[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1363 **I**

Unique Paper Code : 2352011101

Name of the Paper : Algebra (DSC-1)

Name of the Course : **B.Sc. (H) Mathematics**

Semester : 1

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.

2. Attempt all questions by selecting **two** parts from each question.

3. **All** questions carry equal marks.

1. (a) (i) Find a cubic equation with real coefficients two of whose roots are 1 and $3+2i$. Also state the result being used.

(ii) Find an upper limit (using both the theorems) to the roots of

$$x^7 + 2x^5 + 4x^4 - 8x^2 - 32 = 0.$$ (3.5+4)

(b) Solve $3x^3 + 11x^2 + 12x + 4 = 0$, being given that roots are in Harmonic progression. (7.5)

(c) Find all the rational roots of $6y^3 - 11y^2 + 6y - 1 = 0$. (7.5)

2. (a) Compute $z^n + \frac{1}{z^n}$ if $z + \frac{1}{z} = \sqrt{3}$. (7.5)

(b) Find |z|, arg z, Arg z, arg(-z) for z = $(7-7\sqrt{3}\ i)(-1\ -i)$. (7.5)

(c) Solve the equation $z^7 - 2iz^4 - iz^3\ 2 = 0$. (7.5)

3. (a) Solve $28x^3 + 9x^2 - 1 = 0$ by Cardan's method. (7.5)

(b) If a, b and c are non-zero integers with a and c relatively prime, prove that gcd (a, bc) = gcd (a, b) (7.5)

(c) (i) Find gcd of 1800 and 756 and express it in the form ma + nb for some integers m and n.

(ii)If a and b are relatively prime integers, prove that

$$\gcd(a + b, a - b) = 1 \text{ or } 2.$$ (4+3.5)

4\. (a) Solve the following pair of congruences, if possible. If no solution exists, explain why not.

$$2x + y \equiv 1 \ (mod\ 6)$$

$$x + 3y \equiv 3 \ (mod\ 6)$$ (7.5)

(b) If $a \equiv x \ (mod\ n)$ and $b \equiv y \ (mod\ n)$, then

(i) $a + b \equiv x + y \ (mod\ n)$

(ii) $ab \equiv xy \ (mod\ n)$ (3.5+4)

(c) State fundamental theorem of arithmetic. Suppose a and b are integers and p is a prime such that p|ab. Prove that $p|a$ or $p|b$. (2.5+5)

(a) Describe symmetries of a non-square rectangle with diagrams. Also, construct the corresponding Cayley table. (3.5+4)

(b) Define an Abelian group. Show that in a group G if ab = ac then b = c (called left cancellation property). Further, show that in a group G if ab =

P.T.O.

ca implies b = c for all a, b, c in G then G is Abelian (that is, left-right cancellation property implies Abelian). (2+2.5+3)

(c) Show that the set $G = \{\begin{bmatrix} a & a \\ a & a \end{bmatrix} \mid a \in \mathbb{R}, a \neq 0\}$

is a group under matrix multiplication. (7.5)

6. (a) State two-step subgroup test. Let G be an Abelian group and H, K be subgroups of G then show that

$$HK = \{hk \mid h \in H, k \in K\}$$

is a subgroup of G. (2+5.5)

(b) Define order of an element 'a', $O(a)$,, in a group G. Prove that in any group G, $O(bab^{-1}) = O(a)$ for all $a, b \in G$.. (2+5.5)

(c) Write all the generators of the cyclic group Z_{24}. Further describe all the subgroups of Z_{24} and find all generators of the subgroup of order 8 in Z_{24}. (3+3+1.5)

[This question paper contains 8 printed pages.]

Your Roll No..............

I

Sr. No. of Question Paper : 1382

Unique Paper Code : 2352011102

Name of the Paper : Elementary Real Analysis

Name of the Course : **B.Sc. (H) Mathematics (NEP-UGCF 2022)**

Semester : I – DSC-2

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **all** questions by selecting **three** parts from each question.

3. Part of the questions to be attempted together.

4. **All** questions carry equal marks.

5. Use of Calculator is not allowed.

P.T.O.

1. (a) If $a \cdot b = 0$, then either $a = 0$ or $b = 0$.

(b) State the order properties of $\mathbb{R}$. Using it prove that if a, b, c are real numbers such that $a > b$, then $a + c > b + c$.

(c) Find all values of x satisfying $|x - 2| \leq x + 1$.

(d) Write the definition of Supremum and Infimum of a set. Give an example of a set having supremum and infimum, where the set

(i) contains its supremum and infimum

(ii) does not contain its supremum and infimum

2. (a) State and prove Archimedean property.

