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S. No. of Question Paper : 5783

Unique Paper Code : 2352011201

Name of the Paper : Linear Algebra

Name of the Course : B.Sc. (H) Mathematics

Semester : II / DSC

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all questions by selecting any two parts from each question.

All questions carry equal marks.

Use of calculator is not allowed.

1. (a) Prove that if x , y and z are mutually orthogonal vectors in $\mathbf{R}^n$, then

$$\|x + y + z\|^2 = \|x\|^2 + \|y\|^2 + \|z\|^2.$$

Also, verify the same for the vectors in $\mathbf{R}^4$, where $x = (1, 0, -1, 0)$, $y = (1, \sqrt{2}, 1, 1)$ and $z = (1, -\sqrt{2}, 1, 0)$.

- (b) Solve the following system of linear equations using Gaussian Elimination Method :

$$3x_1 - 2x_2 + 4x_3 = -54$$

$$-x_1 + x_2 - 2x_3 = 20$$

$$5x_1 - 4x_2 + 8x_3 = -83.$$

- (c) Using the Gauss-Jordon method, find the minimal integer values for the variables that will balance the following chemical equation :



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2. (a) Define a linear combination of the vectors in $\mathbb{R}^n$. Express the vectors $[1, 11, -4, 11]$ as a linear combination of the vectors $x = [2, -4, 1, -3]$, $y = [7, -1, -1, 2]$ and $z = [3, 7, -3, 8]$.

- (b) Consider the matrix :

$$A = \begin{bmatrix} 5 & -8 & -12 \\ -2 & 3 & 4 \\ 4 & -6 & -9 \end{bmatrix}$$

- (i) State the Cayley-Hamilton Theorem.
 (ii) Find the eigenvalues and eigenvectors of A.
 (iii) Check whether the matrix A is diagonalizable or not ?

- (c) Let

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & 1 \\ -2 & 1 & 5 \\ 3 & 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 & -5 \\ 2 & 3 & 0 \\ 4 & 1 & 1 \end{bmatrix}$$

Compute $R(AB)$ and $(R(A))B$ to verify that they are equal, i.e.,

$R(AB) = (R(A))B$, for the operations :

(i) $R : \langle 3 \rangle \leftarrow -3 \langle 2 \rangle + \langle 3 \rangle$,

(ii) $R : \langle 2 \rangle \leftrightarrow \langle 3 \rangle$.

3. (a) Let $\mathbb{R}^2 = \{(a_1, a_2) : a_1, a_2 \in \mathbb{R}\}$. Prove that $\mathbb{R}^2$ is a vector space over $\mathbb{R}$ with addition and scalar multiplication defined as;

$$(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 + b_2)$$

$$\alpha(a_1, a_2) = (\alpha a_1, \alpha a_2) \text{ where } (a_1, a_2), (b_1, b_2) \in \mathbb{R}^2, \alpha \in \mathbb{R}.$$

- (b) Define basis of a vector space. Prove that the following subset S of $\mathbf{R}^3$ is a spanning set of $\mathbf{R}^3$, where $S = \{(2, 2, 3), (-1, -2, 1), (0, 1, 0)\}$.
- (c) Do the polynomials $x^3 - 2x^2 + 1$, $4x^3 - x + 3$ and $3x - 2$ generate $P_3(\mathbf{R})$?
4. (a) If $\{v_1, v_2, \dots, v_n\}$ is a basis of a vector space $V(F)$, then prove that every element of V can be uniquely expressed as a linear combination of $v_1, v_2, \dots, v_n$.
- (b) Define linearly dependent and independent set in $\mathbf{R}^3$. Show that set of vectors $\{(2, -2, 3), (0, -4, 1), (3, 1, -4)\}$ in $\mathbf{R}^3$ is linearly dependent.
- (c) Let W be a subspace of a finite-dimensional vector space V . Prove that W is finite-dimensional and $\dim(W) \leq \dim(V)$. Moreover, if $\dim(W) = \dim(V)$, then $V = W$.
5. (a) Let $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ is a function. For each of the following parts, state why T is not a linear transformation.
- (i) $T(a, b) = (1, b)$
- (ii) $T(a, b) = (a, b^2)$
- (iii) $T(a, b) = (\sin a, 0)$
- (iv) $T(a, b) = (|a|, b)$
- (v) $T(a, b) = (1 + a, b)$.
- (b) Let V, W and Z be finite-dimensional vector spaces with ordered basis α, β and γ , respectively. Let $T : V \rightarrow W$ and $U : W \rightarrow Z$ be linear transformations. Then $[UT]_\alpha^\beta = [U]_\alpha^\beta [T]_\alpha^\beta$.
- (c) Prove that two same dimensional finite vector spaces over the same field are isomorphic.

P.T.O.

6. (a) Let V and W be vector spaces over same field F , and suppose that $\{v_1, v_2, \dots, v_n\}$ is a basis for V . For $w_1, w_2, \dots, w_n$ in W , there exists exactly one linear transformation $T : V \rightarrow W$ such that $T(v_i) = w_i$ for $i = 1, 2, \dots, n$.

- (b) Let $L : M_2 \times_2(\mathbb{R}) \rightarrow \mathbb{R}^3(\mathbb{R})$ be a linear transformation given by

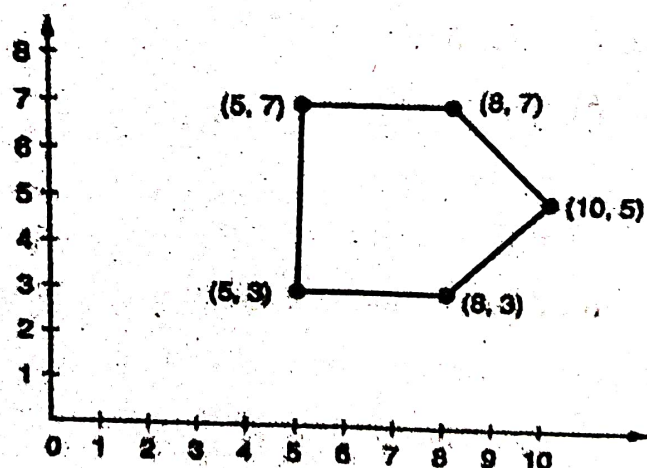
$$L\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = [a - c + 2d, 2a + b - d, -2c + d] \text{ with}$$

$$B = \left(\begin{bmatrix} 2 & 5 \\ 2 & -1 \end{bmatrix}, \begin{bmatrix} -2 & 2 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -3 & 4 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} -1 & -3 \\ 0 & 1 \end{bmatrix} \right);$$

$$C = ([7, 0, -3], [2, -1, -2], [2, 0, 1]).$$

Check whether B and C are bases for $M_2 \times_2(\mathbb{R})$ and $\mathbb{R}^3(\mathbb{R})$, respectively, if yes, then find matrix A for L with respect to the given bases B and C .

- (c) For the graphic in given figure, use ordinary coordinates to find the new vertices after performing each indicated operations :



- (i) Translation along the vector $[4, -2]$
- (ii) Rotation about the origin through $\theta = 30^\circ$
- (iii) Reflection about the line $y = 3x$.

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Roll No.

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S. No. of Question Paper : 5803

Unique Paper Code : 2352011202

Name of the Paper : Calculus-DSC 5

Name of the Course : Bachelor of Science (Honours Course)
Mathematics NEP UGCF-2022

Semester : II

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting *three* parts from each question.

All questions carry equal marks.

Use of calculator is not allowed.

1. (a) Using $\epsilon - \delta$ definition of limit, show that :

$$\lim_{x \rightarrow c} \frac{1}{x} = \frac{1}{c},$$

if $c > 0$.

- (b) Show that if $f : (a, \infty) \rightarrow \mathbf{R}$ ($a > 0$) is such that

$$\lim_{x \rightarrow \infty} xf(x) = L,$$

where $L \in \mathbf{R}$, then

$$\lim_{x \rightarrow \infty} f(x) = 0.$$

P.T.O.

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- (c) Let $A \subset \mathbf{R}$ and $f : A \rightarrow \mathbf{R}$ has a limit at $c \in \mathbf{R}$, where c is a cluster point of A . Then prove that f is bounded on some neighbourhood of c .
- (d) Give example of functions f and g such that f and g do not have limits at a point c , but both $f + g$ and fg have limits at c .
2. (a) If f is continuous on a closed interval $[a, b]$, then prove that f is uniformly continuous on $[a, b]$.
- (b) Show that the function $f(x) = x^2$ is uniformly continuous on $[-4, 4]$.
- (c) Show that :

$$\lim_{x \rightarrow \infty} \frac{x+2}{\sqrt{x}} \quad (x > 0)$$

does not exist in $\mathbf{R}$.

- (d) Use $\varepsilon - \delta$ definition of limit to show that :

$$\lim_{x \rightarrow 6} \frac{x^2 - 3x}{x + 3} = 2.$$

3. (a) Let

$$f(x) = \begin{cases} x^2, & x \geq 0, \\ 0, & x < 0. \end{cases}$$

Calculate f' on $\mathbf{R}$. Is f' continuous as well as differentiable on $\mathbf{R}$? Justify your answer.

- (b) Let $f(x) = \sin |x|$, where $x \in \mathbf{R}$. Determine the set of points where f is not differentiable. Justify your answer.

(c) If $f : \mathbf{R} \rightarrow \mathbf{R}$ is differentiable at $a \in \mathbf{R}$, then show that :

$$(i) \quad \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = f'(a),$$

$$(ii) \quad \lim_{h \rightarrow 0} \frac{f(a+h) - f(a-h)}{2h} = f'(a).$$

(d) Suppose f is defined on an open interval (a, b) . Let $x_0 \in (a, b)$. If f assumes its maximum at x_0 and if f is differentiable at x_0 , then show that $f'(x_0) = 0$.

4. (a) State and prove Rolle's theorem.

(b) Let f be defined on $\mathbf{R}$ and suppose that $|f(x) - f(y)| \leq |x - y|^3$ for all $x, y \in \mathbf{R}$. Prove that f is a constant function on $\mathbf{R}$.

(c) Show that $\sin x \leq x$ for all $x \geq 0$.

(d) State Intermediate value theorem for derivatives. Suppose f is differentiable on $\mathbf{R}$ and $f(0) = 1$, $f(1) = 2$ and $f(2) = 2$.

(i) Show that $f'(x) = \frac{1}{2}$ for some $x \in (0, 2)$,

(ii) Show that $f'(x) = \frac{1}{3}$ for some $x \in (0, 2)$.

5. (a) If

$$y = \log \left(x + \sqrt{1 + x^2} \right),$$

show that

$$(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + n^2y_n = 0.$$

P.T.O.

- (b) Trace the curve :

$$4x^2 = y^2(x^2 - 4).$$

- (c) Determine the intervals of concavity and points of inflexion (if any) of the curve :

$$y = 2x^4 - 3x^2 + 2x + 1.$$

- (d) State Taylor's Theorem. Find the Taylor's series expansion of $\sin 2x$ about $x = 0$.

