2
	0	.10	.04	.02
	1	.08	.20	.06
	2	.06	.14	.30

(i) Compute P(X ≤ 1 and Y ≤ 1).

(ii) Give a word description of the event {X ≠ 0 and Y ≠ 0}, and compute the probability of this event.

(iii) Compute the marginal pmf of X and use it to find P(X ≤ 1).

(b) You have two lightbulbs for a particular lamp. Let X = the lifetime of the first bulb and Y = the lifetime of the second bulb (both in thousands of hours). Suppose that X and Y are independent and that each has an exponential distribution with parameter $\lambda = 1$.

(i) What is the joint pdf of X and T?

(ii) What is the probability that each bulb lasts at most 1000 h (i.e., $X \leq 1$ and $Y \leq 1$)?

(iii) What is the probability that the total lifetime of the two bulbs is at most 2?

(c) (i) Define the correlation coefficient of two random variables X and Y, and establish that

$$-1 \leq \text{Corr}(X, Y) \leq 1.$$

(ii) Verify that if $Y = aX + b$ $(a \neq 0)$, then $\text{Corr}(X, Y) = +1$ or -1.

2. (a) (i) Let X_1 and X_2 be continuous random variables with joint pdf f(x, y). For any real constants a, b and c, prove that

$$E(aX + bY + c) = aE(X) + bE(Y) + c.$$

(ii) For any two random variables X and Y, prove that $E[E(Y|X)] = E(Y)$.

(b) For a Calculus I class, the final exam score Y and the average X of the four earlier tests have abivariate normal distribution with mean $\mu_1 = 73$, standard deviation $\sigma_1 = 12$, mean $\mu_2 = 70$, standard deviation $\sigma_2 = 15$. The correlation is $\rho = .71$. Determine

0) $\mu_{Y|X=x}$

(ii) $\sigma_{Y|X=x}$

(iii) $P(Y > 90|X = 80)$, i.e., the probability that the final exam score exceeds 90 given that the average of the four earlier tests is 80.

(iv) Suppose a student's Calculus I grade is determined by 4X + Y, the total score across five tests. Find the mean of 4X + Y.

(c) A pizza place has two phones. On each phone the waiting time until the first call is exponentially distributed with mean one minute. Each phone is not influenced by the other. Let X be the shorter of the two waiting times and let Y be the longer. The joint pdf of X and Y is

$f(x,y) = 2e^{-(x+y)}$, $0 < x < y < \infty$.

(i) Determine the marginal density of X.

(ii) Determine the conditional density of Y given $X = x$. Are X and Y independent?

(iii) Determine the conditional mean of Y given $X = x$.

3. (a) Let X be the number of packages being mailed by a randomly selected customer at a shipping facility. Suppose the distribution of X is as follows:

x	1	2	3	4
p(x)	.4	.3	.2	.1

Consider a random sample of size n = 2 (two customers) and let $\bar{X}$ be the sample mean number of packages shipped. Obtain the sampling distribution of $\bar{X}$ and use it to calculate $P(\bar{X} \leq 2.5)$.

(b) The inside diameter of a randomly selected piston ring is a random variable with mean value 12 cm and standard deviation .04 cm.

(i) If X is the sample mean diameter for a random sample of n = 16 rings, then where is the sampling distribution of $\bar{X}$ centered, and what is the standard deviation of the $\bar{X}$ distribution?

(ii) If the distribution of diameter is normal, then calculate $P(11.99 \leq \bar{X} \leq 12.01)$ when n = 16.

(c) State the Gosset's theorem.

Suppose $X_1, \ldots, X_{27}$ are independent and identically distributed (iid) N(5,4) random variables. Let $\bar{X}$ and 5 denote their sample mean and sample standard deviation, respectively. Calculate $P(|\bar{X} - 5| > 0.4S)$.

4. (a) Show that for the sample variance S^2, $\frac{(n-1)S^2}{n}$ is a biased estimator of σ^2 for any population distribution. What is the bias?

(b) (i) Suppose $X_1,..., X_{13}$ form a random sample from a normal distribution with mean 5 and standard deviation 8. Calculate $P(\bar{X} < 9.13)$.

(ii) Let $X_1...,X_n$ be a random sample of component lifetimes from an exponential distribution with parameter λ. Use the factorization theorem to show that sufficient statistic for λ is $T = \Sigma X_i$.

(c) (i) Define the Fisher information $I(\theta)$ in a single observation X from a distribution with probability density function $f(x; \theta)$.

(ii) The number of attempts required to successfully transmit a message across a noisy channel can be modelled by a geometric distribution, whose probability mass function

is $(1 - p)^{x-1} p$ for $x = 1,2,3,....$ To estimate the unknown parameter p we obtain the random sample $X_1,....,X_n$ from this geometric distribution. Find the Fisher information I(p) in a single observation X.

5. (a) A random sample of 50 patients who had been seen at an outpatient clinic was selected, and the waiting time to see a physician was determined for each one, resulting in a sample mean time of 40.3 min and a sample standard deviation of 28.0 min.

(i) Calculate and interpret a 95% upper confidence bound for true average waiting time.

(ii) Based on the sample mean and standard deviation, why is it doubtful that the population of waiting times is normally distributed? Does that invalidate the confidence bound you calculated in part (i)?

$[t_{.05,49} = 1.677]$

(b) A new design for the braking system on a certain type of car has been proposed. For the current system, the true average braking distance at 40 mph under specified conditions is known to be 120ft. It is proposed that the new design be implemented only if sample data strongly indicates a reduction in true average braking distance for the new design.

(i) Define the parameter of interest and state the relevant hypotheses.

(ii) Suppose braking distance for the new system is normally distributed with $\sigma = 10$. Let $\bar{X}$ denote the sample average braking distance for a random sample of 36 observations. Which of the following rejection regions is appropriate:

$R_1 = \{\bar{x}: \bar{x} \geq 124.80\}$,

$R_2 = \{\bar{x}: \bar{x} \leq 115.20\}$,

$R_3 = \{\bar{x}: either\ \bar{x} \geq 125.13\ or\ \bar{x} \leq 114.87\}$?

What is the significance level for the appropriate region? How would you change the region to obtain a test with $\alpha = .001$? $[z_{0.001} = 3.08]$

(c) The article "Development of Novel Indus trial Laminated Planks from Sweetgum Lumber" provides the following data on the modulus of rupture (psi) for a sample of planks:

6807.99	7637.06	6663.28	6165.03	6991.41
6992.23	6981.46	7569.75	7437.88	6872.39
7663.18	6032.28	6906.04	6617.17	6984.12
7093.71	7659.50	7378.61	7295.54	6702.76
7440.17	8053.26	8284.75	7347.95	7422.69
7886.87	6316.67	7713.65	7503.33	7674.99

Perform a hypothesis test at the .01 level to determine if the true modulus of rupture for this type of plank differs from 7500 psi.

$[t_{.005,29} = 2.756]$

6. (a) Natural cork in wine bottles is subject to deterioration, and as a result wine in such bottles may experience contamination. The article "Effects of Bottle Closure Type on Consumer Perceptions of Wine Quality" reported that in a tasting of commercial chardonnays, 16 of 91 bottles were

considered spoiled to some extent by cork associated characteristics. Does this data provide strong evidence for concluding that more than 15% of all such bottles are contaminated in this way? Carry out a test of hypotheses using a significance level of. 10. $[z_{0.10} = 1.28]$

(b) The article "Characterization of Highway Run off in Austin, Texas, Area" gave a scatterplot, along with the least squares line, of x = rainfall volume (m^3) and y = runoff volume (m^3) for a particular location. The accompanying values were read from the plot.

x	5	12	14	17	23	30	40	47	55	67	72	81	96	112	127
y	4	10	13	15	15	25	27	46	38	46	53	70	82	99	100

What proportion of the observed variation in runoff volume can be attributed to the simple linear regression relationship between runoff and rainfall?

(c) The article "Racial Stereotypes in Children's Television Commercials" reported the following frequencies with which characters of different ethnicities appeared in recorded commercials aired on Philadelphia television stations.

Ethnicity	African-American	Asian	Caucasian	Hispanic
Frequency	57	11	330	6

Census data at the time reported the population proportions for these four ethnic groups was .177, .032, .734, and .057, respectively. Does the data suggest that the true proportions in commercials are different from the census proportions? Carry out a test of appropriate hypotheses using a significance level of. 10. $[\chi^2_{10,3} = 16.26]$

(9)

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper	:	**5772** **K**
Unique Paper Code	:	2352014701
Name of the Paper	:	Linear Analysis
Name of the Course	:	**Bachelor of Science (Honours Course) Mathematics**
Semester	:	VII

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **five** parts from Question No. **1**.
3. Attempt any **two** parts from Question No. **2** to **6**.

1. (a) Show that norm is a continuous mapping from $(X, \|\cdot\|)$ to $\mathbb{R}$.

(b) Fix $a = (\alpha_j) \in l^2$, define a linear functional $f: l^2 \to \mathbb{R}$ by $f(x) = \sum_{j=1}^{\infty} \alpha_j \xi_j$, where $x = (\xi_j)_{j=1}^{\infty}$. Find the norm of the functional f.

(c) Give definition of the matrix norm. Show that a norm on M_n defined as

$$\|A\|_{\infty} = \max_{1 \le i,\, j \le n} |a_{ij}|, \quad \text{for } A = (a_{ij}) \in M_n$$

is not a matrix norm.

(d) State parallelogram equality for an inner product space. Give an example to show that die norm on l^p does not satisfy the parallelogram equality when $p \neq 2$.

(e) In an inner product space X, show that $y \perp x_n$ and $x_n \to x$ together imply $x \perp y$.

(f) Show that the graph G(T) of a linear operator $T: X \to Y$ is a vector subspace of $X \times Y$.

(g) Let X and Y be Banach spaces and $T: X \to Y$ an injective bounded linear operator. Show that $T^{-1}: \mathcal{R}(T) \to X$ is bounded if and only if $\mathcal{R}(T)$ is closed in Y. (3×5=15)

2. (a) Show that the normed space $(l^2, \|\cdot\|_2)$ is a Banach space.

(b) Give the definition of equivalent norms on a normed space X. Show that the below given norms on $\mathbb{R}^n$ are equivalent :

$$\|x\|_1 = |x_1| + |x_2| + |x_3| + \cdots + |x_n|,$$

$$\|x\|_2 = \sqrt{|x_1|^2 + |x_2|^2 + |x_3|^2 + \cdots + |x_n|^2},$$

where $x = (x_1, x_2, x_3, \ldots \ldots \ldots x_n) \in \mathbb{R}^n$.

(c) Prove that in a finite dimensional normed space X every linear operator on X is bounded. Is the finite dimensional condition necessary for the assertion? (7.5,7.5,7.5)

3. (a) Let X and Y be normed spaces and $T: \mathcal{D}(T) \to Y$ be a linear operator. Then prove that T is continuous if and only if T is bounded.

(b) Prove that the dual space of $\mathbb{R}^n$ is isomorphic to $\mathbb{R}^n$.

(c) Find the norm of the linear functional $f : C[-1, 1] \to \mathbb{R}$ defined as

$$f(x) = \int_{-1}^{0} x(t)\,dt - \int_{0}^{1} x(t)\,dt. \qquad (7.5,7.5,7.5)$$

4. (a) State the Cauchy-Schwarz inequality and, by using it prove the following :

Let $\{x_n\}$ and $\{y_n\}$ be two sequences in an inner product space such that $x_n \to x$ and $y_n \to y$, then show that $\langle x_n, y_n \rangle \to \langle x, y \rangle$.

(b) Let $\{e_k\}$ be any orthonormal sequence in an inner product space X. Show that for any $x, y \in X$,

$$\sum_{k=1}^{\infty} |\langle x, e_k \rangle \langle y, e_k \rangle| \leq \|x\| \, \|y\| .$$

(c) Show that in an inner product space X, $x \perp y$ if and only if $\|x + \alpha y\| \geq \|x\|$ for all scalars α.

(7.5,7.5,7.5)

5. (a) If z is any fixed element of an inner product space X, show that $f(x) = \langle x, z \rangle$ defines a bounded linear functional on X and $\|f\| = \|z\|$.

(b) Let $\{T_n\}$ be a sequence of bounded self-adjoint linear operators $T_n : H \to H$ on a Hilbert space H. Suppose that $\|T_n - T\| \to 0$, where $\|\cdot\|$ is the norm on the space B(H, H), the space of all bounded linear operators from H to H. Show that the limit operator T is a bounded self-adjoint linear operator on H.