(b) Let 5 be a non-empty subset of $\mathbb{R}$ and $a > 0$, then show that

$$\sup(aS) = a \sup S$$

(c) Let (x_n) be a sequence in $\mathbb{R}$ and let $x \in \mathbb{R}$. If (a_n) is a sequence of positive real numbers with

$\lim_{n\to\infty}(a_n)=0$ and for some constant $K > 0$ and some

$m \in \mathbb{N}$ we have $|x_n - x| \leq Ka_n$ for all $n \geq m$, then

prove that $\lim_{n\to\infty}(x_n)=x$.

(d) Using the definition of limit, show that

$$\lim_{n\to\infty}\left(\frac{4n+5}{3n+4}\right)=\frac{4}{3}.$$

3. (a) Let (x_n) and (y_n) be sequences of real number

such that $\lim_{n\to\infty}(x_n)=x$ and $\lim_{n\to\infty}(y_n)=y$, then show

that $\lim_{n\to\infty}(x_n+y_n)=x+y$.

(b) Let (x_n) be a sequence of positive real numbers such that $L = \lim_{n \to \infty} \left(\frac{x_{n+1}}{x_n} \right)$ exists. Show that if $L < 1$, then (x_n) converges and $\lim_{n \to \infty} (x_n) = 0$.

(c) State Squeeze theorem and show that if

$$z_n = \left(2^n + 3^n\right)^{\frac{1}{n}} \text{ then } \lim_{n \to \infty} z_n = 3.$$

(d) Let $X = (x_n)$ be a sequence of real numbers defined by $x_1 = 1$ and

$$x_{n+1} = \sqrt{2 + x_n} \quad \text{for } n \in \mathbb{R}.$$

Show that the sequence (x_n) is convergent and find its limit.

4. (a) Prove that if a sequence (x_n) is a monotone decreasing and bounded below sequence of real numbers, then it is convergent.

(b) State Bolzano Weierstrass Theorem for Sequences. Show that the sequence $((-1)^n)$ is divergent.

(c) Find limit inferior and limit superior of the following sequences:

(i) $\left(\sin\left(\frac{n\pi}{4}\right)\right)$

(ii) $(3 + (-1)^n)$

(d) Show that every Cauchy sequence of real numbers is bounded. Is the converse true? Justify your answer.

5. (a) State and prove Cauchy Criterion for convergence of a series $\sum_{n=1}^{\infty} a_n$.

(b) Test the convergence of the following series :

(i) $\sum_{n=1}^{\infty} \frac{n}{e^n}$

(ii) $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

(c) Prove that $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$, $p > 0$ is convergent for $p > 1$ and divergent for $p \leq 1$.

(d) Show that if the series Σu_n converges, then $\lim_{n \to \infty} u_n = 0$. Is the converse true? Justify your answer.

6. (a) State the Alternating Series test. Show that the alternating series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}$ is convergent.

(b) Test the convergence of the series

$$\frac{1}{e}+\frac{4}{e^2}+\frac{27}{e^3}+\frac{256}{e^4}+\frac{3,125}{e^5}+\cdots.$$

(c) Define a conditionally convergent series and an absolutely convergent series. Test the series $\sum_{n=1}^{\infty}\frac{(-1)^n \sin n}{n^{3/2}}$ for absolute or conditional convergence.

(d) State D'Alembert's Ratio test for a series. Find if the series,

$$\frac{1}{2}+\frac{1.2}{3.5}+\frac{1.2.3}{3.5.7}+\frac{1.2.3.4}{3.5.7.9}+\cdots$$ is convergent.

CH. NO 94494

This question paper contains 6 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1401**

Unique Paper Code : **2352011103**

Name of the Paper : **DSC 3-Probability and Statistics**

Name of the Course : **B.Sc. (H) Mathematics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

All questions carry equal marks.

Use of non-programmable scientific calculator and statistical tables is permitted.

1. (*a*) Construct a stem-and-leaf display for the given batch of exam scores, repeating each stem twice. What feature of the data is highlighted by this display ?

74	89	80	93	64	67	72	70	66	85	89	81
	81	71	74	182	85	63	72	81	81	95	84
81	80	70	69	66	60	83	85	98	84	68	90
82	69	72	87	88							

(*b*) The following data gives the reasonable award (in $ 1000s) in 27 cases identified by the court :

37	60	75	115	135	140	149	150	238	290	340	410
600	750	750	750	1050	1100	1139	1150	1200	1200	1250	
1576	1700	1825	2000								

P.T.O.

(i) What are the values of the quartiles and what is the value of the fourth spread ?