6. (a) Trace the curve :

$$r = 2 \sin 2\theta.$$

- (b) Determine the position and nature of the double points of the curve :

$$x^3 + 2x^2 + 2xy - y^2 + 5x - 2y = 0.$$

- (c) Find the horizontal asymptotes of the graph of the function given by

$$y = x^5 \left[\sin \frac{1}{x} - \frac{1}{x} + \frac{1}{6x^3} \right].$$

- (d) Find the equations of tangents at origin to the curve :

$$x^4 + y^4 = a^2(x^2 - y^2).$$

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S. No. of Question Paper : 5823

Unique Paper Code : 2352011203

Name of the Paper : Ordinary Differential Equations

Name of the Course : B.Sc. (Hons.) Mathematics

Semester : II/DSC

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the questions are compulsory.

Attempt any two parts from each question.

Each part carries 7.5 marks.

Use of non-programmable scientific calculator is allowed.

1. (a) Solve the initial value problem :

$$(2y \sin x \cos x + y^2 \sin x) dx + (\sin^2 x - 2y \cos x) dy = 0, y(0) = 3.$$

(b) Solve :

$$(x^2 - 3y^2) dx + 2xy dy = 0.$$

(c) Solve :

$$x^2 y'' = 2xy' + (y')^2$$

by reduction of order.

2. (a) If the population of a country double in 50 years, in how many years will it triple under the assumption that the rate of increase is proportional to the number of inhabitants ?

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(b) A metal bar at a temperature of 100°F is placed in a room at a constant temperature of 0°F . If after 20 minutes the temperature of the bar is half, find an expression for the temperature of the bar at any time.

(c) Assume that the rate at which radioactive nuclei decay is proportional to the number of nuclei in the sample. In a certain sample 10% of the original number of radioactive nuclei have undergone disintegration in a period of 200 years. What percentage of the original radioactive nuclei will remain after 1000 years ?

3. (a) Show that e^{2x} and e^{3x} are linearly independent solutions of the differential equation :

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0.$$

Find the particular solution satisfying the initial condition,

$$y(0) = 0, y'(0) = 0.$$

(b) Solve the differential equation using the method of variation of parameters

$$\frac{d^2y}{dx^2} + 4y = \tan 2x.$$

(c) Find the general solution of the differential equation using the method of undetermined coefficients :

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2e^x.$$

4. (a) Use operator method to find the general solution of the following linear system :

$$\frac{dx}{dt} + 2y + x = e^t$$

$$\frac{dy}{dt} + 2x + y = 3e^t.$$

- (b) Solve the initial value problem, assuming $x > 0$

$$x^3 \frac{d^3 y}{dx^3} - x^2 \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} - 2y = x^3.$$

- (c) A mass of 3 kg is attached to the end of a spring that is stretched 20 cm by a force of 15 N. It is set in motion with initial position $x_0 = 0$ m and initial velocity $v_0 = -10$ m/s. Find the amplitude, period and frequency of the resulting motion.

5. (a) Define carrying capacity. Derive the logistic equation :

$$\frac{dX}{dt} = rX \left(1 - \frac{X}{K} \right)$$

where r and K are reproduction rate and carrying capacity respectively.

- (b) Suppose a soft drink manufacturing company has a 1000 litre tank containing sugar water solution. Initially s_0 kg of sugar is dissolved. Sugar solution flows into the tank at the rate of 10 litre/min with concentration $cin(t)$ kg/l. Assume that the solution in the tank is thoroughly mixed and the sugar solution flows out at the same rate at which it flows in i.e. the volume of sugar water mixture in the tank remains constant. Find a differential equation for the amount of sugar in the tank at any time t . Also solve it.

- (c) Suppose a population can be modelled using the differential equation :

$$\frac{dx}{dt} = 0.2x - 0.001x^2$$

with the initial population size of $x_0 = 100$ at a time step of 1. Find the predicted population after two months.

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6. (a) The given differential equation :

$$\frac{dN}{dt} = aN \left(1 - \frac{N}{N_T} \right)$$

represents the model for spread of technology where $N(t)$ is the number of ranchers who adopted an improved pasture technology in Uruguay and N_T is the total population of ranchers. It is assumed that rate of adoption is proportional to both the number who have adopted the technology and the fraction of population of ranchers who have not adopted the technology.

- (i) Which term corresponds to the fraction of the population who have not yet adopted the improved pasture technology ?
 (ii) According to the Banks (1994), $N_T = 17015$, $a = 0.490$ and $N_0 = 141$. Determine how long it takes for the improved pasture technology to spread to 80% of the population.

- (b) Derive the epidemic model of influenza

$$\frac{dS}{dt} = -\beta SI, \quad \frac{dI}{dt} = \beta SI - \gamma I, \quad \frac{dR}{dt} = \gamma I$$

where β and γ are positive constants.

- (c) Define equilibrium points. Find all equilibrium points of Predator-Prey model

$$\frac{dX}{dt} = \beta_1 X \left(1 - \frac{X}{K} \right) - c_1 XY, \quad \frac{dY}{dt} = c_2 XY - \alpha_2 Y$$

where the positive constants c_1 , c_2 are interaction parameter, α_2 is predator per capita death rate and β_1 is prey per capita birth rate. Discuss the direction of trajectories of Predator-Prey model.

SL No of QP : 5534
 Unique Paper Code : 2352012401
 Name of the Paper : Sequences and Series of Functions
 Name of the Course : B.Sc. (H) Mathematics
 Semester : IV
 Duration : 3 hours
 Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. All questions carry equal marks.
3. Attempt any two parts from each question.

1. (a) Define uniform convergence for sequence of functions (f_n) defined on $\mathbb{R}$. Show that a sequence (f_n) of bounded functions on $A \subseteq \mathbb{R}$ converges uniformly on A to f if and only if $\|f_n - f\|_A \rightarrow 0$.
 (b) Show that the sequence $f_n(x) = \frac{nx}{1+n^2x^2}$ converges uniformly in the interval $[a, \infty)$ where $a > 0$ but it does not converge uniformly on the interval $[0, \infty)$.
 (c) Let $(f_n), (g_n)$ be sequences of bounded functions on A that converge uniformly on A to f, g , respectively. Show that $(f_n g_n)$ converges uniformly on A to fg .
2. (a) Let $f_n(x): [0,1] \rightarrow \mathbb{R}$ be defined for $n \geq 2$ by

$$f_n(x) := \begin{cases} n^2x & \text{for } 0 \leq x \leq \frac{1}{n} \\ -n^2\left(x - \frac{2}{n}\right) & \text{for } \frac{1}{n} \leq x \leq \frac{2}{n} \\ 0 & \text{for } \frac{2}{n} \leq x \leq 1 \end{cases}$$

Evaluate pointwise limit f of f_n and show that (f_n) does not converge uniformly to f on the interval $[0,1]$.

- (b) Let (f_n) be a sequence of integrable functions on $[a, b]$ and suppose that (f_n) converges uniformly to f on $[a, b]$. Show that f is integrable on $[a, b]$ and

$$\int_a^b f = \lim_{n \rightarrow \infty} \int_a^b f_n.$$

- (c) Show that the sequence $f_n(x) = n^2 x^2 e^{-nx}$ converges uniformly on $[a, \infty)$ where $a > 0$ but does not converge uniformly on the interval $[0, \infty)$.

3. (a) State and prove Weierstrass M-test for uniform convergence of series of functions. Hence, check for uniform convergence of series $\sum_{n=1}^{\infty} \frac{1}{x^2+n^2}$, $x \in \mathbb{R}$.
- (b) Discuss the pointwise convergence of the series $\sum_{n=1}^{\infty} \frac{x^n}{x^n+1}$, $x \geq 0$. Also, show that the given series is not uniformly convergent on $[0,1)$ but is uniformly convergent on $[0, \frac{1}{2})$.
- (c) Prove that if $\sum f_n$ converges uniformly to f on a domain $D \subseteq \mathbb{R}$ and each f_n is continuous on $D \subseteq \mathbb{R}$ to $\mathbb{R}$ then f is continuous on D .
4. (a) Let f_n be a real valued function on $[a, b]$ that has derivative f'_n on $[a, b]$ for each $n \in \mathbb{N}$. Suppose that the series $\sum f_n$ converges for at least one point of $[a, b]$ and the series of derivatives $\sum f'_n$ converges uniformly on $[a, b]$. Show that there exists a real valued function f on $[a, b]$ such that $\sum f_n$ converges uniformly to f on $[a, b]$. Also, that f has a derivative on $[a, b]$ and $f' = \sum f'_n$.
- (b) State and prove the Cauchy criterion for the uniform convergence of series of functions $\sum f_n$. Use it to prove the non uniform convergence of $\sum_{n=1}^{\infty} \frac{1}{n^2 x^2}$ for $|x| < 1$.
- (c) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{x^{n+1}}$ is convergent for $x > 1$ and divergent for $0 \leq x \leq 1$. Also, show that the series converges uniformly on $[a, \infty)$, $a > 1$.
5. (a) (i) Find the exact interval of convergence for the power series $\sum_{n=1}^{\infty} a_n x^n$ where $a_n = \frac{3^n}{n4^n}$.
- (ii) Let $f(x) = |x|$ for $x \in \mathbb{R}$. Is there a power series $\sum_{n=0}^{\infty} a_n x^n$ such that $f(x) = \sum_{n=0}^{\infty} a_n x^n$ for all x ? Justify.
- (b) Let $\sum_{n=0}^{\infty} a_n x^n$ be a power series with radius of convergence $R > 0$. If $0 < R_1 < R$, then show that the power series converges uniformly on $[-R_1, R_1]$ to a continuous function.
- (c) Show that $\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$ for $|x| < 1$. Apply Abel's theorem to show that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$.
6. (a) Define logarithm function L on $(0, \infty)$. Show that the function L satisfies the following:
- (i) $L'(x) = \frac{1}{x}$ for $x > 0$.
- (ii) $L(xy) = L(x) + L(y)$ for $x > 0, y > 0$.
- (b) Show that $\sum_{n=1}^{\infty} n x^n = \frac{x}{(1-x)^2}$ for $|x| < 1$. Evaluate $\sum_{n=1}^{\infty} \frac{n}{3^n}$.
- (c) Show that there does not exist a sequence of polynomials converging uniformly on $\mathbb{R}$ to $f(x) = e^x$.

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Your Roll No.....

Sr. No. of Question Paper : 5594

J

Unique Paper Code : 2352012402

Name of the Paper : Multivariate Calculus

Name of the Course : B.A. / B.Sc. (H)

Semester : IV (DSC)

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of calculator is **NOT** allowed.

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1. (a) Let $f(x,y) = \frac{\sin(x^2 + y^2)}{x^2 + y^2}$ for $(x,y) \neq (0,0)$. Find

the value of $f(0,0)$ for which $f(x,y)$ is continuous at $(0,0)$.

(b) Compute the slope of the tangent line to the graph

of $f(x,y) = \frac{x^2 + y^2}{xy}$ at $P(1, -1, -2)$ in the direction

parallel to

(i) XZ plane

(ii) YZ plane.

(c) Find $\frac{\partial w}{\partial r}$ where $w = e^{2x-y+3z^2}$ and $x = r + s - t$,

$y = 2r - 3s$, $z = \cos(rst)$.

2. (a) Let $f(x, y, z) = ye^{x+z} + ze^{y-x}$. At the point $P(2, 2, -2)$, find the unit vector pointing in the direction of most rapid increase of $f(x, y, z)$.