(c) Let $T : H \to H$ be a bounded linear operator on a Hilbert space H.

(i) If T is self-adjoint, then show that $\langle Tx, x \rangle$ is real for all $x \in H$.

(ii) If H is complex and $\langle Tx, x \rangle$ is real for ail $x \in H$, then show that the operator T is self-adjoint.

(7.5,7.5,7.5)

6. (a) Let f be a bounded linear functional on a subspace Z of a normed space X. Then prove that there exists a bounded linear functional $\tilde{f}$ on X which is an extension of f to X and has the same norm,

$$\|\tilde{f}\|_X = \|f\|_Z$$

where

$$\left\|\tilde{f}\right\|_X = \sup_{\substack{x\in X\\ \|x\|=1}} \left|\tilde{f}(x)\right|, \text{ and } \left\|\tilde{f}\right\|_Z = \sup_{\substack{x\in Z\\ \|x\|=1}} \left|\tilde{f}(x)\right|,$$

(and $\|f\|_Z = 0$ in the trivial case $Z = \{0\}$).

(b) Prove that every Hilbert space is reflexive.

(c) Define open mapping. Prove that a bounded linear operator T from a Banach space X onto a Banach space Y is an open mapping. Also, prove that if T is bijective then T^{-1} is continuous. (7.5,7.5,7.5)

(10)

[This question paper contains 2 printed pages.]

Your Roll No.....

K

Sr. No. of Question Paper	:	**10808**
Unique Paper Code	:	2353010020
Name of the Paper	:	Optimization
Name of the Course	:	**B.Sc. (Hons.) Mathematics**
Semester	:	VII

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.
4. **All** questions carry equal marks.
5. Use of non-programmable scientific calculators is permitted.

Ques. 1 (a) (i) Examine the convexity of the function $f(x_1, x_2) = 4x_1^2 + x_2^2 + 4x_1x_2$.
(ii) Over what domain is the function $f(x) = x^2(x^2 - 1)$ convex?

(b) Let $f_1, f_2, \ldots, f_k: R^n \to R$ be convex functions. Consider the function f defined by $f(x) = \max\{f_1(x), f_2(x), \ldots, f_k(x)\}$. Show that f is convex.

(c) Define the epigraph of a function. Show that the epigraph of the function $f(x) = +\sqrt{(1 - x^2)}$, $0 \le x \le 1$ is convex? Justify your answer.

Ques.2 (a) Let S be a nonempty convex set in R^n and let $f: S \to R$ be a convex function. Consider the problem to minimize $f(x)$ subject to $x \in S$. Suppose that $\bar{x} \in S$ is a local optimal solution to the problem. Then show that $\bar{x}$ is a global optimal solution. Further prove that, if f is strictly convex, then $\bar{x}$ is the unique global optimal solution.

(b) Define a quasiconvex, quasiconcave and quasimonotone functions. State a necessary and sufficient characterization of a quasiconvex function in terms of its lower level sets.

(c) Give example of the following
(i) a quasiconvex function which is not pseudoconvex,
(ii) a pseudoconvex function which is not convex.

Ques.3 (a) Suppose that $f: R^n \to R$ is twice differentiable at $\bar{x}$. If $\bar{x}$ is a local minimum, then show that $\nabla f(\bar{x}) = 0$ and $H(\bar{x})$ is positive semidefinite.

(b) Consider the univariate function $f(x) = xe^{-2x}$. Find all local minima/maxima and inflection points. Also, what can you claim about a global minimum and a global maximum for f ?

(c) Consider the following problem:

$$\text{Minimize } -x_1$$

Subject to

$$x_2 - (1 - x_1)^3 = 0$$
$$-x_2 - (1 - x_1)^3 = 0$$

Verify that the Fritz John conditions hold true at the optimal point (1,0). Also find the Lagrange multipliers such that (1 ,0) becomes the Fritz John point.

Ques.4 (a) Consider the following problem

$$\text{Minimize } x_1^2 + 2x_2^2$$

Subject to $x_1 + x_2 - 2 = 0.$

Find a point satisfying the KKT conditions and verify that it is an optimal solution.

P.T.O.

(b) Let x be a feasible solution to the problem (P) defined as:

$Minimize\ f(x)$

Subject to $g_i(x) \leq 0, for\ i = 1,2,\ldots.m$

$h_i(x) = 0, for\ i = 1,2,\ldots.l$

$x \in X$

Where X is a nonempty open set in R^n; $f: R^n \to R$; $g_i: R^n \to R\ for\ i = 1,2,\ldots..m$; $h_i: R^n \to R\ for\ i = 1,2,\ldots l$.

Let (u, v) be a feasible solution to its dual (D) with objective value $\theta(u, v)$.

Prove that $f(x) \geq \theta(u, v)$.

(c) Consider the following primal problem

$$Minimize\ -2x_1 + x_2$$

Subject to $x_1 + x_2 - 3 = 0.$

$(x_1, x_2) \in X$

Where $X = \{(0,0), (0,4), (4,4), (4,0), (1,2), (2,1)\}$. Find the dual objective function. Further find the optimal solutions of primal and dual problems and show that there exists a duality gap.

Ques.5 Consider the following Quadratic Programming Problem (QPP):

(a) $$Minimize\ (x_1 - x_2)^2 + x_2(x_2 - 6) - 2x_1$$

$$subject\ to \quad x_1 + x_2 \leq 2$$

$$-x_1 + 2x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

Express the objective in the standard QPP form $c^T x + x^T D x$. Is D positive definite? Write the KKT conditions for this problem.

(b) Use Wolfe's Method to solve the following Quadratic Programming Problem:

$$Maximize\ z = x_1 + x_2 - x_1^2 + 2x_1x_2 - 2x_2^2$$

$$subject\ to. \quad 2x_1 + x_2 \leq 1$$

$$x_1, x_2 \geq 0.$$

(c) Starting from the point (0,0), perform two iterations of the steepest descent method to $minimize\ f(x_1, x_2) = 2x_1^2 + 2x_1x_2 + x_2^2 + x_1 - x_2$ over $(x_1, x_2) \in R^2$. Is the obtained solution after two iterations optimal?

Ques.6 (a) Use Newton's method to $minimize\ f(x_1, x_2) = 2x_1^2 + 2x_1x_2 + 3x_2^2, (x_1, x_2) \in R^2$ starting with the point (-2 , -3)

(b) Consider the linear fractional programming problem

$$Maximize\ \frac{c^T x + \alpha}{d^T x + \beta}$$

$$subject\ to \quad Ax = b$$

$$x \geq 0.$$

Where $x, c, d \in R^n, A \in R^{m \times n}, b \in R^m, \alpha, \beta \in R$. Let x_B be a basic feasible solution(b.f.s) with basis B. Derive the conditions to ensure that the current b.f.s can be improved to get another b.f.s with improved objective value using simplex algorithm.

(c) Solve the following linear fractional programming problem by the simplex algorithm

$$Maximize\ \frac{2x_1 + 3x_2}{x_1 + x_2 + 7}$$

$$subject\ to\ 3x_1 + 5x_2 \leq 15$$

$$4x_1 + 3x_2 \leq 12$$

$$x_1, x_2 \geq 0.$$

(11)

[This question paper contains 2 printed pages.]

Your Roll No.....

Sr. No. of Question Paper	:	**6362** **K**
Unique Paper Code	:	2352571101
Name of the Paper	:	DSC – Topics in Calculus
Name of the Course	:	**Bachelor of Arts / Bachelor of Science (Programme) with Mathematics as non- major / minor**
Semester	:	I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.
4. **All** questions carry equal marks.

1.

a) Test the continuity at $x = 0$ of the following function:

$$f(x) = \sin\frac{1}{x} \quad \text{for } x \neq 0, \qquad f(0) = 0.$$

b) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then find the value of

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}.$$

c) Find the nth differential coefficient of

$$y = \sin^5 x \cos^3 x.$$

2.

a) Write the statement of Leibnitz's Theorem, and differentiate n times the following differential equation with respect to x

$$(1 - x^2)y_2 - xy_1 + a^2y = 0.$$

b) If $u = xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$, then prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0.$$

c) Discuss the differentiability at $x = 0$ of the following function:

$$f(x) = x\frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}} \quad \text{for } x \neq 0, \qquad f(0) = 0.$$

P.T.O.

3.

a) Evaluate

$$\lim_{x\to 0} \frac{x\cos x - \log(1+x)}{(x^2)}.$$

b) State Lagrange's Mean Value Theorem and use it to show that

$$\frac{x}{1+x} < log(1+x) < x, \quad x > 0.$$

c) State Rolle's Theorem and discuss the applicability of Rolle's Theorem for the function

$$f(x) = |x|, \quad x \in [-1, 1].$$

4.

a) In the Cauchy's Mean Value Theorem, consider $f(x) = 1/x^2$ and $g(x) = 1/x$ in the interval $[a, b]$, then show that c is the harmonic mean of a and b; where $a, b, c \in (-\infty, \infty)$.

b) Evaluate

$$\lim_{x\to\infty} \left(1+\frac{a}{x}\right)^x.$$

c) State Taylor's Theorem with Lagrange's form of remainder. Prove that

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^{n-1}}{(n-1)!} + \frac{x^n\, e^{\theta x}}{n!}$$

Where $0 < \theta < 1$.

5.

a) Find the asymptotes of

$$y^3 + x^2y + 2xy^2 - y + 1 = 0.$$

b) Find the double points and points of inflexion, if any, of the curve

$$\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1.$$

c) Evaluate

$$\int_0^{\pi/8} cos^3 4x dx.$$

6.

a) By using reduction formula, evaluate

$$\int_0^1 x^2(1-x^2)^{3/2}\, dx.$$

b) Trace the curve

$$y^2(a+x) = x^2(3a-x).$$

c) Show that the curve

$$x^4 - 2ax^2y - axy^2 + a^2y^2 = 0$$

has a cusp of the second kind at the origin.

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[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 6530 **K**

Unique Paper Code : 2352571101

Name of the Paper : DSC: Topics in Calculus

Name of the Course : **B.A. / B.Sc. (Prog.) with Mathematics as Non-Major/ Minor**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **Two** parts from each question.
3. **All** questions carry equal marks.

1. (a) Discuss the continuity of the function f: $\mathbb{R} \to \mathbb{R}$, defined by

$$f(x) = |x| + 1|$$

at $x = 0$ and $x = 1$.

P.T.O.

(b) Find the n^{th} derivative of

$$y(x) = \frac{x+1}{(x-1)^2}$$

(c) If

$$u(x,y,z) = \log(x^3 + y^3 + z^3 - 3xyz).$$

then show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$$

2. (a) Examine the continuity and differentiability of the function $f: \mathbb{R} \to \mathbb{R}$ such that

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0; \\ 0, & x = 0 \end{cases}$$

at $x = 0$.

(b) If

$$y(x) = a\cos(\log x) + b\sin(\log x),$$

where a and b are constants, then prove that

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0.$$

(c) Define a homogenous function of degree n. Further, prove that if

$$u(x,y) = cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right).$$

then

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}cot(u) = 0$$

3. (a) State and prove Lagrange's Mean Value Theorem. Give its geometrical interpretation.

(b) Show that for $x > 0$,

$$\frac{x}{1+x} < \log(1+x) < x$$

(c) Evaluate

(i) $\lim_{x\to 0}(\text{cosec}\, x - \cot x)$

(ii) $\lim_{x\to 0}\left(\frac{1}{e^x-1} - \frac{1}{x}\right)$

(iii) $\lim_{x\to 1}(1-x)\tan\left(\frac{\pi x}{2}\right)$

4. (a) Verify Rolle's Theorem for

(i) $x^3 - 6x^2 + 11x - 6$, $x \in [1,3]$

(ii) $\sin(x)$, $x \in [0,\pi]$

(b) Using Maclaurin's series, expand $f(x) = \sin x$ and $g(x) = \cos x$.

(c) Evaluate $\lim_{x\to 0} \tan\left(\frac{\pi(\tan x - x)}{x^2 \tan x}\right)$.

5. (a) Find all the asymptotes of the curve

$$x^3 + 2x^2y - xy^2 - 2y^3 + 3xy + 3y^2 + 1 = 0.$$

(b) Find the nature of the origin on the curve

$$a^4y^2 = x^4(x^2 - a^2).$$

(c) If $u_n = \int_0^{\frac{\pi}{2}} \cos^n(x)\, dx$, show that $u_n = \left(\frac{n-1}{n}\right) u_{n-2}$. Hence, evaluate u_5.