(ii) How large or small does an observation have to be to qualify as an outlier ? As an extreme outlier ?

(c) The value of Young's modulus (GPa) was determined for cast plates consisting of certain intermetallic substrates, resulting in the following sample observations :

116.6 115.9 114.6 115.2 115.8.

(i) Calculate the sample mean, the sample variance and the sample standard deviation.

(ii) Subtract 100 from each observation to obtain a sample of transformed values. Now calculate the sample variance of these transformed values and compare it with sample variance for the original data.

2. (a) A certain system can experience three different types of defects. Let A_i (i = 1,2,3) denote the event that the system has a defect of type i. Suppose that $P(A_1) = .12$, $P(A_2) = .07$, $P(A_3) = .05$, $P(A_1 \cup A_2) = .13$, $P(A_1 \cup A_3) = .14$, $P(A_2 \cup A_3) = .10$, $P(A_1 \cap A_2 \cap A_3) = .01$

(i) What is the probability that the system has both type-1 and type-2 defects but not a type-3 defect ?

(ii) What is the probability that the system has at most of these defects ?

(b) State Bayes' Theorem. At a certain gas station, 40% of the customers use regular gas (A_1), 35% use plus gas (A_2) and 25% use premium (A_3). Of those customers using regular gas, only 30% fill their tanks (event B). Of those customers using plus, 60% fill their tanks, whereas of those using premium, 50% fill their tanks. If the next customer fills the tank, what is the probability that regular gas is requested ?

(*c*) If A and B are independent event, show that :

(*i*) A' and B are also independent

(*ii*) A and B' are also independent

(*iii*) A' and B' are also independent.

3. (*a*) Starting at a fixed time, observe the gender of each new-born child at a certain hospital until a boy is born. Assume that the successive births are independent and define the random variable X by X = number of births observed.

Find :

(*i*) The probability mass function (*pmf*) of X

(*ii*) The cumulative distribution function (*cdf*) of X.

(*b*) A certain brand of upright freezer is available in three different rated capacities : 450L, 500L and 550L. Let X = the rated capacity of a freezer of this brand sold at a certain store. Suppose that X has *pmf* $p(450) = .2$, $p(500) = .5$, $p(550) = .3$.

(*i*) Compute E(X), $E(X^2)$ and V(X).

(*ii*) If the price of a freezer having capacity X is 2.5X – 650, what is the expected price paid by the next customer to buy a freezer ?

(*c*) Suppose that the number of drivers who travel between a particular origin and destination during a designated time period has a Poisson distribution with parameter $\mu = 20$. What is the probability that the number of drivers travelling between the origin and the destination during the designated time will be :

(*i*) between 10 and 20, both inclusive ?

(*ii*) within 2 standard deviations of the mean value ?

4. (*a*) "Time headway" in traffic flow is the elasped time between the time that one car finishes passing a fixed point and the instant that the next car begins to pass that point. Let X = the time headway for two randomly chosen consecutive cars. Suppose that in a different traffic environment, the distribution of time headway has the form

$$f(x) = \begin{cases} \dfrac{K}{x^4}, & x > 1 \\ 0, & x \le 1. \end{cases}$$

(*i*) Determine the value of K for which $f(x)$ is a *pdf*.

(*ii*) Obtain the *cdf*.

(*iii*) Determine $P(X > 2)$ and $P(2 < X < 3)$.

(*b*) In a road-paving process, asphalt mix is delivered to the hopper of the paver by truck that haul the material from the batching plant. Let the random variable X = truck haul time can be modelled with a normal distribution with mean value 8.46 min and standard deviation .913 min time. What is the probability that haul time :

(*i*) is at least 10 min ?

(*ii*) exceeds 15 min ?

(*iii*) remains between 8 and 10 min ?

(*c*) Suppose component lifetime is exponentially distributed with parameter λ. After putting the component into service, leave for a period of t_0 hours and then return to find the component still working. What is the probability that it lasts at least an additional t hours ?

5. (*a*) Suppose that 25 percent of all students at a large public university receive financial aid. For a random sample of students of size 50, use normal approximation of binomial distribution to find the probability that :

(*i*) at most 10 students receive aid,

(*ii*) between 5 and 15 (both inclusive) of the selected students receive aid.

(*b*) The stress range in certain railway bridge connection is exponentially distributed with an average of 6 MPa (megapascals). Find the probability that :

(*i*) The stress range is at most 10 MPas,

(*ii*) The stress range is between 5 and 10 MPas (both inclusive).

(*c*) Let X be the temperature in degree Celsius at which a certain chemical reaction takes place and let Y be the same temperature in degree Fahrenheit. It is known that conversion of unit from one of the two units to the other unit follows the rule $Y = 1.8X + 32$.