(b) Find all the critical points of $f(x, y) = (x - 1)(y - 1)(x + y - 1)$ and classify each as a point of relative maximum, point of relative minimum or a saddle point.

(c) Find the minimum value of the function $f(x, y, z) = x^2 + y^2 + z^2$ subject to $4x^2 + 2y^2 + z^2 = 4$.

3. (a) (i) Find the volume of the solid bounded below by the rectangle R in the xy -plane and above by the graph of $z = 2x + 3y$; $R: 0 \leq x \leq 1, 0 \leq y \leq 2$.

(ii) Evaluate $\int_0^1 \int_x^1 e^{y^2} dy dx$.

(b) (i) Sketch the region of integration and write

an equivalent integral with the order of

integration reversed $\int_0^4 \int_{y/2}^{\sqrt{y}} f(x, y) dx dy$.

(ii) Use a double integral for finding the volume of the solid region bounded above by the paraboloid $z = 6 - 2x^2 - 3y^2$ and below by the plane $z = 0$.

(c) Use a double integral to find the area bounded by the curve $r = 1 + \sin \theta$.

4. (a) Use cylindrical co-ordinates to compute the integral

$\iiint_D z(x^2 + y^2)^{-1/2} dx dy dz$ where D is the solid bounded above by the plane $z = 2$ and bounded below by the surface $2z = x^2 + y^2$.

(b) (i) Compute the iterated triple integral

$$\int_0^1 \int_0^y \int_0^{\ln y} e^{z+2x} dz dx dy.$$

(ii) Evaluate the iterated integral

$$\int_0^{\pi/2} \int_0^{\pi/4} \int_0^{\cos \phi} \rho^2 \sin \phi \, d\rho d\theta d\phi.$$

(c) Evaluate $\iint_R e^{(2y-x)/(y+2x)} dA$

where R is the trapezoid with vertices (0,2), (1,0), (4,0) and (0,8).

5. (a) A wire has the shape of the curve

$$x = \sqrt{2} \sin t \quad y = \cos t \quad z = \cos t \quad \text{for } 0 \leq t \leq \pi$$

If the wire has density $\delta(x, y, z) = xyz$ at each point (x, y, z) , what is its mass?

(b) Show that the vector field

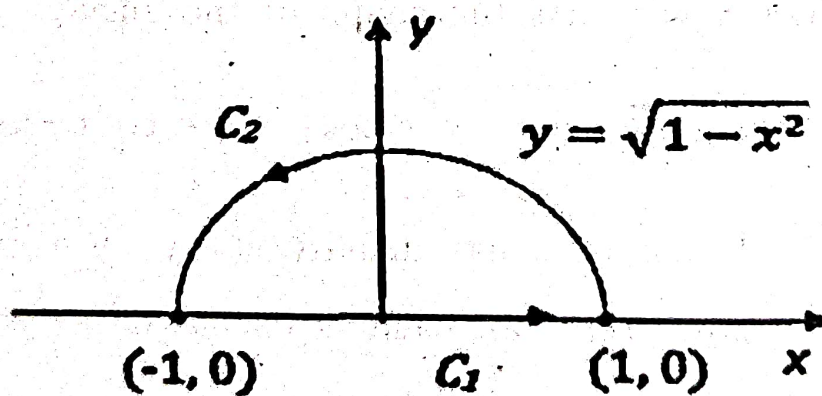
$$F = (e^x \sin y - y)\hat{i} + (e^x \cos y - x - 2)\hat{j}$$

is conservative and hence find a scalar potential function f for F .

(c) Verify Green's theorem for the line integral

$$\oint_C (-y \, dx + x \, dy)$$

where C is the closed path shown in the figure below



Path C

6. (a) Evaluate the surface integral

$$\iint_S g \, dS$$

where $g(x,y,z) = xz + 2x^2 - 3xy$ and S is that portion of the plane $2x - 3y + z = 6$ that lies over the unit square $R: 2 \leq x \leq 3, 2 \leq y \leq 3$.

(b) Evaluate

$$\oint_C \left(\frac{1}{2}y^2 \, dx + z \, dy + x \, dz \right)$$

where C is the curve of intersection of the plane $x + z = 1$ and the ellipsoid $x^2 + 2y^2 + z^2 = 1$, oriented counterclockwise.

(c) Use the divergence theorem to evaluate

$$\iiint_S \mathbf{F} \cdot \mathbf{N} \, dS$$

Where $F = (x^5 + 10x y^2 z^2) \hat{i} + (y^5 + 10y x^2 z^2) \hat{j} + (z^5 + 10z x^2 y^2) \hat{k}$, and S is the closed hemispherical surface $z = \sqrt{1 - x^2 - y^2}$ together with the disc $x^2 + y^2 \leq 1$ in xy -plane and N is the outward unit normal vector field.

This question paper contains 4 printed pages]

Roll No.

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S. No. of Question Paper : 5673

Unique Paper Code : 2352012403

Name of the Paper : Numerical Analysis

Name of the Course : B.Sc. (Hons.) Mathematics

Semester : IV

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

All six questions are compulsory. Attempt any two parts from each question.

All questions carry equal marks.

Use of non-programmable scientific calculator is allowed.

1. (a) Define order of convergence of a sequence $\{p_n\}$ which converges to a number p . The sequence generated by the formula $x_{n+1} = \frac{x_n^3 + 3x_n a}{3x_n^2 + a}$ converges to $\sqrt{a}$. Determine the order of convergence and the asymptotic error constant.
- (b) Perform four iterations of Bisection method to approximate the root of the equation $x^4 - x - 10 = 0$, in the interval $(1, 2)$.
- (c) Explain geometrically the Newton's method to determine a root p of an equation $f(x) = 0$. Hence, using initial approximation $p_0 = 2$, perform two iterations to obtain approximation to the value of $\sqrt[3]{17}$.

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P.T.O.

2. (a) Use Newton's method to approximate the root of the equation $\ln(1+x) - \cos(x) = 0$, in the interval $(0, 1)$. Use error tolerance $= 10^{-4}$, and initial approximation $p_0 = 0.5$.
- (b) Define a fixed point of a function. Let g be a continuous function on the closed interval $[a, b]$, with $g : [a, b] \rightarrow [a, b]$. Furthermore, suppose that g' is continuous on the open interval (a, b) , and there exists a positive constant k such that $|g'(x)| \leq k < 1$, $\forall x \in (a, b)$. Prove that if $g'(p) \neq 0$, then for any $p_0 \in [a, b]$, the sequence $\{p_n\}$, where $p_n = g(p_{n-1})$, converges only linearly to the fixed point p .
- (c) Perform four iterations of Secant method to approximate the root of $x^4 - 7x^2 + 3 = 0$, in the interval $(0, 1)$, using initial approximations $p_0 = 0$ and $p_1 = 1$.
3. (a) For the following system of equations :

$$8x + 5y + 2z = 8,$$

$$6x + 9y + 2z = 11,$$

$$4x + 3y + 8z = 1,$$

starting with the initial approximation as $(0, 0, 0)$, using Gauss Jacobi method, find approximate solution by performing three iterations.

- (b) For the following system of equations :

$$20x + y - 2z = 17,$$

$$3x + 20y - z = -18,$$

$$2x - 3y - 20z = 25,$$

starting with initial approximation as $(0, 0, 0)$, using Gauss Seidel method, find the approximate solution by performing three iterations.

- (c) Find the LU decomposition by taking $u_{ii} = 1$ for the matrix :

$$\begin{bmatrix} 1 & 4 & 3 \\ 2 & 7 & 9 \\ 5 & 8 & -2 \end{bmatrix}.$$

4. (a) Construct the Newton form of interpolating polynomial for a function f passing through the points $(0, 1)$, $(1, 3)$ and $(3, 55)$. Hence, evaluate $f(2.5)$.
- (b) Let $f(x) = \ln(1 + x)$, $x_0 = 1$ and $x_1 = 1.1$. Use Lagrange interpolation to calculate an approximate value for $f(1.04)$ and obtain a bound on the truncation error at $x = 1.04$.
- (c) Obtain the piecewise linear interpolating polynomial for the function $f(x)$ defined by the data :

x	$f(x)$
1	3
2	7
4	21
8	73

5. (a) Derive the central difference approximation to the second order derivative of a function f at a point $x = x_0$

$$f''(x_0) \approx \frac{f(x_0 + h) - 2f(x_0) + f(x_0 - h)}{h^2}.$$

P.T.O.

- (b) Verify that the following difference approximation

$$f'(x_0) \approx \frac{3f(x_0) - 4f(x_0 - h) + f(x_0 - 2h)}{2h}.$$

for the first order derivative provides the exact value of the derivative, regardless of the values of h , for the functions $f(x) = 1$, $f(x) = x$ and $f(x) = x^2$, but not for the function $f(x) = x^3$.

- (c) By using Trapezoidal rule, approximate the value of the integral

$$\int_2^3 (x + \ln x) dx. \text{ Further, calculate the absolute error, theoretical error}$$

bound and compare them.

6. (a) Define the degree of precision of a quadrature rule. Further, calculate the degree of precision of Simpson's rule.
- (b) Use Euler's method to determine the approximate solution of the initial value problem $y'(x) = xy^3 - y$, $0 \leq x \leq 2$ such that $y(0) = 2$, by taking the step size as $h = 1/2$.
- (c) Use Modified Euler's method to determine the approximate solution of the initial value problem : $y'(x) = 3x - y/x$, $1 \leq x \leq 2$, such that $y(1) = 2$, by taking the step size as $h = 1/2$.

(7)
[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5514

J

Unique Paper Code : 2352013601

Name of the Paper : Advanced Group Theory

Name of the Course : B.Sc. (H) Mathematics

Semester : VI – DSC-16

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory and are of 15 marks each.
3. Attempt any five parts from Question 1. Each part is of 3 marks.
4. Attempt any two parts from each of the Questions 2 to 6. Each part is of 7.5 marks.

1. (i) Prove that the left regular action of a group G on itself is a faithful action.
(ii) Prove that the kernel of an action of a group G on a set A is a normal subgroup of G .
(iii) Let σ be an m -cycle in the symmetric group S_n . Find $|C_{S_n}(\sigma)|$, the order of centralizer of σ .

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P.T.O.

- (iv) Is a group of order 210 simple? Justify.
 - (v) Define commutator subgroup G' of a group G . Prove that G' is normal in G .
 - (vi) Prove that the center of a group is a characteristic subgroup.
 - (vii) Define a solvable group. Prove that every abelian group is solvable.
2. (a) For a group G , consider the mapping $G \times G \rightarrow G$ given by $g.a = gag^{-1}$. Prove that this defines a group action of G on itself. Also, find the kernel of this action and the stabilizer G_x of an element $x \in G$.
- (b) Let $G = D_8$ be the dihedral group of order 8 and let $A = \{1, r, r^2, r^3\}$, where r denotes a clockwise rotation of the square by $\frac{\pi}{2}$ radians. Show that the normalizer $N_{D_8}(A) = D_8$ and the centralizer $C_{D_8}(A) = A$.
- (c) Let p be a prime number and let G be a group of order p^α for some $\alpha \geq 1$. Prove that G has a nontrivial center $Z(G)$. Deduce that every group of order p^2 is abelian.
3. (a) Let the symmetric group S_n act on the set $A = \{1, \dots, n\}$ by $\alpha.a = \alpha(a)$, $\forall \alpha \in S_n, a \in A$. Prove that this action is transitive.