6. (a) Determine the position and character of the double points on the curve

$$(x - 2)^2 = y(y - 1)^2.$$

(b) Obtain the reduction formula for $\int_0^{\frac{\pi}{2}} \sin^m(x) \cos^n(x) dx$, where m and n are even positive integers. Hence evaluate $\int_0^{\frac{\pi}{2}} \sin^2(x) \cos^3(x) dx$.

(c) Trace the curve $y = x^3 - 3ax^2$.

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[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 6761 **K**

Unique Paper Code : 2352201102

Name of the Paper : DSC : Elements of Discrete Mathematics

Name of the Course : **B.A. (Prog.)**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory.
4. Marks are indicated.

P.T.O.

1. (a) Determine the following :

(i) Compute the truth table of the statement

$(p \Rightarrow q)$ and $(\sim p) \vee q$

(ii) If $p \Rightarrow q$ is false, then determine the truth value of $(\sim(p \wedge q)) \Rightarrow q$. Explain your answer. (7.5)

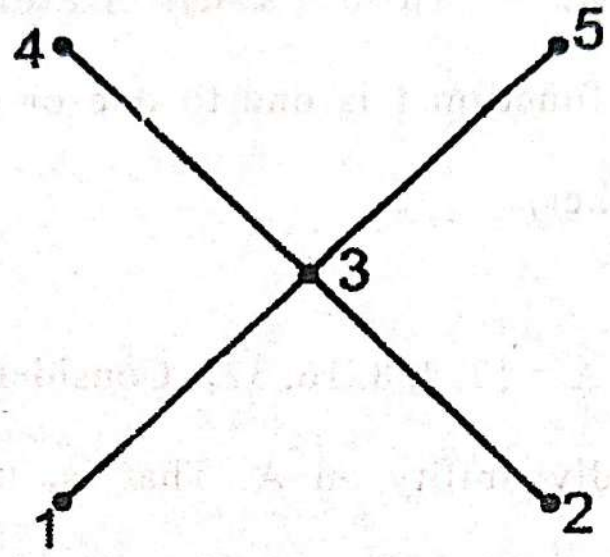
(b) Let A = Z (the set of all integers). Define the relation R on A as a R b if and only if $a \leq b + 1$. Determine whether the relation R on A is reflexive, irreflexive, symmetric, asymmetric, antisymmetric or transitive. Is R an equivalence relation on A? (7.5)

(c) Using mathematical induction, prove that $n! \geq 2^{n-1}$ for every positive integer n. (7.5)

2. (a) Let $f : R \times R \to R \times R$ be a function defined by $f(a,b) = (a+b, a-b)$. Determine whether the function f is one to one or onto (or both or neither). (7.5)

(b) Let $A = \{2, 4, 8, 16, 32\}$. Consider the partial order of divisibility on A. That is, if a and $b \in A$, $a \leq b$ if and only if a divides b. Draw the Hasse diagram of the Poset $(A, \leq)$. How many maximal and minimal elements are there in $(A, \leq)$? (7.5)

(c) Find the lower and upper bounds along with greatest lower and least upper bound of the set $B = \{1, 2, 3, 4, 5\}$, if they exist, in the following Hasse diagram. (7.5)



3. (a) Let A be the set of positive integers and B be the set of even positive integers. Let $\leq$ and $\leq'$ be the partial order "less than or equal to" on A and B respectively. Show that the function f: A $\rightarrow$ B given by f(a) = 2a is an order-isomorphism from $(A, \leq)$ to $(B, \leq')$. (7.5)

(b) Let $(L, \wedge, \vee)$ be an algebraic lattice. Define a relation '$\leq$' on L by $a \leq b$ if and only if $a \vee b = b$. Prove that $(L, \leq)$ is a lattice ordered set. (7.5)

(c) Define sublattice of a lattice. Prove that every non-empty subset of a chain L is a sublattice of L. (7.5)

4. (a) Prove that a join-homomorphism between lattices is an order homomorphism. Also, show by example that the converse need not hold. (7.5)

(b) Determine whether the Diamond lattice and the Pentagon lattice are distributive lattices. (7.5)

(c) Define a complemented lattice. Prove that if L is a distributive lattice, then each $x \in L$ has at most one complement. (7.5)

5. (a) (i) State and prove De Morgan's Laws for a Boolean algebra B. (4.5)

(ii) Show that the lattice ({1,2,3,6,9,18}, gcd, lcm) does not form a Boolean algebra for the set of positive divisors of 18. (3)

(b) (i) Find the Boolean Polynomials that induces the function and combine them to bring to a simpler form. (4)

b_1	b_2	b_3	$f(b_1,b_2,b_3)$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

(ii) Are $x(y + z)' + x' + z'$ and $(xz)'$ equivalent? (3.5)

(c) Let $X = (1,2,3\}$ and $P(X)$ be a power set of X. Show that $(P(X), \cup, \cap)$ form a Boolean Algebra. (7.5)

6. (a) Use Karnaugh map to simplify –

(i) $f(x, y, z) = x'y'z' + x'yz' + x'yz + xyz'$ (3.5)

(ii) $f(x, y, z) = x'y'z'w' + x'yz'w' + xyz'w' + xy'z'w' + x'y'zw' + x'yzw' + xyzw' + xy'zw'$ (4)

(b) Draw the symbolic representati ve of the circuit given by-

$f(x, y, z) = (x + y)'(x' + (y + z))\,(y' + z')$

(7.5)

(c) Use Quine-Mc-Cluskeys Method to simplify-

$wx'y'z + w'x'yz + wxyz' + wx'yz + w'x'y'z + wx'yz'$ (7.5)

(14)

22/02/2025
(M)

[This question paper contains 2 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5959 **K**

Unique Paper Code : 2342012303
Name of the Paper : Numerical Optimization
Name of the Course : ----
Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. The paper has **two** sections **A** and **B**. **Section A** is compulsory.
3. Attempt any **four** questions from **Section B**. Each question in this section is of **15** marks.
4. Use of a calculator is allowed.

Section A

1 (a) In case of graphical solution of an Linear Programming Problem (LPP), how can one determine whether the given problem has a **(3)**

i. Unique solution
ii. Multiple solution
iii. Infeasible solution

(b) Define convex function. Check whether the following function is convex or not. **(5)**

$$f(x,y) = \{x^2 + y^2 \geq 6\}$$

(c) Find the point of global and local maxima for the function $f(x) = x^3 - 6x$ on the interval $-2 \leq x \leq 3$ **(5)**

(d) Show that the function $f(x) = 8x_1 + 12x_2 + x_1^2 - 2x_2^2$ has only one stationary point, and the point is neither a maximum or minimum, but a saddle point. **(5)**

(e) Find out the optimum value of $f(x_1, x_2) = -\log(1 - x_1 - x_2) - \log x_1 - \log x_2$ using Newton's method till four iterations. **(6)**

(f) Find out the optimum value of $f(x) = 7x - \log x$ using Newton's method till four iterations. **(6)**

Section B

2 A company produces two types of computers (i.e. A and B). Each computer of the first type requires twice as much production time as the second type. If all computers are of the second type only, the company can produce a total of 500 computer a day. The market limits daily sales of the first and second type to 150 and 250 computers respectively. Assuming that the profit per computer is Rs. 8 (in thousands) for type A and Rs. 5 (in thousands) for type B, formulate the problem as a Linear Programming. **(15)**

3 (a) Find out the optimum value of the function $f(x) = (x-5)^2$ using line search optimization algorithm where input range of $x \in [-10, 20]$ and also draw the graph at the input range. **(7)**

(b) Find the points on a circle $x^2 + y^2 = 16$, which are closet and farthest from (1,2) using Lagrangian method. **(8)**

4 (a) Use line search method to minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1x_2 + 2{x_1}^2 + x_2^2$ where $(x_1, x_2) \in \mathbb{R}^2$ **(7)**

(b) Use the conjugate gradient method to minimize $f(x_1, x_2) = 2x_1^2 - 2x_1x_2 + 2x_2^2$, where $b = [1,0]^T$ & $(x_1, x_2) \in \mathbb{R}^2$ **(8)**

5 (a) Explain any two methods used for finite-difference approximating derivatives? **(7)**

(b) Consider the following Linear Programming Problem: **(8)**

$max\ Z = 3x + 2y$

subject to the following constraints:
$x + 2y \leq 720$
$2x + y \leq 780$
$x \leq 320$
$x, y \geq 0$

Solve the given problem using Simplex method.

6 (a) Find out the error of function $f(x) = 3x^3 + 2$ using approximating gradient method. **(7)**

(b) Find the rate of change of the function $f(x, y) = x^3y$ at $x_0 = (1, -2)$ in the direction of vector $u = (1/2, \sqrt{3}/2)$ **(8)**

7 (a) What is *smoothness*? Discuss the general formulation of constrained optimization. **(7)**

(b) Find *min* x^2 over [-5,15] by the golden section rule, using epsilon=1.5. **(8)**

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[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 6729 **K**

Unique Paper Code : 2352202002

Name of the Paper : Theory of Equations and Symmetries

Name of the Course : **BACHELOR OF ARTS (PROGRAMME) WITH MATHEMATICS AS MAJOR – DSC**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts of each question.
3. **All** questions are compulsory.
4. **All** questions carry equal marks.
5. Use of Calculator is not allowed.

P.T.O.

1. (a) Form a equation of the lowest degree with real coefficient which has $(1 + 3i)$ and $(2 - i)$ as two of its roots.

   (b) Find all the integral roots of

   $$x^4 - x^3 - 19x^2 + 49x - 30 = 0.$$

   (c) Without actual division, find the remainder when $3x^4 - 5x^3 + 10x^2 + 11x - 61$ is divided by $x - 3$.

2. (a) If $x_r = \cos\left(\frac{\pi}{2^r}\right) + i\sin\left(\frac{\pi}{2^r}\right)$, then prove that $x_1, x_2, x_3 \ldots \infty = -1$.

   (b) Find all the values of $\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{3/4}$.

   (c) Use De-Moivre's theorem to solve the equation $z^4 + 1 = 0$ for $z \in \mathbb{C}$.

3. (a) Solve the cubic equation $x^3 - 9x + 28 = 0$ by Cardan's method.

   (b) Find a necessary condition for the roots of equation $a_0x^3 + 3a_1x^2 + 3a_2x + a_3 = 0$ to be in the geometric progression.

(c) Solve the equation $x^4 - 15x^2 + 20x - 6 = 0$ by the method of Descartes.

4. (a) Solve the equation $x^3 - 18x - 35 = 0$ using Cardan's method.

(b) Transform the equation $25x^4 + 5x^3 - 7x^2 + 1 = 0$ into one with integral coefficients and unity for the coefficient of the first term.

(c) Find the solution of equation $x^4 + 3x^2 + 2x + 12 = 0$ by Descartes method.

5. (a) If α, β, γ are the roots of equation $x^3 + px^2 + qx + r = 0$ then prove that

$$\sum \alpha^2\beta = 3r - pq.$$

(b) Solve the equation $x^3 - 5x^2 - 16x + 80 = 0$ for the sum of two of its roots being zero.

(c) If α, β, γ be the roots of equation $x^3 + qx + r = 0$, then find the value of the symmetric function

$$\sum\left(\frac{\beta}{\gamma} + \frac{\gamma}{\beta}\right).$$

6. (a) Find the equation whose roots are the roots of the equation

$$x^4 - 5x^3 + 7x^2 - 17x + 11 = 0,$$

each diminished by 4.

(b) Find the value of the symmetric function $\Sigma\alpha^3$ for the equation

$x^3 + 5x^2 - 6x + 3 = 0$, whose roots are α, β, γ.

(c) Use Newton's formula to prove that for a cubic equation $x^3 + a_1x^2 + a_2x + a_3 = 0$,

$$s_4 = a_1^4 - 4a_1^2a_2 + 4a_1a_3 + 2a_2^2;$$

where $s_k = \Sigma(\alpha_1)^k$, α_1, α_2, α_3 the roots of the given cubic equation.

[This question paper contains 4 printed pages.]

Your Roll No.....

K

Sr. No. of Question Paper : 6482

Unique Paper Code : 2352572301

Name of the Paper : Differential Equations

Name of the Course : **B.A. / B.Sc. (Prog.) with Mathematics as Non-Major/ Minor**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks

1. (a) Determine the constant A such that the following equation is exact, and solve the resulting exact equation :

$$\left(\frac{Ay}{x^3}+\frac{y}{x^2}\right)dx+\left(\frac{1}{x^2}-\frac{1}{x}\right)dy=0$$

P.T.O.