(*i*) If the median of the Y distribution is 50, find the median of X distribution.

(*ii*) If third quartile for X distribution is 15, find the third quartile for Y distribution.

6. (*a*) The lifetime of a certain type of battery is normally distributed with mean value 10 hours and standard deviation 1 hour. There are four batteries in a package. What lifetime value is such that the total lifetime of all batteries in a package exceeds that value for only 5 percent of all packages ?

(*b*) The efficiency ratio for a steel specimen immersed in a phosphating tank is the weight of the phosphate coating divided by the metal loss (both in mg/ft 2). The following data is on tank temperature (x) and efficiency ratio (y) :

Temp.(x)	170	172	173	174	174	175	176
Ratio(y)	.84	1.31	1.42	1.03	1.07	1.08	1.04
Temp.(x)	177	180	180	180	180	180	181
Ratio(y)	1.80	1.45	1.60	1.61	2.13	2.15	.84
Temp.(x)	181	182	182	182	182	184	184
Ratio(y)	1.43	.90	1.81	1.94	2.68	1.49	2.52
Temp.(x)	185	186	188				
Ratio(y)	3.00	1.87	3.08				

(i) Determine the equation of the estimated regression line.

(ii) Calculate a point estimate for true average efficiency ratio when tank temperature is 182.

(c) If x and y are two variables such that $\Sigma x = 25.7$, $\Sigma x^2 = 88.31$, $\Sigma y = 14.40$, $\Sigma y^2 = 26.4324$, $\Sigma xy = 46.856$, where summation runs over $n = 15$ values of x and y, then compute the coefficient of correlation between x and y. Does the value of the coefficient of correlation between the variables change when each value of x and y in the data is doubled ?

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5904 **I**

Unique Paper Code : 32351302

Name of the Paper : BMATH306 - Group Theory-1

Name of the Course : **B.Sc. Maths (Hons) (2019 onwards) LOCF**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.
2. Attempt all questions by selecting two parts from each question.
3. All parts of a question are to be attempted together.
4. Marks of the questions are indicated.
5. Use of Calculator not allowed.

1. (a) Let $G = \left\{ \begin{pmatrix} a & a \\ a & a \end{pmatrix} : a \in \mathbb{R} \sim \{0\} \right\}$. Show that G is an Abelian group.

P.T.O.

(b) Let G be group and H be a non-empty subset of G. Prove that H is a subgroup of G if

$ab^{-1} \in H, \forall a, b \in H$.

Hence prove that $H = \{A \in GL(2, \mathbb{R})$: det A is a power of 2$\}$ is a subgroup of $GL(2, \mathbb{R})$.

(c) Define a cyclic group. Show that every cyclic group is Abelian. Is the converse true? Justify.

(2×6=12)

2. (a) Prove that an Abelian group with two elements of order two must have a subgroup of order 4.

(b) Describe the group of symmetries of an equilateral triangle.

(c) Let G be a group and $a \in G$. If $|a| = n$, show that

$< a >= \{e, a, a^2, \ldots\ldots, a^{n-1}\}$ & $a^i = a^j \Leftrightarrow$ n divides $(i - j)$.

(2×6½=13)

3. (a) Let G = $<a>$ be a cyclic group of order n. Then show that G = $<a^k> \Leftrightarrow$ gcd (k, n) = 1.

(b) Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 1 & 3 & 5 & 4 & 7 & 6 & 8 \end{pmatrix}$ and

$\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{pmatrix}$

(i) Write α and β as product of disjoint cycles.

(ii) Compute $\alpha\beta$ and α^{-1}.

(c) Suppose that a has order 15. Compute all the left cosets of $< a^5 >$ in $<a>$. (2x6=12)

4. (a) Define left coset of a subgroup H of a group G. Find all the left cosets of H={1,11} in U(30).

(b) (i) Prove that a subgroup H of a group G is normal in G $\Leftrightarrow xHx^{-1} \subseteq H, \forall x \in G$.

(ii) Let H be a subgroup of a group G such that $x^2 \in H, \forall x \in G$. Show that H is a normal subgroup of G.

(c) Leto: $\phi: G \longrightarrow G'$ be a group homomorphism and let H be a subgroup of G. Prove that

(i) if H is cyclic, then $\phi(H)$ is cyclic.

(ii) if H is normal, then $\phi(H)$ is normal in $\phi(G)$. (2×6½=13)

5. (a) Let G be a finite group and H be a subgroup of G. Show that |H| divides |G|.

P.T.O.

(b) Determine all group homomorphisms from $\mathbb{Z}_n$ to $\mathbb{Z}_n$.