(b) Let the dihedral group D_8 act via its natural action on the set $A = \{1, 2, 3, 4\}$ consisting of four vertices of a square. Label these vertices 1, 2, 3, 4 in a clockwise direction. Let r be the rotation of the square clockwise by $\frac{\pi}{2}$ radians and s be the reflection in the line which passes through vertices 1 and 3. Find the stabilizer of all the four vertices of square.

(c) Let G be a finite group and let $g_1, g_2, \dots, g_r$ be representatives of the distinct conjugacy classes of G not contained in the centre $Z(G)$ of G . Then prove the class equation:

$|G| = |Z(G)| + \sum_{i=1}^r |G : C_G(g_i)|$, where $C_G(g_i)$ is the centralizer of the element g_i in G . Verify the class equation for the symmetric group S_3 .

4. (a) Let G be a finite group and p be a prime dividing the order of G . Let P be a Sylow p -subgroup of G , and let n_p denote the number of Sylow p -subgroups of G . Assume that G acts transitively on the set of its Sylow p -subgroups by conjugation.

(i) Describe the orbit of P under this action.

(ii) Show that $n_p = |G : N_G(P)|$, where $N_G(P)$ denotes the normalizer of P in G .

- (b) Let G be a group of order 12. Prove that either G has a normal Sylow 3-subgroup or G is isomorphic to alternating group A_4 .
- (c) State Sylow's first theorem. Let G be a group of order 77. Prove that G has a normal Sylow 7-subgroup and a normal Sylow 11-subgroup. Further, prove that G is cyclic.
5. (a) State and prove the Index theorem. Use this theorem to show that there is no simple group of order 216.
- (b) Let G be a finite group, and let p be the smallest prime dividing the order of G . Prove that if G has a subgroup of index p , then it must be a normal subgroup of G . Is it always true that G has a subgroup of index p ? Justify.
- (c) Prove that no simple group has order pqr , where p, q and r are distinct primes.
6. (a) Prove that a group G is solvable if and only if $G^{(n)} = \{e\}$ for some positive integer n , where $G^{(n)}$ is the n^{th} derived subgroup of G .
- (b) Find a composition series for the symmetric group S_4 .
- (c) Prove that any finite p -group is nilpotent.

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[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5554

J

Unique Paper Code : 2352013602

Name of the Paper : Advanced Linear Algebra

Name of the Course : **Bachelor of Science**
(Honours Course)
Mathematics

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Attempt any **TWO** parts from each Question.

1. (a) Let T be a linear operator on $\mathbb{R}^2$ defined as

$$T \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2a + b \\ a - 3b \end{pmatrix}.$$

For the ordered basis $\beta = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$ and

$\beta' = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$, of $\mathbb{R}^2$, find the change of

coordinate matrix Q that changes β' -coordinates into β -coordinates. Also, verify that

$$[T]_{\beta'} = Q^{-1} [T]_{\beta} Q. \quad (2.5+5)$$

- (b) Let $V = P_1(\mathbb{R})$ and for $p(x) \in V$, let $f_1, f_2 \in V^*$ be defined as

$$f_1(p(x)) = \int_0^1 p(t) dt$$

and $f_2(p(x)) = \int_0^2 p(t) dt.$

Prove that $\{f_1, f_2\}$ is a basis for V^* and find a basis of V for which it is the dual basis.

(3+4.5)

(c) Let V be finite dimensional vector space. Define the annihilator S^0 of a subset S of V and prove that S^0 is a subspace of V^* . If W_1 and W_2 are subspaces of V , prove that

$$(W_1 + W_2)^0 = W_1^0 \cap W_2^0. \quad (3.5+4)$$

2. (a) Let $A = \begin{pmatrix} i & 1 \\ 2 & -i \end{pmatrix} \in M_{2 \times 2}(C)$. Determine all eigen

values of A and for each eigen value λ of A , find the set of eigen vectors corresponding to λ . Also, find a basis for C^2 consisting of eigenvectors of A .

(2.5+5)

(b) Let T be a linear operator on $P_2(\mathbb{R})$ defined as

$$T(f(x)) = f(0) + f(1)(x + x^2).$$

Test the linear operator T for diagonalizability. If T is diagonalizable, then find a basis β for V such that $[T]_\beta$ is a diagonal matrix. (2.5+5)

(c) Let T be a diagonalizable linear operator on a finite dimensional vector space V and $\lambda_1, \lambda_2, \dots, \lambda_k$ be the distinct eigen values of T . Prove that

$$V = E_{\lambda_1} \oplus E_{\lambda_2} \oplus \dots \oplus E_{\lambda_k}, \text{ where } E_{\lambda_i} \text{ is the eigen space of } \lambda_i, \text{ for all } i. \quad (7.5)$$

3. (a) Let T be a linear operator on the vector space $V = \mathbb{R}^4$ defined as

$$T(a, b, c, d) = (a + b, b - c, a + c, a + d)$$

Find an ordered basis of the T -cyclic subspace W of V generated by $z = e_1$. Also, find the characteristic polynomial of T_W . (3+4.5)

(b) Let T be a linear operator defined on a finite dimensional vector space V . Prove that the characteristic polynomial and the minimal polynomial of T have the same zeros. (7.5)

(c) Let T be a linear operator on $V = M_{n \times n}(\mathbb{R})$ defined as $T(A) = A^t$. Find the minimal polynomial of T . Hence show that T is diagonalizable. (7.5)

4. (a) Let V be an inner product space, prove that the following inequality holds

$$|\langle x, y \rangle| \leq \|x\| \|y\|, \text{ for all } x, y \in V.$$

Also, verify that the inequality holds for $x = (1, 2i, 1 + i)$, $y = (5 + i, 1, 2)$ in C^3 . (5+2.5)

(b) Let V be an inner product space and let S be an orthogonal subset of V consisting of nonzero vectors. Prove that S is linearly independent.

(7.5)

- (c) Let $W = \text{span} (\{(i, 0, 1)\})$ in C^3 . Find the orthonormal bases for W and $W^\perp$. (7.5)

5. (a) Let $V = P(R)$ with the inner product defined as

$$\langle f(x), g(x) \rangle = \int_0^1 f(t) g(t) dt, \quad \forall f(x), g(x) \in V.$$

Find the orthogonal projection of the vector $h(x) = 4 + 3x - 2x^2$ on the subspace $W = P_1(R)$. (7.5)

- (b) (i) Let V be a finite dimensional inner product space and β be an orthonormal basis for V . If T is a linear operator on V , show that

$$[T^*]_\beta = ([T]_\beta)^*.$$

- (ii) For the inner product space $V = C^2$ and linear operator

$$T(z_1, z_2) = (2z_1 + iz_2, (1 - i)z_1),$$

evaluate T^* at $z = (3 - i, 1 + 2i)$. (4.5+3)

- (c) Find the best fit linear function for the data $\{(-3,9), (-2,6), (0,2), (1,1)\}$ using the least squares approximation. Also, compute the error E .

(5+2.5)

6. (a) Let T be a normal operator defined on a finite dimensional real inner product space V whose characteristic polynomial splits. Prove that V has an orthonormal basis of eigen vectors of T . Hence prove that T is self-adjoint.

(5+2.5)

- (b) For the following matrix A , find an orthogonal matrix P and a diagonal matrix D such that $P^t A P = D$.

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}. \quad (7.5)$$

- (c) (i) State the Spectral theorem.

- (ii) Let T be a linear operator on a finite dimensional complex inner product space V . Show that T is unitary if and only if T is normal and $|\lambda| = 1$ for every eigen value λ of T . (2+5.5)

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This question paper contains 4 printed pages]

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S. No. of Question Paper : 5574

Unique Paper Code : 2352013603

Name of the Paper : Complex Analysis

Name of the Course : B.Sc. (Hons.) Mathematics (NEP-UGCF 2022)

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory and carry equal marks.

Each question contains three parts (a), (b) and (c), attempt only two parts.

1. (a) (i) Sketch the set $S = \left\{ z \in \mathbb{C} : |z| \leq 1, -\frac{\pi}{2} \leq \arg z \leq \frac{\pi}{2} \right\}$. 3

(ii) Under the map $f(z) = z^2$, draw the image of the set : 4.5

$$S = \left\{ z \in \mathbb{C} : |z| = 1, -\frac{\pi}{4} \leq \arg z \leq \frac{\pi}{4} \right\}.$$

(b) Under the map $f(z) = e^z$ draw the image of the rectangular region

$$R = \left\{ z = x + iy \in \mathbb{C} : 1 \leq x \leq 2, -\frac{\pi}{4} \leq y \leq \frac{\pi}{2} \right\}. \quad 7.5$$

P.T.O.

(c) Does $\lim_{z \rightarrow 0} \left(\frac{z}{\bar{z}} \right)^2$ exist ? Justify your answer. 7.5

2. (a) Show that the function $f(z) = \bar{z}$ is nowhere differentiable on $\mathbb{C}$. 7.5

(b) (i) Giving proper justification, write two differences between the functions : 3.5

$$f(x) = e^x, x \in \mathbb{R} \text{ and } f(z) = e^z, z \in \mathbb{C}$$

(ii) Solve the equation $e^z = -2$. 4

(c) Find out the values of $\sin^{-1} 2$. 7.5

3. (a) (i) Let a function f be integrable along a contour C . Prove that : 3.5

$$\int_{-C} f(z) dz = - \int_C f(z) dz.$$

(ii) Define a simple arc. Give an example to explain that a same set of points may represent different arcs. 4

(b) Let C be any simple closed contour, described in the positive sense in the z plane, then compute the value of the integral :

$$\int_C \frac{s^3 + 2s}{(s - z_0)^3} ds,$$

(i) when z_0 is inside C , and

(ii) when z_0 is outside C . 7.5

(c) (i) Let $w(t)$ be a piecewise continuous complex valued function defined on an interval $a \leq t \leq b$, then show that : 3.5

$$\left| \int_a^b w(t) dt \right| \leq \int_a^b |w(t)| dt.$$

- (ii) Let C be the arc of the circle $|z| = 2$ such that $0 \leq \arg z \leq \frac{\pi}{2}$.
Show that :

$$\left| \int_C \frac{z+4}{z^3-1} dz \right| \leq \frac{6\pi}{7}.$$

4. (a) State and prove Cauchy integral formula. 7.5
- (b) Let f be an entire function such that $|f(z)| \leq A|z|$ for all z , where A is a fixed positive number. Show that $f(z) = bz$, where b is a complex constant. 7.5
- (c) (i) Evaluate the following integral along the line segment, say L , from 0 to $\pi + 2i$: 3.5

$$\int_L \cos(z/2) dz.$$

- (ii) Evaluate the following integral :

$$\int_C \frac{z dz}{(9-z^2)(z+i)},$$

where C is the positively oriented circle $|z| = 2$.