(b) Solve the initial-value problem :

$2x(y + 1)dx - (x^2 + 1)dy = 0,\ y(1) = -5$

(c) Solve the differential equation

$(2xy^2 + y)dx + (2y^3 - x)dy = 0$

2. (a) Find the orthogonal trajectories of the family of ellipses having center at the origin, a focus at the point (c, 0), and semimajor axis of length 2c.

(b) In a certain bacteria culture the rate of increase in the number of bacteria is proportional to the number present.

(i) If the number triples in 5 hours, how many will be present in 10 hours?

(ii) When will the number present be 10 times the number initially present?

(c) Show that e^{-x}, e^{3x} and e^{4x} are linearly independent solutions of the differential equation

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 12y = 0$$

Also, write the general solution.

3. (a) Using the method of undetermined coefficients, find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2e^{2x}$$

(b) Solve the initial value problem

$$y'' - y' + 3y = 0, \quad y(0) = 6, \; y'(0) = -7$$

(c) Using the method of Variation of Parameters, find the general solution of the differential equation

$$y'' + y' - 6y = \sin(2x)$$

4. (a) Find the general solution of the given differential equation

$$x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} - 3y = 4x$$

(b) Write the general solution of the 10^{th} order homogenous differential equation whose auxiliary equation has the following roots :

3, 3, 3, –3, 5 + i, 5 –i, 0, 0, 1 + i, 1 – i.

Also, verify that e^{2x} and e^{3x} are solutions to the differential equation

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

(c) Find the general solution of the linear system :

$$\frac{dx}{dt} + \frac{dy}{dt} - x = -2t$$

$$\frac{dx}{dt} + \frac{dy}{dt} - 3x - y = t^2$$

5. (a) Eliminate the arbitrary function f from

$$z = x + y + f(xy),$$

and obtain the partial differential equation. Also specify the order.

(b) Find the general solution of the partial inferential equation

$$yu_y - xu_x = 1$$

(c) Reduce the equation

$$yu_x + u_y = x$$

into canonical form and obtain the general solution.

6. (a) Apply the method of separation of variable $u(x, y) = f(x) + g(y)$ to solve the equation

$$u_x^2 + u_y^2 = 1$$

(b) Solve the Cauchy problem

$$yu_x + xu_y = 0, \quad u(0,y) = e^{-y^2}$$

(c) Classify the partial differential equation

$$yu_{xx} - xu_{yy} = 0, \quad x > 0, \quad y > 0.$$

Also, obtain the general solution by reducing it to canonical form.

(17) AB-08-24-30 C-154

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 7045 **K**

Unique Paper Code : 2352572301

Name of the Paper : Differential Equations

Name of the Course : **B.A. / B.Sc. (Prog.) with Mathematics as Non-Major/ Minor**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt all questions by selecting two parts from each question.
3. **All** questions carry equal marks.

1. (a) Determine the most general function N(x, y) such that the following equation is exact, and solve the resulting exact equation:

$$(x^3 + xy^2)dx + N(x, y)dy = 0$$

P.T.O.

3. (a) Find the solution of the differential equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 0, \quad y(0) = -3, \quad y'(0) = -1.$$

(b) Using the method of undetermined coefficients, find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = \cos(4x)$$

(c) Using the method of Variation of Parameters, find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + y = \cot(x).$$

4. (a) Find the general solution of the given differential equation

$$x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 4x - 6$$

(b) Find the general solution of the linear system

$$\frac{dx}{dt} + \frac{dy}{dt} - 2y - 4y = e^t$$

$$\frac{dx}{dt} + \frac{dy}{dt} - y = e^{4t}$$

(c) Show that $x = 2e^{5t}, y = e^{5t}$ and $x = e^{-t}, y = -e^{-t}$ are two linearly independent solutions on every interval $a \leq t \leq b$ of the homogeneous linear system :

$$\frac{dx}{dt} = 3x + 4y$$

$$\frac{dy}{dt} = 2x + y$$

Also, write the general solution.

5. (a) Form partial differential equation by eliminating arbitrary constants a and b from the equation

$$2z = (ax + y)^2 + b$$

(b) Find the general solution of the partial differential equation

$$(x - y)y^2u_x + (x - y)x^2u_y = (x^2 + y^2)u$$

(c) Solve the Cauchy problem

$$u_x + xu_y = 0, \quad u(0, y) = \sin y$$

6. (a) Solve

$$y^2u\frac{\partial u}{\partial x} + u^2x\frac{\partial u}{\partial y} = -xy^2$$

(b) Use $v = \ln u$ and then $v(x, y) = f(x) + g(y)$ to solve the equation

$$x^2u_x^2 + y^2u_y^2 = u^2$$

using the method of separation of variables.

(c) Reduce the equation: $u_x + xu_y = y$ to canonical form and hence find the general solution.

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[This question paper contains 2 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 7140 **K**

Unique Paper Code : 2353572001

Name of the Paper : COMBINATORICS

Name of the Course : **B.Sc. (Prog.) – DSE**

Semester : III

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of a Calculator is not allowed.

Q1. (a) How many non-empty different collections can be formed from 6 identical bananas and 7 identical mangoes?

(b) How many ways are there to arrange the seven letters in the word 'SUCCESS'? In how many of these arrangements do the two Cs appear together?

(c) If there are n objects, with r_1 of type 1, r_2 of type 2, . . . , and r_m of type m, where $r_1 + r_2 + \cdots + r_m = n$, then prove that the number of arrangements $P(n; r_1, r_2, \ldots, r_m)$ of these n objects is given by

$$P(n; r_1, r_2, \ldots, r_m) = \binom{n}{r_1}\binom{n-r_1}{r_2}\binom{n-r_1-r_2}{r_3}\cdots\binom{n-r_1-r_2\cdots-r_{m-1}}{r_m} = \frac{n!}{r_1!r_2!\ldots r_m!}.$$

Q2. (a) In bridge, the 52 cards of a standard card deck are randomly dealt 13 apiece to four players. What is the probability that each player has one Ace?

(b) Let M be a multi-set with r distinct objects $x_1, x_2, \ldots, x_r$ each with infinite multiplicity. Let $\bar{C}(r, k)$ denote the number of k-combinations of M. Then, prove that

$$\bar{C}(r, k) = \binom{k+r-1}{k}.$$

(c) Let X be a finite set and let P_i ; $i = 1,2,\ldots,n$ be a set of n properties satisfied by (some of) the elements of X. Let A_i denote the set of those elements of X that satisfy the property P_i. Show that the size of the set $\overline{A_1} \cap \overline{A_2} \cap \cdots \cap \overline{A_n}$ of all those elements that do not satisfy any one of these properties is given by

$$\overline{A_1} \cap \overline{A_2} \cap \cdots \cap \overline{A_n} = |X| - \sum_{i=1}^{n}|A_i| + \sum_{1\le i<j\le n}|A_i \cap A_j| - \cdots$$
$$+\{(-1)^k \sum_{1\le i_1<i_2<\cdots<i_k\le n}|A_{i_1} \cap A_{i_2} \cdots \cap A_{i_k}|\}$$
$$+ \cdots + (-1)^n|A_1 \cap A_2 \cdots \cap A_n|.$$

Q3. (a) Find a generating function for modelling the number of distributions of 16 chocolate bunny rabbits into four Easter baskets with at least three rabbits in each basket. Which coefficient do we want?

(b) How many ways are there to select 25 toys from seven types of toys with between two and six of each type?

P.T.O.

(c) Define exponential generating function. Find the number of different r arrangements of objects chosen from unlimited supplies of n types of objects.

Q4. (a) Find and solve a recurrence relation for the number of matches played in a tournament with n players, where n is a power of 2.

(b) Find a formula for a_n satisfying the relation $a_n + 2a_{n-2} + a_{n-4} = 0$ with initial condition $a_0 = 0, a_1 = 1, a_2 = 2$ and $a_3 = 3$.

(c) Find a functional equation for $g(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n + \cdots$ where a_n satisfies the recurrence relation $a_n = n + a_{n-1}$ for all $n \geq 1$.

Q5. (a) An elf has a staircase of n stairs to climb. Each step it takes can cover either one or two or three stairs. Find a recurrence relation for a_n, the number of different ways for the elf to ascend the n stairs.

(b) For a natural number n, let $p(n)$ denote the number of partitions of n. Then find $p(5)$ and $p(7)$.

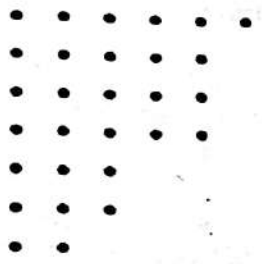
(c) Define the conjugate of a partition $\pi = (a_1, a_2, \ldots, a_k)$. Find the conjugate of the partitions $\pi_1 = (10,5,5,3,2,2,1)$ *and* $\pi_2 = (7,5,4,2,1,1)$.

Q6. (a) Show that

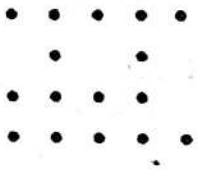
$$P_{\text{all part partitions}}(x) = \prod_{j=1}^{\infty}(1 - x^j)^{-1}.$$

(b) Define Ferrers Diagram. Which of the following is/are Ferrers Diagrams? Give Reasons.

(i)



(ii)



Also, draw Ferrers diagram for the partition $\pi = (4,3,3,2,1)$.

(c) Define Durfee Square of Ferrers Diagram. Show that for any $m \in \mathbb{Z}^+$, Durfee Square D_m is also a Ferrers Diagram of some partition. Draw the Ferrers Diagram of the partition $\pi = (10,5,4,3,3,3,3,2)$ and highlight the Durfee Square in the same.

[This question paper contains 2 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 14158 **K**

Unique Paper Code : 2352572301

Name of the Paper : Differential Equations

Name of the Course : **B.A./B.Sc. (Prog.) with Mathematics as Non-Major/Minor**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting two parts from each question.
3. **All** questions carry equal marks.

Q1. (a) Determine the most general function $N(x, y)$ such that the following equation is exact, and solve the resulting exact equation:

$$(x^3 + xy^2)\,dx + N(x, y)\,dy = 0$$

(b) Solve the differential equation

$$y^2\,dx - (1 - 3xy)\,dy = 0$$

(c) Solve the differential equation

$$(2x + \tan y)\,dx + (x - x^2 \tan y)\,dy = 0$$

Q2. (a) Find the orthogonal trajectories of the family of circles which are tangent to the y axis at the origin.

(b) A large tank initially contains 100 gal of brine in which 10 lb of salt is dissolved. Starting at t = 0, pure water flows into the tank at the rate of 5 gal/min. The mixture is kept uniform by stirring and the well-stirred mixture leaves the tank at the rate of 2 gal/min. How much salt is in the tank at the end of 15 minutes and what is the concentration at that time?

(c) Consider the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0.$$

(i) Show that e^x and xe^x are linearly independent solutions of this equation on the interval $-\infty < x < +\infty$.

(ii) Find the solution that satisfies the condition $y(0) = 1, y'(0) = 4$. Explain why this solution is unique.

Q3. (a) Find the solution of the differential equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 0,\ y(0) = -3,\ y'(0) = -1.$$

(b) Using the method of undetermined coefficients, find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = cos(4x)$$

P.T.O.

(c) Using the method of Variation of Parameters, find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + y = cot(x)$$

Q4. (a) Find the general solution of the given differential equation

$$x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 4x - 6$$

(b) Find the general solution of the linear system

$$\frac{dx}{dt} + \frac{dy}{dt} - 2x - 4y = e^t$$
$$\frac{dx}{dt} + \frac{dy}{dt} - y = e^{4t}$$

(c) Show that $x = 2e^{5t}, y = e^{5t}$ and $x = e^{-t}, y = -e^{-t}$ are two linearly independent solutions on every interval $a \le t \le b$ of the homogeneous linear system:

$$\frac{dx}{dt} = 3x + 4y$$
$$\frac{dy}{dt} = 2x + y$$

Also, write the general solution.

Q5. (a) Form partial differential equation by eliminating arbitrary constants a and b from the equation

$$2z = (ax + y)^2 + b$$

(b) Find the general solution of the partial differential equation

$$(x - y)y^2u_x + (x - y)x^2u_y = (x^2 + y^2)u$$

(c) Solve the Cauchy problem

$$u_x + xu_y = 0, \quad u(0, y) = \sin y$$

Q6. (a) Solve

$$y^2u\frac{\partial u}{\partial x} + u^2x\frac{\partial u}{\partial y} = -xy^2$$

(b) Use $v = ln\, u$ and then $v(x,y) = f(x) + g(y)$ to solve the equation

$$x^2u_x^2 + y^2u_y^2 = u^2$$

using the method of separation of variables.

(c) Reduce the equation: $u_x + xu_y = y$ to canonical form and hence find the general solution.

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[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 6439 **K**

Unique Paper Code : 2352203501

Name of the Paper : Elements of Real Analysis

Name of the Course : **B.A. / B.Sc. (P) with Mathematics as Non-Major/ Minor**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory and carry equal marks.
3. Attempt any **two** parts from each question.

1. (a) Show that for all $x, y \in \mathbb{R}$, $\big||x|-|y|\big| \leq |x-y|$.

(b) Show that for all $x, y \in \mathbb{R}$,

$$0 \le x < y \Rightarrow \frac{x}{1+x} < \frac{y}{1+y}.$$

(c) Define infimum of non-empty subsets of $\mathbb{R}$. Find supremum and infimum of the following sets :

(i) $\left\{\frac{1}{x} : 1 < x < 2\right\}$

(ii) $\left\{1 + \frac{1}{2^n} : n \in \mathbb{N}\right\}$

2. (a) Prove that a set cannot have more than one least upper bound.

(b) Prove that a sequence cannot converge to more than one real number.

(c) Prove that if $x > 1$, then $\lim_{n\to\infty} \frac{n}{x^n} = 0$.

3. (a) Show that $\lim_{n\to\infty} \frac{n^2 + 3n + 5}{2n^2 + 5n + 7} = \frac{1}{2}$.

(b) Using the definition of limit of a sequence, show that

$$\lim_{n\to\infty} \frac{3+2\sqrt{n}}{\sqrt{n}} = 2.$$

(c) Prove that $\lim_{n\to\infty} \frac{1}{\sqrt{n!}} = 0.$

4. (a) Show that the sequence $\langle S_n \rangle$ defined by $S_1 = \sqrt{2}$, $S_{n+1} = \sqrt{(2S_n)}$ converges to 2.

(b) Prove that a sequence converges if and only if it is a Cauchy sequence.

(c) Prove that every sequence has a monotonic subsequence.

5. (a) Test the convergence of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n \operatorname{Sin} n\alpha}{n^3}, \alpha \text{ being real.}$$

(b) State Leibnitz test for convergence of an alternating series of real number

$$\sum_{n=1}^{\infty}(-1)^n u_n, \text{ where } u_n > 0.$$

Apply it to test the convergence and absolute convergence of the series

$$1-\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}-\frac{1}{4\sqrt{4}}+\cdots.$$

(c) Show that the series $\sum_{n=2}^{\infty}\frac{1}{n(\log n)^p}$ is convergent if $p > 1$ and divergent if $0 < p \leq 1$.

6. (a) Test for convergence the series

$$\frac{1}{\sqrt{1.2}}+\frac{1}{\sqrt{2.3}}+\frac{1}{\sqrt{3.4}}+\cdots.$$

(b) Test for convergence the series whose n^{th} term is $\frac{\sqrt{n+1}-\sqrt{n-1}}{n}$.

(c) Test the conditional and absolute convergence of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt{n}}$.

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[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 6652 **K**

Unique Paper Code : 2353200007

Name of the Paper : COMBINATORICS

Name of the Course : **B.A. (Prog.) – DSE**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of a Calculator is not allowed.

P.T.O.

1. (a) There are 7 books of Hindi, 3 books of English and 9 books of German. All books are different. How many ways are there to pick an (unordered) pair of two books not both in the same language?

   (b) How many different 9-digit binary sequences are there with six 1's and three 0's?

   (c) A committee has to be formed from eight women and five men. In how many ways can the committee be formed if it must contain equal numbers of women and men, and the committee can be of any positive size?

2. (a) In bridge, the 52 cards of a standard deck are randomly dealt 13 each to four players. What is the probability that one specific player has all 13 spades?

   (b) If a fair coin is flipped 11 times, what is the probability of 9 or more heads?

   (c) Prove that the number of derangements D_n on n letters is given by

$$D_n = n!\left\{1 - 1 + \frac{1}{2!} - \cdots + (-1)^n \frac{1}{n!}\right\}.$$

3. (a) Find an exponential generating function for the number of distributions of r distinct objects into n different boxes with exactly m nonempty boxes.

(b) Build a generating function for a_r, the number of integer solutions to the equation:

$$e_1 + e_2 + e_3 + e_4 + e_5 + e_6 = r, \quad 0 \leq e_i \leq 6.$$

(c) Verify the binomial identity

$$\binom{2n}{n} = \binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \cdots + \binom{n}{n}^2.$$

4. (a) An elf has a staircase of n stairs to climb. Each step it takes can cover either one stair or two stairs. Find a recurrence relation for a_n, the number of different ways for the elf to ascend the n-stair staircase.

(b) Solve the recurrence relation $a_n = 3a_{n-1} - 2a_{n-2}$ with initial conditions $a_0 = a_1 = 1$.

(c) Find functional equation for the generating function whose coefficients satisfy the following relation:

$$a_n = 2a_{n-1} + 2^n, \; a_0 = 1.$$

5. (a) Find the coefficient of x^{12} in

$$(x^2 + x^3 + x^4 + x^5 + x^6 + x^7)^3.$$

(b) Define a partition of a positive integer n into k parts. Write the set of all partitions of 7. Hence, deduce the value of p(7).

(c) Define the conjugate of a partition $\pi = (a_1, a_2, \ldots, a_k)$. Find the conjugate of the partitions $\pi_1 = (9,4,4)$ and $\pi_2 = (7,5,4,2,1,1)$.

6. (a) Show that

$$P_{\text{all part partitions}}(x) = \prod_{j=1}^{\infty} (1 - x^j)^{-1}.$$

(b) Define Ferrers Diagram. Write any two partitions of 12 into 5 parts and draw their corresponding Ferrers diagram.

(c) Show that for any $m \in \mathbb{Z}^+$, Durfee Square D_m is also a Ferrers Diagram of some partition. Draw the Ferrers Diagram of the partition $\pi = (10, 5, 5, 3, 2, 2, 1)$ and highlight the Durfee Square in the same.

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[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 6653 **K**

Unique Paper Code : 2353200008

Name of the Paper : ELEMENTS OF NUMBER THEORY

Name of the Course : **Discipline Specific Elective**

Semester : V (Part 3)

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting **two** parts from each question.
3. **All** questions carry equal maria.
4. Use of Calculator not allowed.

P.T.O.

1. (a) If n is odd integer, show that $n^4 + 4n^2 + 11$ is of the form 16k, for some integer k.

   (b) By using division algorithm show that the cube of any integer has one of the forms 9k, 9k + 1, 9k + 8, for some integer k.

   (c) Prove that gcd (a,b) = 1, then gcd (a+b, ab) = 1.

2. (a) State Fundamental Theorem of Arithmetic and prove that for any integer $k \neq 0$, gcd (ka, kb) = |k| gcd (a, b).

   (b) Show that 41 divides $2^{20} - 1$.

   (c) Solve the following simultaneous congruences :

   $x \equiv 2(\text{mod}3)$, $x \equiv 3(\text{mod}5)$, $x \equiv 2(\text{mod}7)$.

3. (a) If gcd(a, 35) = 1, show that $a^{12} = 1(\text{mod}35)$.

   (b) State Wilson's Theorem and use it to find the remainder when 15! is divided by 17.

   (c) Prove that $\tau(n)$ is an odd integer if and only if n is a perfect square.

4. (a) Find the highest power of 5 dividing 1000! and highest power of 2 dividing 50!.

(b) Show that $\phi(n)$ is an even integer for $n>2$ and calculate $\phi(5040)$.

(c) Prove that:

$$1^{p-1} + 2^{p-1} + 3^{p-1} + \cdots + (p-1)^{p-1} \equiv -1 \pmod{p},$$

where p is an odd crime

5. (a) Employ Hills Cipher to encrypt the message BUYNOW using the congruences

$$C_1 \equiv 2P_1 + 3P_2 \pmod{26}$$

$$C_1 \equiv 5P_1 + 8P_2 \pmod{26}.$$

(b) Prove that the integer $n = 2^{10}(2^{"}-1)$ is not a perfect number by showing that

$$\sigma(n) \neq 2n.$$

(c) Show that the Mersenne number M_{23} is not a prime number.

6. (a) Prove that the Eermat number $F_5 = 2^{2^5} + 1$ is divisible by 641.

(b) If the Caesar cipher produced KDSSBELUWKGDB, what is the plaintext message?

(c) Show that the sum of the squares of the first n Fibonacci numbers is given by the formula

$$u_1^2 + u_2^2 + u_3^2 + \cdots + u_n^2 = u_n u_{n+1}.$$

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[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 7041 **K**

Unique Paper Code : 2352203501

Name of the Paper : Elements of Real Analysis

Name of the Course : **B.A. / B.Sc. (P) with Mathematics as Non-Major / Minor**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory and carry equal marks.
3. Attempt any **two** parts from each question.

1. (a) Let F be any ordered field and $x, y, z \in F$. Show that if $x < y$ and $y < z$, then $x < z$.

   (b) Prove that for all $x, y \in \mathbb{R}$,

   $$\min\{x,y\} = -\max\{-x, -y\}.$$

P.T.O.

(c) Define an upper bound of a non-empty subset of R. Find supremum and infimum of the following sets :

(i) $\left\{\frac{x+1}{x} : x > 2\right\}$

(ii) $\left\{2 - \frac{1}{n} : n \in \mathbb{N}\right\}$

2. (a) Let F be an Archimedean ordered field, $A \subseteq F$ & $u \subseteq F$. Then $u = \sup A \Leftrightarrow$ for all $\epsilon > 0$ and $x \in A$, $x < u + \epsilon$, and there exists $x \in A$ such that $x > u - \epsilon$.

(b) Show that if $\{x_n\}$ is a convergent sequence and $c \in \mathbb{R}$, then $\lim_{n\to\infty} cx_n = c \lim_{n\to\infty} x_n$.

(c) Prove that if $|a| < 1$, then $\lim_{n\to\infty} a^n = 0$.

3. (a) Prove that a sequence cannot converge to more than one limit.

(b) Show that $\lim_{n\to\infty} n^{\frac{1}{n}} = 1$.

(c) Show that the sequence $\langle f_n \rangle$ where

$f_n = 1 + \frac{1}{3} + \frac{1}{3^2} + \cdots\cdots \frac{1}{3^{n-1}}$, converges. Find $\lim_{n\to\infty} f_n$.

4. (a) Prove that a monotone sequence converges if and only if it is bounded.

(b) Show that the sequence $\langle s_n \rangle$, where $s_n = \frac{(-1)^{n-1}}{n}$ converges to zero.

(c) Prove that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ exists and lie between 2 and 3.

5. (a) State Cauchy's n^{th} Root test for the convergence of a positive term series. Apply it to test for convergence the series

$$\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-1} + \cdots.$$

(b) Test for convergence the series

(i) $\sum \sin\frac{1}{n^2}$

(ii) $\sum \frac{1}{\sqrt{n}} \tan\frac{1}{n}$

(c) Test for convergence the series

(i) $\sum_{n=1}^{\infty} \frac{2^{n-1}}{3^n+1}$

(ii) $\sum_{n=1}^{\infty} \frac{r^n}{n!}$

6. (a) State D'Alembert's Ratio test for the convergence of positive term series. Use it to test the convergence of the series $\sum_{n=1}^{\infty} \frac{5^n}{n^2+5}$.

(b) Test for convergence the series

$$1+\frac{2}{5}x+\frac{6}{9}x^2+\frac{14}{17}x^3\frac{30}{33}x^4+\cdots$$

(c) Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\log(n+1)}$ is conditionally convergent.

(2)

[This question paper contains 4 printed pages.]

Your Roll No.....

K

Sr. No. of Question Paper : 6388

Unique Paper Code : 2342574702

Name of the Paper : Design and Analysis of Algorithm

Name of the Course : Discipline Specific Core DSC B.A.(P) CS M/N.M.