5. (a) State Laurent's series theorem. Give two series expansions in powers of z for the function $f(z) = \frac{z+1}{z-1}$ specifying the regions in which those expansions are valid. 7.5
- (b) If a power series $\sum_{n=0}^{\infty} a_n (z-z_0)^n$ converges when $z = z_1$ ($z_1 \neq z_0$), then prove that it is absolutely convergent at each point z in the open disk $|z-z_0| < R_1$ where $R_1 = |z_1 - z_0|$. 7.5

P.T.O.

- (c) Define residue. Evaluate the residue of the function $f(z) = \frac{e^z - 1}{z^4}$ and use it to find the integral $\int_C \frac{e^z - 1}{z^4} dz$, where C is the positively oriented unit circle $|z| = 1$. 7.5

6. (a) Define an isolated singular point. For the below given functions, find the isolated singular point and determine whether it is a removable singular point, an essential singular point, or a pole : 7.5

(i) $\frac{\sin z}{z}$ and

(ii) $\frac{e^{2z}}{(z-1)^2}$.

- (b) Let z_0 be an isolated singular point of a function f . Then show that the following two statements are equivalent :

(i) z_0 is a pole of order m ($m = 1, 2, \dots$) of f ;

(ii) $f(z)$ can be written in the form $f(z) = \frac{\varphi(z)}{(z - z_0)^m}$, where $\varphi(z)$ is analytic

and non-zero at z_0 .

Further, establish that

$$\text{Res}_{z=z_0} f(z) = \frac{\varphi^{(m-1)}(z_0)}{(m-1)!} \text{ when } m = 1, 2, 3, \dots$$

where $\varphi^{(0)} \equiv \varphi$.

- (c) Using residues, evaluate the following integral : 7.5

$$\int_0^{2\pi} \frac{d\theta}{5 + 4 \sin \theta} d\theta.$$

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This question paper contains 4 printed pages]

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S. No. of Question Paper : 5377

Unique Paper Code : 2354001202

Name of the Paper : Introduction to Linear Algebra

Name of the Course : Common Prog Group

Semester : II

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all questions by selecting two parts from each question.

All questions carry equal marks.

1. (a) Let x and y be vectors in $\mathbf{R}^n$. Prove that $\|x + y\| \leq \|x\| + \|y\|$.
- (b) Solve the following system of linear equations using Gauss-Elimination method :

$$x_1 + 2x_2 + 3x_3 = 1$$

$$x_1 + 3x_2 + 5x_3 = 2$$

$$2x_1 + 5x_2 + 9x_3 = 3.$$

- (c) Solve the following system of linear equations using Gauss-Jordon method :

$$2x_1 + 3x_2 - x_3 = 9$$

$$x_1 + x_2 + x_3 = 9$$

$$3x_1 - x_2 - x_3 = -1.$$

P.T.O.

2. (a) Define rank of a matrix. Find the rank of the following matrix :

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 2 & 5 \\ 0 & -1 & 1 \end{bmatrix}.$$

- (b) Find the characteristic polynomial of the matrix :

$$A = \begin{bmatrix} 7 & 1 & -1 \\ -11 & -3 & 2 \\ 18 & 2 & -4 \end{bmatrix}.$$

Also, find all the eigenvalues and eigenspace for any *one* eigenvalue.

- (c) Is the matrix :

$$A = \begin{bmatrix} -3 & -1 & -2 \\ -2 & 16 & -18 \\ 2 & 9 & -7 \end{bmatrix}$$

diagonalizable ? Justify.

3. (a) Show that the set $\mathbf{R}^2$, with the usual scalar multiplication but with vector addition given by $[x, y] \oplus [u, z] = [x + y, 0]$ is not a vector space.

- (b) Use Simplified Span Method to find a simplified general form for all the vectors in $\text{Span}(S)$, where :

$$S = [x^3 - 1, x^2 - x, x - 1] \text{ is the subset of } \mathbf{P}_3.$$

- (c) Show that the subset $\{[-1, 2, -3], [3, 1, 4], [2, -1, 6]\}$ of $\mathbf{R}^3$ forms a basis of $\mathbf{R}^3$.

4. (a) Prove or disprove that the subset :

$$S = \left\{ \begin{bmatrix} a & b \\ -a & 0 \end{bmatrix}; a, b \in \mathbb{R} \right\}$$

is a subspace of M_{22} under usual matrix operations.

- (b) Use Independent Test Method to find whether the set :

$$S = \{[2, -1, 3], [4, -1, 6], [-2, 0, 2]\}$$

of vectors is linearly independent or linearly dependent.

- (c) Determine whether the vector $X = [7, 1, 18]$ is in the row space of the matrix :

$$A = \begin{bmatrix} 3 & 6 & 2 \\ 2 & 10 & -4 \\ 2 & -1 & 4 \end{bmatrix}.$$

If so, then express $[7, 1, 18]$ as a linear combination of the rows of A.

5. (a) Define the linear transformation. Determine whether the following function is a linear transformation :

$$T_1 : M_{32} \rightarrow P_4 \text{ given by } T_1 \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix} = a_{11}x^4 - a_{21}x^2 + a_{31}.$$

- (b) Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear operator that performs a counterclockwise through an angle 30° . Find the matrix representation for T with respect to the ordered basis $B = \{[1, 1], [2, 3]\}$ for $\mathbb{R}^2$.

P.T.O.

4. (a) Prove or disprove that the subset :

$$S = \left\{ \begin{bmatrix} a & b \\ -a & 0 \end{bmatrix}; a, b \in \mathbb{R} \right\}$$

is a subspace of M_{22} under usual matrix operations.

- (b) Use Independent Test Method to find whether the set :

$$S = \{|2, -1, 3|, |4, -1, 6|, |-2, 0, 2|\}$$

of vectors is linearly independent or linearly dependent.

- (c) Determine whether the vector $X = [7, 1, 18]$ is in the row space of the matrix :

$$A = \begin{bmatrix} 3 & 6 & 2 \\ 2 & 10 & -4 \\ 2 & -1 & 4 \end{bmatrix}$$

If so, then express $[7, 1, 18]$ as a linear combination of the rows of A.

5. (a) Define the linear transformation. Determine whether the following function is a linear transformation :

$$T_1 : M_{32} \rightarrow P_4 \text{ given by } T_1 \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix} = a_{11}x^4 - a_{21}x^2 + a_{31}.$$

- (b) Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear operator that performs a counterclockwise through an angle 30° . Find the matrix representation for T with respect to the ordered basis $B = \{|1, 1|, |2, 3|\}$ for $\mathbb{R}^2$.

P.T.O.

(30)

- (c) Let $T : P_2(\mathbb{R}) \rightarrow P_3(\mathbb{R})$ be given by :

$$T(p(x)) = \int p(x) dx.$$

Find the matrix for T with respect to standard bases for $P_2(\mathbb{R})$ and $P_3(\mathbb{R})$. Use this matrix to calculate $T(3x^2 - 2x + 5)$ by matrix multiplication.

6. (a) Suppose $L : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear operator with :

$$L([1, 2]) = [1, 1] \text{ and } L([2, 0]) = [2, -4].$$

Give a formula for $L([x, y])$ for any $[x, y] \in \mathbb{R}^2$. Also, find a basis for $\text{Ker}(L)$ and $\text{Range}(L)$.

- (b) Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the linear operator given by :

$$T \left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = \begin{bmatrix} 1 & -1 & 5 \\ -2 & 3 & -13 \\ 3 & -3 & 15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Find a basis for $\text{Ker}(T)$ and a basis for $\text{Range}(T)$. Also, verify that :

$$\dim(\text{Ker}(T)) + \dim(\text{Range}(T)) = \dim(\mathbb{R}^3).$$

- (c) Let $T : M_{23}(\mathbb{R}) + M_{22}(\mathbb{R})$ be the linear transformation given by :

$$T \left(\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \right) = \begin{bmatrix} a+b & a+c \\ d+e & d+f \end{bmatrix}.$$

Show that T is onto but not one-to-one.

(11)
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S. No. of Question Paper : 5967

Unique Paper Code : 2352571201

Name of the Paper : Elementary Linear Algebra

Name of the Course : B.A./B.Sc. (Prog.) with Mathematics as
Non-Major/Minor-DSC

Semester : II

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting any *two* parts from each question.

All questions carry equal marks.

Use of simple calculators is allowed.

1. (a) Define $\|x\|$ for any vector x in $\mathbf{R}^n$. Prove that :

$$x \cdot y = \frac{1}{4}(\|x + y\|^2 - \|x - y\|^2)$$

for any vectors x and y in $\mathbf{R}^n$.

2+5.5

- (b) Use the Gaussian elimination method to solve the system of linear equations :

$$x_1 + x_2 + x_3 = 5$$

$$2x_1 + x_2 - x_3 = 2$$

$$2x_1 - x_2 + x_3 = 2.$$

7.5

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- (c) Find the rank of the matrix :

$$A = \begin{bmatrix} 1 & 3 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{bmatrix}.$$

7.5

2. (a) Determine whether the vector $[7, 1, 18]$ is in the row space of the matrix :

$$\begin{bmatrix} 3 & 6 & 2 \\ 2 & 10 & -4 \\ 2 & -1 & 4 \end{bmatrix}.$$

7.5

- (b) Use the Gauss-Jordan method to convert the matrix A to reduced row echelon form :

$$A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 5 \end{bmatrix}.$$

7.5

- (c) Find the eigenvalues and the corresponding eigenspaces for the following matrix :

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 3 & -2 \\ 0 & 0 & -4 \end{bmatrix}.$$

7.5

3. (a) Use the Diagonalization Method to determine whether the following matrix A is diagonalizable. If so, specify the matrices D and P. Check your work by verifying that $D = P^{-1}AP$.

$$A = \begin{bmatrix} -3 & 3 & -1 \\ 2 & 2 & 4 \\ 6 & -3 & 4 \end{bmatrix}.$$

7.5

(b) State True or False for the following statements. Give a justification for your choice :

(i) The set of polynomials of degree seven is a vector space under the operations of usual addition and scalar multiplication of polynomials.

(ii) The set of non-singular 2×2 matrices, under the operations of usual matrix addition and scalar multiplication, is a vector space. 3.5+4

(c) Show that the set of vectors of the form $[a, b, 0, c, a - 2b + c]$ in $\mathbb{R}^5$ forms a subspace of $\mathbb{R}^5$ under the operations of vector addition and scalar multiplication. 7.5

4. (a) Use the Simplified Span Method to find a simplified general form for all vectors in span (S), where S is a subset of $M_{2 \times 2}$ given by

$$\left\{ \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \right\}. \quad 7.5$$

(b) Determine whether the given subset of P is linearly independent :

$$\{4x^2 + 2, x^2 + x - 1, x, x^2 - 5x - 3\}. \quad 7.5$$

(c) Define a basis for a vector space. Prove that the following set is a basis of $M_{2 \times 2}$:

$$\left\{ \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} -3 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 5 & -2 \\ 0 & -3 \end{bmatrix} \right\}. \quad 2+5.5$$

P.T.O.