Semester : VII

Duration : 3 Hours

Maximum Marks :90

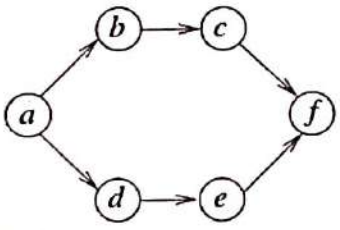
Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Section A is compulsory contain 30 marks
3. Attempt any four questions from Section B.
4. Parts of a question must be answered together.

Section A

1. (a) What is a stable sorting algorithm? List any two stable sorting algorithms. Also mention their time complexity. (4)

   (b) If all elements in an array are identical, which of the following sorting algorithms -Insertion Sort, Quick Sort, Merge Sort will perform best? Justify your answer in terms of number of comparisons and swaps. (3)

   (c) 

(3)

From the above directed graph List all the topological orderings of the graph.

   (d) Define a spanning tree. How many edges does a spanning tree of a graph with n vertices have?If the graph is disconnected, can we find a Minimum Spanning tree of the graph? (5)

P.T.O.

(e) Define the following term in the context of graphs giving suitable examples of each: (4)

(i) path

(ii) cycle

(iii) circuit

(f) Solve the following Subset Sum problem using Dynamic Programming: (4)

Set: {3, 5, 7, 10, 12}

Target sum: 15

(g) Hash table size m = 11. Use the division method h(k) = kmod 9. Insert keys in this order: {27, 18, 29, 28, 39, 13, 16}. (5)

Resolve collisions by linear probing (step = +1).

(a) For each key, compute the initial hash index.

(b) Show the final hash table contents.

(h) What is the complexity of linear search and binary search? What is the prerequisite condition to apply binary search to a list of numbers. (3)

Section B

Attempt any 4 out of 6

2. (a) What is an optimal solution? Does greedy method always provide the optimal solutions? Give an example where greedy method might not solve the problem optimally. (5)

(b) Consider capacity of the knapsack as 30 kg. Solve the following problem using (10)

(i) Fractional Knapsack Problem

(ii) 0/1 knapsack problem.

Calculate the total value that can be carried. explain which approach obtains more profit for this problem.

Item	Value	Weight
I1	120	24
I2	60	10
I3	40	6
I4	75	15

3. (a) Perform Quick Sort on the following array: (5)

A = {50, 20, 60, 10, 30} using last element as pivot element.

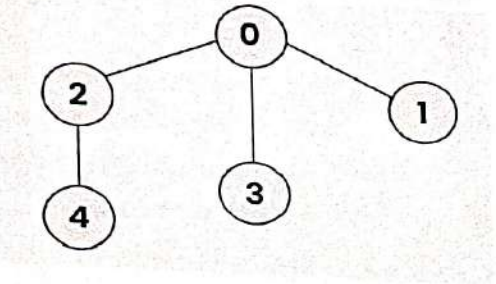
Show the array at all intermediate steps.

(b) Differentiate between Heap and Max-Heap. (10)

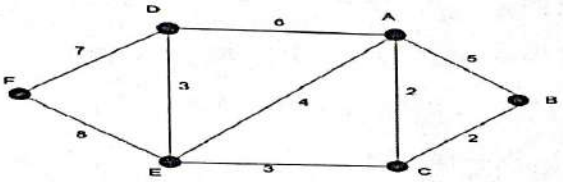
Perform Heap Sort on the following array to build a Max heap. Show the heap after each swap.

[5, 12, 11, 13, 4, 6, 7]

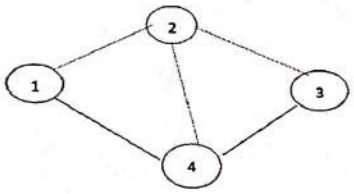
4. (a) Consider the following graph. Generate Depth first traversal (DFS) for thegraph taking 0 as source vertex. (5)



(b) Using Prim's Algorithm, find the cost of minimum spanning tree (MST) of the given graph taking vertex A as the root. Show all the intermediate steps indicating how the algorithm selects each edge. (10)



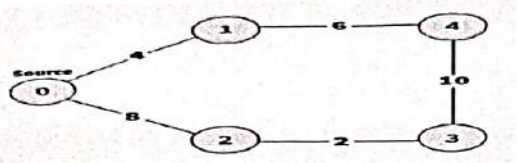
5. (a) Define a bipartite graph. Given the following graph: (5)



Check whether the graph is bipartite. If yes, provide the two partitions; if not, explain why it is not bipartite.

(b) Define the Single Source Shortest Path (SSSP) problem. How is it different from the All-Pairs Shortest Path (APSP) problem? (10)

Apply Dijkstra's algorithm to the following weighted graph and find the shortest path from source vertex 0 to all other vertices.



6. (a) What is a hash function? Explain the desirable properties of a good hash function. Explain chaining technique used for collision resolution in hash tables. (5)

(b) A hash table has size 11. (10)
The two hash functions are:

$$h_1(k) = k \bmod 11$$
$$h_2(k) = 1 + (k \bmod 10)$$

Insert the following keys using double hashing:

25, 36, 49, 51, 62,84

Show the content of all hash table.

7. (a) Apply Strassen's algorithm to multiply the following two 2 x 2 matrices and find CA X B: (5)

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$$

(b) Explain Weighted Interval Scheduling Problem. (10)

For the following jobs, find the optimal schedule:

Job	Start	Finish	Profit
1	1	2	20
2	3	5	20
3	6	9	100
4	2	8	200

Show step-by-step calculation of Memorization table entries and the final selection of jobs.

(1000)

(24)

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 6400 **K**

Unique Paper Code : 2352204701

Name of the Paper : Numerical Methods, DSC

Name of the Course : **Bachelor of Arts / Bachelor of Science (Programme)**

Semester : VII

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the **six** questions are compulsory. Attempt any **two** parts from each question.
3. Each question carries equal marks.
4. Use of Non-Programmable Scientific Calculator is allowed.

1. (a) Round-off the number 46.82135 correct to four significant digits and then calculate the absolute, relative and percentage errors.

   (b) Determine the number of significant digits in the following numbers:

   (i) 235700

   (ii) 0.0036

   (iii) 4.56700

   (iv) 30.00056

   (v) 4.5×10^{8}

(c) Perform three iterations of Bisection Method to find root of $x^2 - 2 = 0$ in $]1,2[$.

2. (a) Using Secant Method, find a real root of the equation $xe^x - 1 = 0$, given that the root lies between 0 and 1. Perform three iterations, by taking $x_0 = 0$ and $x_1 = 1$.

(b) By performing three iterations of Regula-Falsi method, find a root of the equation $3x + \sin(x) - e^x = 0$ in $]0,1[$.

(c) Do three iterations of the Newton Raphson's Method to find cube root of 25, taking initial approximation $x_0 = 3$.

3. (a) Using Gauss Elimination method, solve the following system of linear equations:

$$2x - y + 3z = 9$$

$$x + y + z = 6$$

$$x - y + z = 2$$

(b) Find the Lagrange interpolating polynomial which fits into the given data. Also, approximate the value of f(10)

x	5	6	9	11
f(x)	12	13	14	16

(c) Show that : $\mu = \frac{1}{2}\left[E^{1/2} + E^{-1/2}\right]$

(Note: Symbols have their own meaning)

4. (a) Starting with initial vector (x,y,z) = (0,0,0) perform three iterations of Gauss Seidel Method to solve the following system of equations :

$$20x + y - 2z = 17$$

$$3x + 20y - z = -18$$

$$2x - 3y + 20z = 25$$

(b) Find the unique polynomial P(x) of degree 3 or less using Newton's interpolating Formula for the following data :

x	0	2	3	6
f(x)	648	704	729	792

(c) Calculate the 3rd divided difference of $\frac{1}{x}$, based on the points x_0, x_1, x_2, x_3.

5. (a) Evaluate $I = \int_0^{\pi/2} \sqrt{\sin x}\, dx$ using Simpson's rule.

(b) Use the Taylor's expansion for a function f to derive the central divided difference formula of the first order derivative at x_i with step-size h,

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h}.$$

(c) Given the differential equation $y' = x^2 + y$ with $y(0) = 1$, compute $y(0.02)$ using Euler's method and step size, $h = 0.01$.

6. (a) Approximate y'(1) and y'(2) using backward difference formula if

x	−1	0	1	2	3
y	0.5	1	2	4	8

(b) Consider the differential equation $\frac{dy}{dt} = t^2 - y$ with y(0) = 1. Use Euler's method to compute y(3) with step-size h = 1.

(c) Use the Trapezoidal rule with n = 4 to evaluate $\int_3^7 x^2 \log x \, dx$.

(25) Mathematics

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 8904 **K**

Unique Paper Code : 2353010015

Name of the Paper : Research Methodology

Name of the Course : **Bachelor of Arts / Bachelor of Science (Programme)**

Semester : VII – DSE

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All **six** questions are compulsory, attempt any **two** parts from each question.
3. **All** questions carry equal marks.

1. (a) Explain the format of a mathematical research article.

   (b) What all challenges may appear while doing mathematical research? Explain your answer.

   (c) What do you understand by Mathematical survey?

P.T.O.

2. (a) Explain the concepts of learning and writing mathematics.

(b) Write 8 key points for giving "Good Presentation".

(c) Discuss some common errors one may make while writing mathematics paper? Explain how to correct them with examples.

3. (a) State and explain the difference between the tabular environment and the array environment in LaTeX. Provide a short code snippet demonstrating how to center the content within a column in a table.

(b) Explain the syntax and purpose of defining a new command in LaTeX using \newcommand. Create a new command called \researcher that accepts one mandatory argument (the name of the author) and prints the name in bold text.

(c) Discuss how the hierarchy of sectioning commands (\section, \subsection, \subsubsection) aids in structuring a research document. Write the necessary LaTeX commands to generate a Table of Contents at the beginning of the document.

4. (a) What is PSTricks? Discuss the utility and advantages of using this code-based package for generating technical illustrations directly within the LaTeX environment. Also, write the necessary LaTeX code to draw a square centered at the origin (0,0) with a total side length of 2 units, making sure to define the vertices using the appropriate PSTricks command.

   (b) Describe the use of the commands Ellipsis and Delimiters in LaTeX environment. Provide examples showing how to display these commands.

   (c) Write a comprehensive LaTeX source program to display the following mathematical equations clearly and properly formatted in a display environment:

   (i) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

   (ii) $\int_a^b f(x)\ dx = F(b) - F(a)$

   (iii) $\frac{\partial}{\partial x}(x^2 y + e^{xy}) = 2xy + ye^{xy}$

5. (a) How is arXiv different from Google Scholar? Explain.

(b) Discuss about the various databases used for exploring mathematical journals.

(c) Write the full forms of MAA and AMS. Explain how they are helpful in doing mathematical research.

6. (a) What is the best way to prevent plagiarism and fraud in research? Name three software that help us identify and prevent plagiarism?

(b) What is Q1, Q2, Q3, Q4 journal impact factor?

(c) Why are journal metrics important? How can one assess journal metrics?

26

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 8910 **K**

Unique Paper Code : 2353200012

Name of the Paper : Advanced Linear Algebra

Name of the Course : **B.A./B.Sc. (P)**

Semester : VII – DSE

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of Calculator is not allowed.

P.T.O.

1. (a) Let Tbe a linear operator on $\mathbb{R}^2$ defined by

$$T(a, b) = (a - 3b, 2a + b),$$

and let $\beta = \{(1,0), (0,1)\}$ and $\beta' = \{(1,1), (1,2)\}$ be the ordered bases of $\mathbb{R}^2$. Find the change of coordinate matrix Q that change β'-coordinates into β-coordinates. Also, verify that

$$[T]_\beta = Q^{-1}[T]_{\beta'}Q. \qquad (2.5+5)$$

(b) Let $V = P_1(\mathbb{R})$, the vector space of all real polynomials of degree less than or equal to 1. For $p(x) \in V$, define $f_1, f_2 \in V^*$ by

$$f_1(p(x)) = \int_{-1}^{1} p(t)dt \text{ and } f_2(p(x)) = \int_{0}^{1} p(t)dt.$$

Prove that $\{f_1, f_2\}$ is a basis for V^*, then find a basis for V for which it is the dual basis. (3+4.5)

(c) Let V denotes a finite-dimensional vector space. Define the annihilator S^0 of a subset S of V and prove that S^0 is a subspace of V^*. Further, if W is a subspace of V and $x \notin W$, then prove that there exists $f \in W^\circ$ such that $f(x) \neq 0$.