5. (a) Define $T : M_{2 \times 2} \rightarrow M_{2 \times 2}$ given by $T(A) = A^T$, where $M_{2 \times 2}$ is the vector space of all 2×2 matrices over $\mathbf{R}$. Is T a linear operator? Find range T and kernel T . Is T an isomorphism? 2+4+1.5
- (b) Find the matrix of the linear transformation $T : P_3 \rightarrow P_2$ given by $T(p(x)) = p'(x)$ with respect to the standard ordered bases of P_3 and P_2 respectively. 7.5
- (c) Find the range and kernel of the linear transformation $T : \mathbf{R}^3 \rightarrow \mathbf{R}^2$ defined as $T(a, b, c) = (a - b, 2a)$. Verify the Dimension Theorem for T . 2.5+2.5+2.5
6. (a) Are $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ defined as $T(x, y) = (0, y)$ and $S : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ defined as $S(x, y) = (\sin x, y)$ linear? Justify your answer. 3.5+4
- (b) Let A be a fixed non-singular $n \times n$ matrix. Show that $T : M_{n \times n} \times M_{n \times n}$ given by $T(B) = AB$ is an isomorphism. 7.5
- (c) Define $T : M_{2 \times 2} \rightarrow \mathbf{R}$ as $T(A) = \text{Trace } A$. Is T a linear transformation? Find the matrix of T with respect to the standard ordered bases of $M_{2 \times 2}$ (over $\mathbf{R}$) and $\mathbf{R}$ (over $\mathbf{R}$) respectively. 2.5+5

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Roll No.

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S. No. of Question Paper : 7719

Unique Paper Code : 2352571201

Name of the Paper : Elementary Linear Algebra

Name of the Course : B.A./B.Sc. (Prog.) with

Mathematics as Non-Major/Minor-DSC

Semester : II

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting any *two* parts from each question.

All questions carry equal marks.

Use of simple calculators is allowed.

1. (a) Find a quadratic equation $y = ax^2 + bx + c$ that goes through the points (3, 20), (2, 11) and (-2, 15). 7½
- (b) State Cauchy-Schwarz inequality and verify it for the vectors $x = (2, -1, 4)$ and $y = (-3, 0, 2)$. 2½+5
- (c) Use Gaussian elimination method to solve the system of linear equations : 7½

$$x_1 + 2x_2 + 3x_3 = 14$$

$$3x_1 + x_2 + 2x_3 = 11$$

$$2x_1 + 3x_2 + x_3 = 11.$$

P.T.O.

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2. (a) Find the rank of the matrix :

7½

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}.$$

- (b) Determine the characteristic roots and characteristic vectors of the matrix :

7½

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}.$$

- (c) Use the Gauss-Jordan method to convert the given matrix to reduced row echelon form :

7½

$$\begin{bmatrix} 1 & 2 & -1 & 2 \\ 2 & 5 & -2 & 3 \\ 1 & 2 & 1 & 2 \end{bmatrix}.$$

3. (a) Use the Diagonalization Method to determine whether the following matrix A is diagonalizable. If so, specify the matrices D and P. Check your work by verifying that $D = P^{-1}AP$:

7½

$$A = \begin{bmatrix} -5 & 18 & 6 \\ -2 & 7 & 2 \\ 1 & -3 & 0 \end{bmatrix}.$$

- (b) Let V be a vector space. Prove that for any real number a and vector v in V, $av = 0$ implies $a = 0$ or $v = 0$. Show that the set $\mathbb{R}$ with the usual scalar multiplication but with addition given by $x \oplus y = 2(x + y)$ is not a vector space.

3½+4

- (c) Define subspace of a vector space V . Prove that the set of all real valued functions f defined on the interval $[0, 1]$ such that $f\left(\frac{1}{2}\right) = 0$ is a subspace of the vector space of all real valued functions defined on the interval $[0, 1]$. 2+5½
4. (a) Define span of a subset S in a vector space V . Prove that $\text{span}(S)$ forms a subspace of V . 2+5½
- (b) Determine whether the given subset of P , the vector space of all polynomials with real coefficients, is linearly independent : 7½
- $\{3x^3 + 2x + 1, x^3 + x, x - 5, x^3 + x - 10\}$.
- (c) Define a basis for a vector space V . Check whether the following subset of $\mathbf{R}^4$ forms a basis of $\mathbf{R}^4$: 2+5½
- $\{[2, 1, 0, 0], [0, 1, 1, -1], [0, -1, 2, -2], [3, 1, 0, -2]\}$.
5. (a) Define a linear transformation from one vector space to another. Let $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ be given by $T(x, y) = (y, x)$ and $S : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ given by $S(x, y) = (x + 1, y)$, where $\mathbf{R}^2$ is a vector space over $\mathbf{R}$. Are T and S linear transformations ? Justify your answer. 1½+3+3
- (b) Find the matrix of the linear transformation $T : \mathbf{R}^3 \rightarrow \mathbf{R}^3$ defined as $T(x, y, z) = (x, y, 0)$ with respect to the standard ordered basis of $\mathbf{R}^3$. Also find the matrix of $[T]_\beta$, where $\beta = \{(0, 0, 1), (1, 0, 0), (0, 1, 0)\}$. Are the two matrices equal ? 3½+3+1

(c) State Dimension Theorem for a linear transformation. Verify Dimension Theorem for the linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined as $T(x, y, z) = (0, y)$. 2+5½

6. (a) Is the linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined as $T(x, y) = (x, x + y, y)$ one-to-one ? Is it onto ? Justify your answer. 4+3½

(b) Show that $T : M_{2 \times 2} \rightarrow M_{2 \times 2}$ given by $T(A) = \text{trace } A$ is a linear transformation. Is T an isomorphism ? 3½+4

(c) Find bases for range and kernel of the linear transformation $T : P_4 \rightarrow P_2$ given by :

$$T(a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4) = a_0 + a_1x + a_2x^2. \quad 4+3½$$

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Roll No.

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S. No. of Question Paper : 7753

Unique Paper Code : 2352201202

Name of the Paper : Analytic Geometry

Type of the Paper : DSC

Name of the Course : Bachelor of Arts (Programme) with
Mathematics as Major

Semester : II

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any two parts of each question.

Each part carries 7.5 marks.

Use of calculators is not allowed.

1. (a) Identify and sketch the curve :

$$4y^2 - x^2 + 40y - 4x = -60.$$

(b) Describe the graph of the equation :

$$x^2 + 9y^2 + 2x - 18y + 1 = 0.$$

(c) Sketch the curve and label the focus, vertex and directrix :

$$(y + 1)^2 = -7(x - 4).$$

P.T.O.

2. (a) Find the equation of the parabola with vertex $(5, -3)$; axis parallel to the y -axis and passes through $(9, 5)$.

- (b) Identify and sketch the curve :

$$x^2 + 4xy - 2y^2 - 6 = 0.$$

- (c) (i) Find the vector of length 2 and oppositely directed to $\vec{v} = -3i + 4j + k$.

- (ii) Find unit vector oppositely directed to $6i - 4j + 2k$.

3. (a) For $\vec{u} = \langle 1, 2, -2 \rangle$ and $\vec{v} = \langle 3, 0, 1 \rangle$, find :

(i) $\vec{v} \times \vec{u}$

(ii) $\vec{u} \times \vec{v}$

(iii) $\vec{u} \times \vec{u}$

(iv) $\vec{v} \times \vec{v}$.

- (b) Find the angles between the lines whose direction ratios are :

(i) $5, -12, 13$ and $-3, 4, 5$.

(ii) $1, 1, 2$ and $\sqrt{3} - 1, -\sqrt{3} - 1, 4$.

- (c) Find the parametric equations of line :

(i) passing through $(4, 2)$ and parallel to $\vec{v} = -i + 5j$;

(ii) passing through $(1, 2, -3)$ and parallel to $\vec{v} = 4i + 5j - 7k$;

- (iii) passing through the origin in 3 - space and parallel to
 $\vec{v} = i + j + k$.

4. (a) (i) Find an equation of the plane passing through the point
 $(3, -1, 7)$ and perpendicular to the vector $\vec{n} = \langle 4, 2, -5 \rangle$.
 (ii) Determine whether the planes $3x - 4y + 5z = 0$ and
 $-6x + 8y - 10z - 4 = 0$ are parallel.

- (b) Show that the lines $\frac{x+5}{3} = \frac{y+4}{1} = \frac{z-7}{-2}$ and $3x + 2y + z - 2 = 0$
 $= x - 3y + 2z - 13$ are coplanar and find the equation to the plane
 in which they lie.

- (c) Find the magnitude and the equations of the line of shortest distance
 between the lines :

$$\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{y-5}{-5}.$$

5. (a) Find the equations of the sphere through the circle $x^2 + y^2 + z^2 = 1$,
 $2x + 4y + 5z = 6$ and touching the plane $z = 0$.

- (b) Find the equation of the sphere which touches the sphere :

$$x^2 + y^2 + z^2 - x + 3y + 2z - 3 = 0,$$

at the point $(1, 1, -1)$ and passes through the origin.

- (c) Find the equation of curve whose vertex is the point $(1, 1, 0)$ and
 whose guiding curve is :

$$y = 0, x^2 + z^2 = 4.$$

P.T.O.

6. (a) Find the equation of the right circular cylinder of radius 2 whose axis is the line :

$$\frac{(x-1)}{2} = \frac{(y-2)}{2} = \frac{(z-2)}{2},$$

- (b) Find the equation of a cylinder whose generating lines have the direction cosines (l, m, n) and which passes through the circle $x^2 + z^2 = a^2, y = 0$.
- (c) Show that the general equation of a cone which touches the three co-ordinate planes is :

$$\sqrt{fx} \pm \sqrt{gy} \pm \sqrt{hz} = 0;$$

f, g, h being parameters.

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This question paper contains 3 printed pages]

Roll No.

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S. No. of Question Paper : 5874

Unique Paper Code : 2352572401

Name of the Paper : Abstract Algebra

Name of the Course : B.A./B.Sc. (Programme)

Type of the Paper : DSC

Semester : IV

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all questions by selecting *two* parts from each question.

Both parts of a question to be attempted together.

All questions carry equal marks.

Use of calculator is not allowed.

1. (a) Define a group. Prove that in a group G , left and right cancellation laws hold true. 2+5.5

- (b) Show that the set

$$G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix} \mid a \in \mathbf{R}, a \neq 0 \right\}$$

is a group under matrix multiplication. 7.5

P.T.O.

- (c) Describe each symmetry in D_3 (the set of symmetries of an equilateral triangle). Also construct its corresponding Cayley table. 3.5+4

2. (a) Let G be an Abelian group and H, K be subgroups of G . Then prove that :

$$HK = \{hk \mid h \in H, k \in K\}$$

is a subgroup of G .

7.5

- (b) Define order of an element in a group G . Find the order of each element in the group $U(15)$. 2+5.5

- (c) State and prove One-Step Subgroup Test. 2+5.5

3. (a) Define a cyclic group. Let $|a| = 30$, then find $\langle a^{26} \rangle$, $\langle a^{17} \rangle$, $\langle a^{18} \rangle$ and $|a^{26}|$, $|a^{17}|$ and $|a^{18}|$. 1.5+3+3

(b) Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 & 1 & 7 & 8 & 6 \end{bmatrix}$, $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{bmatrix}$

Compute each of the following :

(i) $\alpha\beta$

(ii) $\beta\alpha$

(iii) $o(\alpha)$ and $o(\beta)$.

2+2+3.5

- (c) Let $G = S_3$ and $H = \{(1), (13)\}$. Then find left and right cosets of H in G . 7.5

4. (a) Let G be group and H be a subgroup of G . Then for $a, b \in G$, either $aH = bH$ or $aH \cap bH = \phi$. 7.5
- (b) Prove that the center $Z(G)$ of a group G is a normal subgroup of G . Let $H = \{(1), (12)\}$. Is H normal in S_3 ? 4+3.5
- (c) Define kernel of a group homomorphism $\phi : G \rightarrow \bar{G}$. Show that $\text{Ker } \phi$ is a subgroup of group G . 1.5+6
5. (a) Find the unity in the ring $\{0, 2, 4, 6, 8\}$ under addition and multiplication modulo 10. In Z_6 , show that $4 \mid 2$ (4 divides 2) and in Z_8 , show that $3 \nmid 7$ (3 divides 7). 2.5+5
- (b) Define characteristic of a ring. Prove that characteristic of an integral domain is either '0' or prime. 2.5+5
- (c) Show that every non-zero element of Z_n is a unit or a zero divisor. 7.5
6. (a) Describe the factor ring $Z/6Z$. Is $Z/6Z$ a field ? Justify. 3+4.5
- (b) Determine all ring homomorphisms from Z_n to itself. 7.5
- (c) Prove that the only ideals of a field F are $\{0\}$ and F itself. 7.5

(5)
This question paper contains 3 printed pages]

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S. No. of Question Paper : 7439

Unique Paper Code : 2352572401

Name of the Paper : Abstract Algebra

Name of the Course : B.A./B.Sc. (Programme) Mathematics as
Non-Major/Minor-DSC

Semester : IV

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting any *two* parts from each question.

Each part carries 7.5 marks.

Use of calculator is not allowed.

1. (a) Let a and n be positive integers and let $d = \gcd(a, n)$. Prove that the equation $ax \bmod n = 1$ has a solution if and only if $d = 1$.
- (b) Define Group. Show that the general linear group of 2×2 matrices over $\mathbf{R}$ is a group.
- (c) Prove that a group G of order 3 must be cyclic.

2. (a) Define Abelian group. Show that a group G is Abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$ for all a, b in G .
- (b) Define order of an element in a group. Find the order of each element of $U(15)$.
- (c) State and prove one-step subgroup test. Use it to prove that if G is an abelian group with identity e , then $H = \{x \in G \mid x^2 = e\}$ is a subgroup of G .
3. (a) If $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 6 & 2 & 5 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 6 & 3 & 5 & 1 & 2 \end{pmatrix}$, then :
- (i) Find $O(\alpha\beta)$
- (ii) Find $O(\beta\alpha)$
- (iii) Find α^{-1} .
- (b) Find all the right cosets of $\{1, 15\}$ in $U(32)$.
- (c) Show that the factor group $\mathbf{Z}/5\mathbf{Z}$ is a cyclic group of order 5.
4. (a) Define normal subgroup. Prove that $SL(2, \mathbf{R})$, the group of 2×2 matrices with determinant 1 is normal subgroup of $GL(2, \mathbf{R})$, the group of 2×2 matrices with non-zero determinant.
- (b) Show that $\phi : \mathbf{Z}_{12} \rightarrow \mathbf{Z}_{12}$ given by $\phi(x) = 4x$ is a group homomorphism.
- (c) Show that the kernel of homomorphism $\phi : \mathbf{R}^* \rightarrow \mathbf{R}^*$ given by $\phi(x) = |x|$ is a cyclic group.

5. (a) Prove that Z_4 is a ring and find its set of units.
- (b) Let x, y and z belong to a ring R . Then show that :
- (i) $(-x)(-y) = xy$
- (ii) $x(y - z) = xy - xz$.
- (c) State ideal test and prove that $2\mathbb{Z} = \{2n \mid n \in \mathbb{Z}\}$ is an ideal of $\mathbb{Z}$.
6. (a) Define zero-divisors and integral domain. Give an example of a commutative ring without zero-divisors that is not an integral domain.
- (b) Let a, b and c belong to an integral domain. If $a \neq 0$ and $ab = ac$, then show that $b = c$.
- (c) Let f be a ring homomorphism from a ring R to a ring S and let A be a subring of R . Then prove that $f(A) = \{f(a) \mid a \in A\}$ is a subring of S .

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[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1791

J

Unique Paper Code : 62357604

Name of the Paper : Differential Equations

Name of the Course : B.A. (Prog.) (CBCS-LOCF)

Semester : VI

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Attempt any two parts from each question.

1. (a) Solve the initial value problem: (6)

$$(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0; y(0) = 2.$$

- (b) Solve the differential equation: (6)

$$(x^2 + xy)dy = (x^2 + y^2)dx.$$

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(c) Solve :

(6)

$$x \frac{dy}{dx} + y = -2x^6 y^4.$$

2. (a) Solve :

(6.5)

$$y = 2px - xp^2.$$

(b) Solve :

(6.5)

$$xy(y - px) = x + py.$$

(c) Solve :

(6.5)

$$\frac{d^3 y}{dx^3} - 4 \frac{d^2 y}{dx^2} - 3 \frac{dy}{dx} + 18y = 0.$$

3. (a) Show that the solutions e^{-x} , e^{3x} and e^{4x} of the differential equation

(6)

$$\frac{d^3 y}{dx^3} - 6 \frac{d^2 y}{dx^2} + 5 \frac{dy}{dx} + 12y = 0$$

are linearly independent on the interval $-\infty < x < \infty$ and write the general solution of the given equation.

(b) Using the method of variation of parameters, solve :

(6)

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + y = \frac{e^x}{x}.$$

(c) Solve :

(6)

$$x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + 5y = x^2 \sin(\ln x).$$

4. (a) Solve the following system of equations: (6.5)

$$\frac{dx}{dt} + 2x - 3y = t,$$

$$\frac{dy}{dt} - 3x + 2 = e^{2t}.$$

(b) Solve :

(6.5)

$$\frac{dx}{z-y} = \frac{dy}{x-z} = \frac{dz}{y-x}$$

(c) Solve :

(6.5)

$$(ydx + xdy)(a - z) + xydz = 0.$$

5. (a) Eliminate the arbitrary function f from the equation (6)

$$z = e^{ax+by} f(ax - by)$$

to find the corresponding partial differential equation.

(b) Find the general solution of the differential equation (6)

$$x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2).$$

(c) Find the complete integral of the partial differential equation (6)

$$2(z + xp + yq) = yp^2.$$

6. (a) (i) Classify the following partial differential equation into elliptic, parabolic or hyperbolic: (2.5)

$$x^2r - y^2t + px - qy = x^2$$

$$\text{where } r = \frac{\partial^2 z}{\partial x^2}, S = \frac{\partial^2 z}{\partial x \partial y}, t = \frac{\partial^2 z}{\partial y^2}, p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}.$$

(ii) Form a partial differential equation by eliminating constants a, b from relation :

$$z = (x - a)^2 + (y - b)^2. \quad (4)$$

(b) Find the general solution of the differential equation (6.5)

$$(x^2 - yz)p + (y^2 - zx)q = z^2 - xy.$$

(c) Find the complete integral of the partial differential equation (6.5)

$$p^2x + q^2y = z.$$

(12)
[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5106

J

Unique Paper Code : 2354000009

Name of the Paper : Abstract Algebra

Name of the Course : COMMON PROG GROUP

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the six questions are compulsory.
3. Attempt any two parts from each question.
4. Each part carries 7.5 marks.
5. Use of Calculator not allowed.

1. (a) Define $SL(2, \mathbb{F})$ and find the inverse of the

element $\begin{bmatrix} 3 & 4 \\ 4 & 4 \end{bmatrix}$ in $SL(2, \mathbb{Z}_5)$.

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(b) Describe the symmetries of a square. Construct the corresponding Cayley Table.

(c) Prove that the set $G = \left\{ \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \mid a, b, c \in \mathbb{R} \right\}$ is a group under matrix multiplication.

2. (a) If H and K are subgroups of a group G , show that $H \cap K$ is a subgroup of G . What can you say about $H \cup K$? Justify.

(b) Let G be a group. Then prove that: $c(a) = c(a^{-1})$ $\forall a \in G$, where $c(a)$ is the centralizer of a in G .

(c) Prove that the center of a group G is a subgroup of G . Find the center of the group D_4 . Justify.

3. (a) Define a cyclic group. Find an example of a noncyclic group, all of whose proper subgroups are cyclic. Justify your answer.

(b) Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 & 1 & 7 & 8 & 6 \end{pmatrix}$,

$\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{pmatrix}$.

Write α , β and $\alpha\beta$ as product of disjoint cycle and also find β^{-1} .

- (c) State the Lagrange's Theorem. Let $|G| = 60$, then what are the possible orders for subgroups of G .
4. (a) Let H be a subgroup of a group G , then show that either $\alpha H = bH$ or $\alpha H \cap bH = \phi$ for every $\alpha, b \in G$. Also, find all the left cosets of $\{1, 11\}$ in $U(30)$.
- (b) Define the factor group. Also, let $G = \mathbb{Z}_{24}$, then find the order of the element $14 + \langle 8 \rangle$ in the factor group $\mathbb{Z}_{24}/\langle 8 \rangle$.
- (c) Consider the map $f: \mathbb{Z} \rightarrow \mathbb{Z}_n$ defined by $f(m) = m \bmod n$. Show that f is a homomorphism. Also, find the $\text{Ker} f$.
5. (a) Show that the set $\mathbb{Z}[x]$ of all polynomials in the variable x with integer coefficients under addition and multiplication is a commutative ring with unity $f(x) = 1$.
- (b) State the subring test. Prove that in a ring R , the set $\{x \in R \mid \alpha x = x\alpha \text{ for all } \alpha \in R\}$ is a subring of R . Find all the subrings of the ring $\mathbb{Z}$.

(c) Prove that in a ring R with unity 1 ,

(i) if 1 has infinite order under addition, then the characteristic of R is 0 ,

(ii) if 1 has finite order n under addition, then the characteristic of R is n .

Hence or otherwise state the characteristics of the rings $\mathbb{Z}$, $M_2(\mathbb{Z})$ and $\mathbb{Z}_3[i]$.

6. (a) State the ideal test. Let R be a commutative ring with unity and let $a \in R$. Then verify that the sets $I = \{ra \mid r \in R\}$ and $S = \{ar \mid r \in R\}$ are ideals of R .

(b) Let $\phi: R \rightarrow S$ be a ring homomorphism. Prove that :

(i) $\phi(nr) = n\phi(r)$ and $\phi(r^n) = (\phi(r))^n$ for any $r \in R$ and any positive integer n .

(ii) $\phi(R)$ is commutative if R is commutative.

(c) Prove that every ring homomorphism ϕ from $\mathbb{Z}_n$ to itself has the form $\phi(x) = ax$, where $a^2 = a$. Also, find $\text{Ker}\phi$.

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This question paper contains 7 printed pages]

Roll No.