(1.5+2+4)

2. (a) Let $A = \begin{pmatrix} i & 1 \\ 2 & -i \end{pmatrix} \in M_{2\times 2}(\mathbb{C})$. Determine all the eigenvalues of A. For each eigenvalue λ of A, find the set of eigenvectors corresponding to λ. Also, find a basis for $\mathbb{C}^2$ consisting of eigenvectors of A. (2.5+5)

(b) For $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \in M_{2\times 2}(\mathbb{R})$, find an expression for A^n, where n is a positive integer. (7.5)

(c) Let $P_2(\mathbb{R})$ denote the vector space of all real polynomials of degree less than or equal to 2. Let T be a linear operator on $P_2(\mathbb{R})$ defined as

$$T(f(x)) = f(1) + f(0)x + f(-1)x^2.$$

Prove that $P_2(\mathbb{R})$ is the direct sum of the eigenspaces of T. (7.5)

3. (a) Let T be a linear operator on a vector space V and z be a nonzero vector of V. Define a T-cyclic subspace of V generated by z. Show that it is a T-invariant subspace of V. Further, for $V = P_3(\mathbb{R})$, the vector space of all real polynomials of degree less than or equal to 3, $T(f(x)) = f''(x)$, and $z = x^3$, find an ordered basis for the T-cyclic subspace of V generated by z. (1+3+3.5)

(b) State the Cayley-Hamilton Theorem for a linear operator on a finite-dimensional vector space V. Let T be the linear operator on $P_2(\mathbb{R})$ defined by $T(f(x)) = f(0) + f(1)(x + x^2)$. Verify the Cayley-Hamilton Theorem for the linear operator T. (2+5.5)

(c) Let

$$A = \begin{pmatrix} 11 & -4 & -5 \\ 21 & -8 & -11 \\ 3 & -1 & 0 \end{pmatrix}.$$

Find a basis for each generalized eigenspace of L_A consisting of a union of disjoint cycles of generalized eigenvectors. Then find a Jordan canonical form J of A. (5.5+2)

4. (a) Define the minimal polynomial of a linear operator on a finite-dimensional vector space. Let T be the linear operator on $P_2(\mathbb{R})$ defined by $T(f(x) = -xf''(x) + f'(x) + 2f(x)$. Find the minimal polynomial of T. (2+5.5)

(b) Let V = C([0,1]), die vector space of real-valued continuous functions on [0,1]. For f, g ∈ V, define :

$$< f, g >= \int_0^1 f(t)g(t)dt.$$

Prove that V with $\langle .,. \rangle$ defined above is an inner product space. For $f(t) = t$ and $g(t) = e^t$, compute $\langle f, g \rangle$. (5.5+2)

(c) Let $x = (2, 1 + i, i)$ and $y = (2-i, 2, 1 + 2i)$ be vectors in $\mathbb{C}^3$. Compute $\langle x,y \rangle$, $\|x\|$, $\|y\|$, and $\|x + y\|$. Then verify both the Cauchy-Schwarz inequality and the triangle inequality for the given vectors x and y. (4+2+1.5)

5. (a) Let $P_3(\mathbb{R})$ be the space of all real polynomials of degree less than or equal to 3 with inner product

$$\langle f,g \rangle = \int_0^1 f(t)g(t)dt.$$

Also, let $S = \{1, x, x^2, x^3\}$. Apply the Gram-Schmidt process to the set S to obtain an orthonormal basis for $P_3(\mathbb{R})$. (7.5)

(b) Find the best-fit linear function for the data $\{(-3,9), (-2,6), (0,2), (1,1)\}$ using least squares approximation. Also, compute the error E. (5+2.5)

(c) Let $P_2(\mathbb{R})$ be the space of all real polynomials of degree less than or equal to 2 with the inner product

$$\langle f, g \rangle = \int_0^1 f(t)g(t)dt.$$

Find the orthogonal projection of $y(t) = 4t + 2t^2 + 5$ onto the subspace $P_1(\mathbb{R})$, where $P_1(\mathbb{R})$ is the space of all polynomials of degree less than or equal to 1. (7.5)

6. (a) Let $V = \mathbb{C}^2$ be an inner product space and be a linear operator on defined as $T(z_1, z_2) = (2z_1 + iz_2, z_1 + 2z_2)$. Check whether T is normal, self-adjoint, or neither. (7.5)

(b) Let $T: \mathbb{R}^3 \to \mathbb{R}^3$, $T(x, y, z) = (y, z, x)$. Write the matrix representation and determine whether Tis an orthogonal operator. Also compute T*(1,2,3). (5+2.5)

(c) State the Singular Value Decomposition theorem for matrices and find die Singular Value Decomposition for the matrix $A = \begin{pmatrix} 1+i & 1 \\ 1-i & -i \end{pmatrix}$.

(2+5.5)

(27)

Sr. No. of QP : 10580
Unique Paper Code: **2343010026**
Name of the Paper: **Machine Learning**
Name of the Course: **DSE**
Semester: **VII**
Duration: **3 Hours**
Maximum Marks: **90 Marks**

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **Section A** is compulsory.
3. Attempt any **four** questions from **Section B.**
4. Parts of a question must be answered together.
5. Use of a **scientific calculator** is allowed.

Section A

Q1. a) Explain the difference between the predictor and response variable. Additionally, provide a real-world example for each. Consider the case study of finding out the number of ice-creams sold in a particular mother dairy shop based on the temperature of the day. Specify the predictor and response variables in this case study. 3

b) Given a Neural network (NN) with two hidden layers where each hidden layer has 4 hidden units. The number of features in the training data set is 4 and the target variable can take three distinct values. Calculate the total number of parameters used in the said NN. 3

c) A medical test was conducted on 500 individuals, out of which 400 were healthy, and 100 were truly sick. Among the sick individuals, only 70 tested positive, while the rest received a negative result. For the healthy individuals, 25 tested positive, and 375 received a negative result. Draw the confusion matrix for the test results and calculate the Accuracy. 3

d) Identify whether the following problems should be handled as supervised learning or unsupervised learning problems. Justify your answer. 3
   (i) Identify whether the mail in the given Inbox is spam or non-spam.
   (ii) Group the similar customers according to the items purchased in the supermarket.
   (iii) Identify the characters in the handwritten paper.

e) Name a classification algorithm which is a 'lazy learner'. Justify the reason why it is called a lazy learner. 3

f) State Naïve Bayes theorem. Enumerate the key assumption underlying the Naive Bayes classifier. 3

g) Given the number of features (p) as 15, find the number of models to be fitted by the best subset selection method and the forward subset selection. 3

h) Construct a decision tree to represent the Boolean function $F(A,B,C)=A \wedge B \wedge C$. 3

i) The precision and recall values for two models M1 and M2 is 0.88 and 0.33 respectively, and 0.61 and 0.56 respectively. Which one of the two models has a better performance? 3

j) Why R^2 statistic is a better performance metric to assess a regression model, as compared to Residual Standard Error? 3

Section B

Q2. a) Using customer transaction data given in part (b) of this question, find the entropy of attributes 'Price' and 'Quantity'. Which of the attribute has more randomness? 5

b) Consider the following customer transaction data, with the "Purchase" attribute representing the class label: 10

Product	Price	Quantity	Day	Purchase
Laptop	high	1	Weekday	Yes
Headphones	low	2	Weekend	No
Mouse	low	3	Weekday	Yes
Keyboard	low	1	Weekend	Yes
Monitor	high	1	Weekday	No
Laptop	high	1	Weekend	Yes
Headphones	low	1	Weekday	No
Mouse	low	2	Weekend	Yes
Keyboard	low	1	Weekday	No
Monitor	high	1	Weekend	Yes

Using this dataset, train a Naive Bayes classifier and predict the class label "Purchase" for a new instance with the following attribute values: Product = Laptop, Price = low, Quantity = 1, Day = Weekday.

Q3. a) When do we use logistic regression classifier over linear regression model? Explain with one example. In a binary classification scenario using logistic regression, the model outputs a probability score of 0.4 for the positive class. If the decision threshold is set at 0.5, what will be the predicted class label? 5

b) Given the following data with one input attribute 'x' and one output variable 'y', fit a linear regression model using the least squares method. Determine the residual error for the best-fit line. Calculate the R^2 for the linear regression model for this data and predict the value of y for x=20. 10

Input (x)	Output (y)
10	45
14	55
19	66
21	79
24	95
26	102

Q4. a) Apply the k-means clustering to cluster the following data into two clusters: 3, 5, 8, 10, 13, 16, 21, 25 Initially, let 8 and 16 be the two initial cluster centroids. Use Euclidean distance to compute the distance between any two points. Compute the data membership and the centroid of each cluster after first iteration. 5

b) Consider the following dataset representing customer behaviour metrics X and Y for an online retail platform: 10

Points	X	Y
P1	2	8
P2	3	5
P3	8	4
P4	4	8
P5	7	5
P6	6	4

Generate the distance matrix using Euclidean distance. Using the Complete Linkage method, generate a dendrogram for the given data. From the dendrogram, identify the clustering scheme with two clusters.

Q5. a) For a maximal margin classifier, which points of the training set are called as support vectors? Discuss the capability of this classifier for data which is not linearly separable. 5

b) Describe the structure of a multilayer neural network with a suitable example. Discuss any two strategies employed to prevent a neural network from overfitting? 5

c) The provided regression coefficients show a multiple logistic regression model used to predict the risk of sudden death. Using these coefficients calculate the predicted probability of death for a man with blood pressure of 110 mmHg, Cholesterol level of 280 mg/100mL, and Glucose level of 120 mg/100mL. 5

Risk Factor	Regression Coefficient
Constant term (β_0)	-17.2
Blood Pressure (mm Hg) (β_1)	0.112
Cholesterol (mg/100 mL) (β_2)	0.0071
Glucose (mg/100 mL) (β_3)	0.0075

Q6. a) How does the hold-out method differ from k-fold cross-validation in context to the performance evaluation of a classifier? Compare the two approaches on the basis of bias-variance trade-off. 5

b) Consider the following customer purchase dataset for an e-commerce platform: 10

Customer ID	Purchase	Age Group	Income Level	Purchase
1	Yes	Kid	High	Yes
2	No	Adult	Low	No
3	Yes	Adult	High	Yes
4	No	Kid	Low	No
5	Yes	Adult	High	No
6	No	Adult	Low	Yes
7	Yes	Kid	High	Yes
8	No	Adult	Low	No

Using the Entropy criterion, construct a decision tree.

Q7. a) What is the maximum number of principal components that can be extracted for the dataset with n observations and p features? Write the formal equation for computing the first principal component. What are the distinguishing characteristics of the first principal component compared to the second principal component? 5

b) Consider the 2 classes of points in a 2-dimensional graph: 5
class +1 : (3,3), (6,3), (6,6)
class -1 : (3,6), (6,9), (3,9).
(i) Plot the above points on 2-dimensional axes.
(ii) Draw the line that separates the two classes and then draw the maximal margin between the classes.
(iii) Identify the support vectors.

c) Discuss the role of activation functions in a neural network. Name the two commonly used activation functions. Show the graphical representation of each, along with their mathematical formula. 5

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Mathematics

[This question paper contains 2 printed pages.]

Your Roll No.....

K

Sr. No. of Question Paper	:	**10812**
Unique Paper Code	:	2353200015
Name of the Paper	:	DSE - INTEGRAL TRANSFORM S
Name of the Course	:	**NEP-UGCF 2022 BACHELOR OF ARTS / BACHELOR OF SIENCE (PROGRAMME)**
Semester	:	VII

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt all **six** questions. **All** questions carry equal marks.

1. Define orthogonal functions. Show that a sequence of functions $\{\sin mx\}$, $m = 1,2,\cdots$ form an orthogonal system on the interval $[-\pi, \pi]$. What is the norm of the system?

OR

Find the Fourier series expansion of the following function

$$f(x) = \begin{cases} -1-x, & -1 < x < 0 \\ 1-x, & 0 < x < 1 \end{cases}$$

2. Find the maximum value of the Fourier series expansion of the following function,

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 0, & x = 0 \\ 1, & 0 < x < \pi. \end{cases}$$

OR

If $f(x) = e^{-x}$, $0 < x < \infty$. Show that the Fourier cosine integral representation is

$$f(x) = \frac{2}{\pi}\int_0^{\infty} \frac{\cos kx}{1+k^2} dk .$$

3. State and prove the convolution theorem of the Fourier transform.

OR

Find the Convolution of

(a) $f(x) = \cos x$ and $g(x) = \exp(-a|x|)$, $a > 0$

(b) $f(x) = \chi_{[a,b]}(x)$ and $g(x) = x^2$

P.T.O.

Where $\chi_{[a,b]}(x)$ is the characteristic function of the interval $[a, b] \leq R$ defined by

$$\chi_{[a,b]}(x) = \begin{cases} 1, & a \leq x \leq b \\ 0, & otherwise \end{cases}$$

4. If $a(>-1)$ is a real number then show that

$\mathcal{L}\{t^a\} = \frac{\Gamma(a+1)}{s^{a+1}}$ for all $(s > 0)$, symbols with usual notations.

Also prove that $(x) = \frac{1}{\Gamma(x)} \int_0^\infty \frac{t^{x-1}}{e^t-1} dt$, where $\xi(x)$ is Riemann Zeta function.

OR

If $f(t) = \text{erf}\left(\frac{a}{2\sqrt{t}}\right)$, then show that $\mathcal{L}\left\{\text{erf}\left(\frac{a}{2\sqrt{t}}\right)\right\} = \frac{1}{s}\left(1 - e^{-a\sqrt{s}}\right)$ where $\text{erf}(t)$ is the error function.

5. Using the Laplace transform solve the IVP

$$\frac{dx}{dt} + 2x = \cos t, t > 0, x(0) = 1.$$

OR

Consider the motion of a string of length π due to a force acting on it. Let the string be fixed at both ends. The motion is thus governed by

$$u_{tt} = c^2 u_{xx} + f(x,t), 0 < x < \pi, t > 0,$$

$$u(x,0) = 0,\ u_t(x,0) = 0,\ 0 < x < \pi,$$

$$u(0,t) = 0,\ u(\pi,t) = 0,\ 0 < t,$$

Using the Fourier transform solve the PDE

6. Using the Laplace transform solve the one-dimensional wave equation

$$u_{tt} = c^2 u_{xx}, 0 \leq x < \infty, t > 0,$$

with the conditions $u(0,t) = Af(t)\ at\ x = 0, t \geq 0,$

$$u(x,t) \to 0\ as\ x \to \infty, t \geq 0,$$

$$u(x,t) = 0 = \frac{\partial u}{\partial t}\ at\ t = 0\ for\ 0 < x < \infty$$

OR

Define the finite Fourier sine and cosine transforms.

Let $f'(x)$ be continuous and $f''(x)$ be piece-wise continuous in $[0, \pi]$. If $F_c(n)$ is the Finite Fourier cosine transform of $f(x)$ then

$$F_c[f''(x)] = \frac{2}{\pi}[(-1)^n f'(\pi) - f'(0)] - n^2 F_c(n)$$

[This question paper contains 2 printed pages.]

Your Roll No.....

K

Sr. No. of Question Paper : **12337**
Unique Paper Code : 2354000015
Name of the Paper : Topics In Multivariate Calculus
Name of the Course : **Common Prog Group**
Semester : VII
Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of calculator is not allowed.

1. (a) Define $\epsilon - \delta$ definition of limit of a function of two variables. Using this definition, show that $\lim_{(x,y)\to(0,0)} (x^2+y^2)^{\frac{3}{2}} = 0$.

(b) (i) Given that the function $f(x,y) = \begin{cases} \frac{3x^3-3y^3}{x^2-y^2} & for\ x^2 \neq y^2 \\ A & otherwise \end{cases}$ is continuous at the origin, what is A?

(ii) Let z be defined implicitly as a function of x and y by the equation $x^2z + yz^3 = x$. Determine z_x and z_y.

(c) When two resistances R_1 and R_2 are connected in parallel, the total resistance R satisfies

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

If R_1 is measured as 300 ohms with a maximum error of 2% and R_2 is measured as 500 ohms with a maximum error of 3%, what is the maximum percentage error in R?

2.(a) (i) Define gradient of a function of two variables.

(ii) In what direction, is the function defined by $f(x,y) = xe^{2y-x}$ increasing, rapidly at the point $P_0(2,1)$, and what is the maximum rate of increase? In what direction is f decreasing most rapidly?

(b) (i) Find a vector that is normal to the level surface $x^2 + 2xy - yz + 3z^2 = 7$ at the point $P_0(1,1,-1)$.

(ii) Let $z = f(x,y)$, where $x = at$ and $y = at$ for constants a and b. Assuming all necessary differentiability, find $\frac{d^2z}{dt^2}$ in terms of the partial derivatives of z.

(c) Find the point on the plane $x + 2y + z = 5$ that is closest to the point $P(0,3,4)$.

3. (a) (i) Evaluate $\iint_R \frac{2xy}{x^2+1} dA$; $R: 0 \le x \le 1, 1 \le y \le 3$ using iterated integration.

(ii) Find the volume of the solid bounded below by the rectangle R in the xy plane and above by the graph of $z = f(x,y)$ where $f(x,y) = 2x + 3y$; R: $0 \le x \le 1$, $0 \le y \le 2$.

P.T.O.

(b) Evaluate $\iint_D x\,dy\,dx$; D is the region between the parabola $y = x^2$ and the line $y = 2x + 3$.

(c) Evaluate $\int_0^2 \int_y^{\sqrt{4-y^2}} e^{x^2+y^2} dxdy$ by converting to polar coordinates

4. (a) Find the volume of the tetrahedron T bounded by the plane $x + y + z = 1$ and the coordinate planes $x = 0,\ y = 0,\ z = 0$.

(b) Use cylindrical coordinates to evaluate $\iiint_D z(x^2 + y^2)^{-1/2} dx\,dy\,dz$ where D is solid bounded above by the plane $z = 2$ and below by the surface $2z = x^2 + y^2$

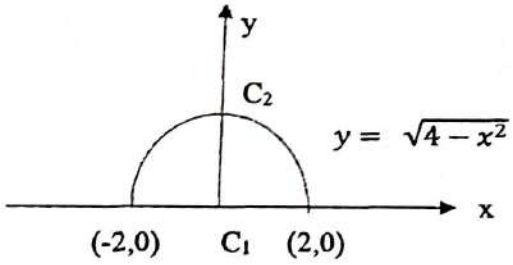
(c) Let D be the region in xy – plane that is bounded by coordinate axes and the line $x + y = 1$. Use suitable change of variables $u = x - y, v = x + y$ to compute the integral $\iint_D \frac{(x-y)^6}{(x+y)^4} dy\,dx$.

5. (a) Evaluate the line integral $\int_C (y\,dx + x\,dy + zdz)$, where C is the helical path given by

$x = \cos t,\ y = \sin t,\ z = t$ for $0 \le t \le \pi/2$.

(b) Verify the vector field $F(x,y) = (x + 2y)\,i + (2x + y)\,j$ is conservative and find a scalar potential f for **F**. Then evaluate the line integral $\int_C \mathbf{F}.d\mathbf{R}$, where C is any smooth path connecting $A(0,0)$ to $B(1,1)$.

(c) Verify Green's Theorem for the line integral $\oint_C (2ydx - xdy)$ where C is the closed path as shown in the figure below



Path C

6. (a) Evaluate $\iint_S (x^2 + y^2) dS$ where S is the surface of the hemisphere $z = \sqrt{1 - x^2 - y^2}$.

(b) Use Stokes' theorem to evaluate $\iint_S (\text{curl } \mathbf{F}.\mathbf{N}) dS$, where $\mathbf{F} = x\,i + y^2 j + xyz\,k$ and S is that part of paraboloid $z = 4 - x^2 - y^2$ with $z \ge 0$. Use the upward unit normal vector.

(c) Use the divergence theorem to evaluate the surface integral $\iint_S \mathbf{F}.\mathbf{N}\,dS$ for $F = 2y\,i - z\,j + 3x\,k$ and S is the surface of the upper five faces of the unit cube $0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le 1$ missing $z = 0$.

[This question paper contains 2 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 13449 **K**

Unique Paper Code : 2354000015

Name of the Paper : Topics In Multivariate Calculus

Name of the Course : **Common Prog Group**

Semester : VII

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of Calculator is not allowed.

1. (a) Define continuity of a function of two variables. Show that f is continuous at (0,0),where

$$f(x,y) = \begin{cases} y\ sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}.$$

(b) (i) Show that $\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2}$ does not exist.

(ii) Compute the slope of the tangent line to the graph of f at the given point P_0 in the direction parallel to the xz −plane and yz −plane, where $f(x,y) = x^2\ sin(x+y)$ and $P_0\left(\frac{\pi}{2}, \frac{\pi}{2}, 0\right)$.

(c) At a certain factory, the daily output is $Q = 60\ K^{\frac{2}{3}} L^{\frac{1}{3}}$ units, where K denotes the capital investment (in units of \$1000) and L the size of the labor force (in worker-hours). The current capital investment is \$900,000 and 1000 worker-hours of labor are used each day. Estimate the change in output that will result if capital investment is increased by \$1000 and labor is decreased by 2 worker-hours.

2. (a) Define differentiability of a function of two variables. Give an example of a non-differentiable function for which f_x and f_y exist.

(b) (i) Let $f(x,y,z) = xy\ sin(xz)$. Find ∇f_0 at the point $P_0(1,-2,\pi)$ and then compute the directional derivative of f at P_0 in the direction of the vector $v = -2i + 3j - 5k$.

(ii) Find an equation for each horizontal tangent plane to the surface $z = 5 - x^2 - y^2 + 4y$.

(c) Given that the largest and smallest values of $f(x,y) = 1 - x^2 - y^2$ subject to the constraint $x + y = 1$ with $x \geq 0$ and $y \geq 0$ exist, use the method of Lagrange multipliers to find the extrema.

P.T.O.

3. (a) (i) Evaluate $\iint_R 2xe^y\, dA$; R: $-1 \le x \le 0,\ 0 \le y \le ln2$ using iterated integration.

(ii) Find volume of the solid bounded below by the rectangle R in the xy plane and above by the graph of $z = f(x,y)$ where $f(x,y) = \sqrt{xy}$ R : $0 \le x \le 1,\ 0 \le y \le 4$.

(b) Evaluate $\iint_D (2y - x)dA$; D is the region between the parabola $y = x^2$ and the line $y = 2x$.

(c) Compute the area in polar form of the region D bounded above by the line $y = x$ and below by the circle $x^2 + y^2 - 2y = 0$.

4. (a) Evaluate $\iiint_D xdV$ where D is the solid in the first octant bounded by the cylinder $x^2 + y^2 = 4$ and the plane $2y + z = 4$.

(b) Using cylindrical coordinates, find the volume of the solid in the first octant that is bounded by the cylinder $x^2 + y^2 = 2y$, the half cone $z = \sqrt{x^2 + y^2}$ and xy-plane.

(c) Let D be the region in xy plane that is bounded by coordinate axes and the line $x + y = 1$. Use suitable change of variables $u = x - y,\ v = x + y$ to compute the integral $\iint_D \left(\frac{x-y}{x+y}\right)^4 dy\, dx$.

5(a) Evaluate the line integral $\int_C (-y\, dx + x\, dy + xzdz)$, where C is the helical path given by $x = cost, y = sint, z = t$ for $0 \le t \le 2\pi$.

(b) Verify the vector field $\mathbf{F}(x,y) = (y - x^2)\boldsymbol{i} + (x + y^2)\mathbf{j}$ is conservative and find a scalar potential f for **F**. Then evaluate the line integral $\int_C \mathbf{F}.d\mathbf{R}$, where C is any smooth path connecting $A(0,0)$ to $B(1,1)$.

(c) Show that the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ has area πab using the line integral.

6(a) Evaluate $\iint_S 2xdS$ where S is the portion of the plane $x + y + z = 1$ with $x \ge 0, y \ge 0$, $z \ge 0$.

(b) Use Stokes' theorem to evaluate $\iint_S (\text{curl } \mathbf{F}.\mathbf{N})\, dS$, where $\mathbf{F} = x\,\boldsymbol{i} + y^2\,\boldsymbol{j} + xyz\,\boldsymbol{k}$ and S is that part of the paraboloid $z = 1 - x^2 - y^2$ with $z \ge 0$. Use the upward unit normal vector.

(c) Use the divergence theorem to evaluate the surface integral $\iint_S \mathbf{F}.\mathbf{N}dS$ for the $\mathbf{F} = (x^2+y^2 - z^2)\,\mathbf{i} + x^2y\,\mathbf{j} + 3z\,\mathbf{k}$ and S is the surface of the upper five faces of the unit cube $0 \le x \le 1,\ 0 \le y \le 1, 0 \le z \le 1$ missing $z = 0$.