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S. No. of Question Paper : 7400

Unique Paper Code : 2352203601

Name of the Paper : DSC - Probability and Statistics

Name of the Course : Bachelor of Arts/Bachelor of Science
(Programme) with Mathematics as non-major/
minor

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any two parts from each question.

All questions carry equal marks.

Use of non-programmable scientific calculator and statistical tables is permitted.

1. (a) Consider the following data :

Class	Frequency
[2, 4)	9
[4, 6)	15
[6, 8)	5
[8, 12)	9
[12, 20)	8
[20, 30)	2

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Construct an appropriate histogram and comment on any interesting features.

- (b) The following data consists of observations on the time until failure (1000s of hours) for a sample of turbochargers from one type of engine. Compute the Median, Upper Fourth (third quartile) and Lower Fourth (first quartile)

590, 815, 575, 608, 350, 1285, 408, 540, 555, 679.

- (c) The following data contain the information about fuel characteristics of various vehicles. Find the value of the sample range, mean and variance.

27.3, 27.9, 32.9, 35.2, 44.9, 39.9, 30.0, 29.7, 28.5, 32.0, 37.6.

2. (a) In a certain residential suburb, 60% of all households get Internet service from the local cable company, 80% get television service from that company, and 50% get both services from that company. If a household is randomly selected, what is the probability that it gets at least one of these two services from the company, and what is the probability that it gets exactly one of these services from the company ?

- (b) State Baye's Theorem A large operator of timeshare complexes requires anyone interested in making a purchase to first visit the site of interest. Historical data indicates that 20% of all potential purchasers select a day visit, 50% choose a one-night visit, and 30% opt for a two-night visit. In

addition, 10% of day visitors ultimately make a purchase, 30% of one-night visitors buy a unit, and 20% of those visiting for two nights decide to buy. Suppose a visitor is randomly selected and is found to have made a purchase. How likely is it that this person made a day visit ?

(c) An aircraft seam requires 25 rivets. The seam will have to be reworked if any of these rivets is defective. Suppose rivets are defective independently of one another, each with the same probability.

(i) If 15% of all seams need reworking, what is the probability that a rivet is defective ?

(ii) How small should the probability of a defective rivet be to ensure that only 10% of all seams need reworking ?

3. (a) Consider whether the next person buying a computer at a certain electronics store buys a laptop or a desktop model.

$$\text{Let } X = \begin{cases} 1, & \text{if the customer purchase a desktop computer} \\ 0, & \text{if the customer purchase a laptop computer} \end{cases}$$

Find the probability mass function (pmf) for X , if 20% of all purchasers during that week select a desktop. Also draw the line graph for the pmf of X .

(b) A certain brand of upright freezer is available in three different rated capacities : 450L, 500L and 550L. Let X = the rated capacity of a

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freezer of this brand sold at a certain store. Suppose that X has (pmf)
 $p(450) = 0.2$, $p(500) = 0.5$, $p(550) = 0.3$.

(i) Compute $E(X)$, $E(X^2)$ and $V(X)$.

(ii) If the price of a freezer having capacity X is $2.5X - 650$, what is the expected price paid by the next customer to buy a freezer ?

(c) For any random variable X , let $E(X) = 5$ and $E[X(X - 1)] = 27.5$.

Compute :

(i) $E(X^2)$

(ii) $V(X)$

(iii) $V(2X + 3)$.

4. (a) "Time headway" in traffic flow is the elapsed time between the time that one car finishes passing a fixed point and the instant that the next car begins to pass that point. Let X = the time headway for two randomly chosen consecutive cars. Suppose that in a different traffic environment, the distribution of time headway has the form

$$f(x) = \begin{cases} \frac{K}{x^4}, & x > 1 \\ 0, & x \leq 1 \end{cases}$$

- (i) Determine the value of K for which $f(x)$ is a pdf.
(ii) Determine $P(X > 2)$.

- (b) Consider the pmf of the magnitude X of a dynamic load on a bridge (in newtons) as

$$f(x) = \begin{cases} \frac{1}{8} + \frac{3}{8}x, & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

- (i) Find the cumulative distribution function (cdf) for X .
- (ii) Compute $P(1 \leq X \leq 1.5)$ and $P(X > 1)$.
- (c) Define Binomial distribution. Find the mean of a binomial distributed random variable X . When a Bernoulli distribution becomes a particular case of Binomial distribution ?
5. (a) If a publisher of nontechnical books takes great pains to ensure that its books are free of typographical errors, so that the probability of any given page containing at least one such error is 0.005 and errors are independent from page to page, what are the probabilities that one of its 600-page novels will contain exactly one page with errors and at most three pages with errors ?
- (b) Suppose that 25% of all students at a large public university receive financial aid. Let X be the number of students in a random sample of size 50 who receive financial aid. Find the probability that at most 10 students receive aid. Also, find the probability that between 5 and 15 (inclusive) of the selected students receive aid.

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- (c) According to the article "Predictive Model for Pitting Corrosion in Buried Oil and Gas Pipelines" (Corrosion, 2009 : 332-342), the lognormal distribution has been reported as the best option for describing the distribution of maximum pit depth data from cast iron pipes in soil. The authors suggest that a lognormal distribution with $\mu = 0.353$ and $\sigma = 0.754$ is appropriate for maximum pit depth (mm) of buried pipelines. What value c is such that only 1% of all specimens have a maximum pit depth exceeding c ?
6. (a) The inside diameter of a randomly selected piston ring is a random variable with mean value 12 cm and standard deviation 0.04 cm. If $\bar{X}$ is the sample mean diameter for a random sample of $n = 16$ rings. How likely is it that the sample mean diameter exceeds 12.01 ?
- (b) Consider the following data of on x = rainfall volume (m^3) and y = runoff volume (m^3) for a particular location.

x	y
5	4
12	10
14	13
17	15
23	15
30	25
40	27
47	46
55	38
67	46

Find the equation of regression line. And estimate the runoff volume (y) when rainfall volume (x) is 50.

- (c) Consider the following data on two variables x and y :

x	y
2.4	13
3.4	21
4.6	18
3.7	16
2.2	20
3.3	17
4.0	21
2.1	16

Compute the coefficient of correlation between the variables x and y . Does the value of the coefficient of correlation between the variables change when each value of x and y in the data is doubled.

This question paper contains 4 printed pages]

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S. No. of Question Paper : 7472

Unique Paper Code : 2352203602

Name of the Paper : DSC-Elementary Mathematical Analysis

Name of the Course : NEP-UGCF B.A. (Prog) with Mathematics as
Major

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any two parts from each question.

All questions are compulsory.

1. (a) Define the limit of a function in terms of ϵ and δ . Use $\epsilon - \delta$ definition,

to show that $\lim_{x \rightarrow -4} (2x + 13) = 5$. 7.5

(b) State sequential criterion for limits. Use it to prove that $\lim_{x_0 \in \mathbb{R}} f(x)$ does

not exist, where : 7.5

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$$

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- (c) Let $f : A \rightarrow \mathbf{R}$ where $A \subseteq \mathbf{R}$ and let x_0 be a cluster point of A . Then, prove that $\lim_{x \rightarrow x_0} f(x) = f(x_0)$ if and only if for each sequence $\langle x_n \rangle$ in $A \setminus \{x_0\}$ that converges to x_0 , the sequence $\langle f(x_n) \rangle$ converges to $f(x_0)$. 7.5
2. (a) Define the continuity of a function at a point in terms of $\epsilon - \delta$. Use $\epsilon - \delta$ definition, to show that $f(x) = 2x^2 + 3x + 5$ is continuous at $x_0 = -1$. 7.5
- (b) State Intermediate Value theorem. Also, prove that $x 2^x = 1$ for some x in $(0, 1)$. 7.5
- (c) Define uniform continuity of a function f on interval I . Use it to prove that the function $f(x) = x^3$ is uniformly continuous on the interval $[0, 4]$. 7.5
3. (a) Let f be defined and bounded on a compact interval $[a, b]$ for each partition P of $[a, b]$. Define the upper and lower Darboux's sums for f over P . Also, show that for all partitions P of $[a, b]$, $\underline{S}(f, P) \leq \bar{S}(f, P)$. 7.5
- (b) Define Riemann Integrability of a bounded function on $[a, b]$. Prove that a constant function $f(x) = c$ is integrable over $[a, b]$ and $\int_a^b f = c(b - a)$. 7.5
- (c) Consider the characteristic function of a closed interval $f = \chi_{[1, 3]}$ given by :

$$f(x) = \begin{cases} 1, & \text{if } 1 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

Prove that f is integrable on $[0, 5]$ and find $\int_0^5 f$.

7.5

4. (a) Use sequential criterion for integrability to show that function $f(x) = x$ is integrable on $[0, 1]$ and also find $\int_0^1 f$. 7.5

(b) Prove that :

(i) If f is integrable on $[a, b]$ and if $x \in [a, b]$, $m \leq f(x) \leq M$, then $m(b - a) \leq \int_a^b f \leq M(b - a)$.

(ii) If f is integrable on $[a, b]$ and $\forall x \in [a, b]$, $|f(x)| \leq M$, then $\left| \int_a^b f \right| \leq M(b - a)$. 7.5

- (c) Let $f(x) = 3x + 2$ defined on the interval $[1, 3]$ and the partition $P = \left\{1, \frac{3}{2}, 2, 3\right\}$. Find $\underline{S}(f, P)$ and $\overline{S}(f, P)$. Also, find $\underline{S}(f, P) - \overline{S}(f, P)$. 7.5

5. (a) Define pointwise convergence and find the pointwise limit of the following sequence of function :

(i) $\langle f_n \rangle$, where $f_n(x) = \begin{cases} \frac{1}{n} & \text{if } |x| \leq \frac{1}{n} \\ |x| & \text{if } \frac{1}{n} < |x| \leq 1 \end{cases}$ on $[-1, 1]$

(ii) $\langle g_n \rangle$, where $g_n(x) = \frac{nx}{1 + n^2 x^2}$ on $\mathbf{R}$. 7.5

- (b) Let $f_n(x) = e^{-nx}$, $x \geq 0$. Then show that $\langle f_n \rangle$ is uniformly convergent on $[a, \infty]$, where $a > 0$ but is not uniformly convergent on $[0, \infty]$. 7.5

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- (c) Let $\langle f_n \rangle$ be a sequence of continuous functions that converges pointwise to a function f on $[a, b]$. Then does it mean that f is continuous on $[a, b]$? Justify your answer and also state the condition on convergence that ensures f is continuous on $[a, b]$. 7.5

6. (a) State Weierstarass M-Test to test the uniform convergence of a series of functions. Hence check the uniform convergence of the following series :

(i) $\sum_{n=1}^{\infty} \frac{\cos nx}{n^3}$ on $\mathbf{R}$

(ii) $\sum_{n=0}^{\infty} \frac{1}{1+n^2x^2}$ on $[1, 2]$. 7.5

- (b) Find the Radius of convergence and Interval of convergence for the following Power Series : 7.5

(i) $\sum_{n=0}^{\infty} \frac{n^2}{2^n} (x-3)^n$

(ii) $\sum_{n=0}^{\infty} \frac{n!}{3^n} x^n$

- (c) Write Power Series expansion of $\ln(1+x)$ and use Abel's theorem to show that $\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$. 7.5