(c) Let H be a subgroup of G and a,b $\in$ G. Prove that

(i) $aH = bH \Leftrightarrow a \in bH$.

(ii) $aH = H \Leftrightarrow a \in H$. (2x6=12)

6 (a) State and prove the First Fundamental Theorem of Group Homomorphisms.

(b) Let G be the symmetric group of permutations on the set {1,2,3..., n}. For each σ in G, define

Sgn(σ) = {-1, if σ is an odd permutation

1, if σ is an even permutation.

Prove that Sgn is a group homomorphism. Find the kernel of Sgn.

(c) (i) Let $\phi: G \to G^*$ be a group homomorphism. Prove that the Ker & is a normal subgroup of the group G.

(ii) Show that : $\phi: (\mathbb{R}, +) \longrightarrow (\mathbb{R}^*, .)$ defined as $\phi(x) = x^2$ is not a group homomorphism. (2×6½=13)

(5)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1083 **I**

Unique Paper Code : 2352012301

Name of the Paper : Group Theory

Name of the Course : **B.Sc. (H) Mathematics UGCF**

Semester : III – DSCC-7

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **all** questions by selecting **two** parts from each question.

3. **All** questions carry equal marks.

4. Use of Calculator not allowed.

P.T.O.

1. (a) Prove that the order of a permutation of a finite set written in disjoint cycle form is the least common multiple of the lengths of the cycles. (7.5)

(b) (i) Let S_n denote the symmetric group of degree n. In S_3, find elements α and β such that $|\alpha| = 2$, $|\beta| = 2$ and $|\alpha\beta| = 3$.

(ii) Let $\beta \in S_7$ and $\beta^4 = (2\ 1\ 4\ 3\ 5\ 6\ 7)$. Then find β. (3,4.5)

(c) (i) Give two reasons to show that the set of odd permutations in S_n is not a subgroup of S_n.

(ii) Define even and odd permutations and show that the set of even permutations in S_n is a subgroup of S_n. (3, 4.5)

2. (a) (i) Let a be an element in a group G such that $|a| = 15$. Find all left cosets of $\langle a^5 \rangle$ in $\langle a \rangle$.

(ii) State and prove Lagrange's theorem.

(3,4.5)

(b) Suppose that G is a group with more than one element and G has no proper, non-trivial subgroups. Prove that $|G|$ is prime. (7.5)

(c) Let $\mathbb{C}^*$ be the group of non-zero complex numbers under multiplication and let $H = \{a + bi \in \mathbb{C}^* \mid a^2 + b^2 = 1\}$. Give a geometrical description of the coset $(3 + 4i)H$. Give a geometrical description of the coset $(c + di)H$. (7.5)

3. (a) (i) Let G be a group and H be its subgroup. Prove that if H has index 2 in G, then H is normal in G.

(ii) If a group G has a unique subgroup H of some finite order, then show that H is normal in G. (3,4.5)

(b) (i) Prove that a factor group of a cyclic group is cyclic. Is converse true? Justify your answer.

(ii) Let G be a group and let Z(G) be the center of G. If G/Z(G) is cyclic, then show that G is Abelian. (3,4.5)

(c) (i) Let ϕ be a group homomorphism from group G_1 to group G_2 and H be a subgroup of G_1. Show that if H is cyclic, then $\phi(H)$ is cyclic.

(ii) How many homomorphisms are there from $\mathbb{Z}_{20}$ to $\mathbb{Z}_8$? How many are there onto $\mathbb{Z}_8$?

(3,4.5)

4. (a) (i) Suppose that ϕ is a homomorphism from U(30) to U(30) and Ker ϕ = {1,11}. If $\phi(7) = 7$, find all the elements of U(30) that map to 7.

(ii) Let ϕ be a homomorphism from a group G_1 to group G_2. Show that $\phi(a) = \phi(b)$ iff aKer ϕ = bKer ϕ. (3,4.5)

(b) (i) Is U(8) isomorphic to U(10)? Justify your answer.

(ii) Show that any infinite cyclic group is isomorphic to the group of integers under addition. (3,4.5)

(c) If ϕ is an onto homomorphism from group G_1 to group G_2, then prove that $G_1/\text{Ker}\,\phi$ is isomorphic to G_2. Hence show that if G_1 is finite, then order of G_2 divides the order of G_1. (7.5)

5. (a) Let G be a group and let $a \in G$. Define the inner automorphism of G induced by a. Show that the set of all inner automorphisms of a group G, denoted by Inn(G), forms a subgroup of Aut(G), the group of all automorphisms of G. Find $\text{Inn}(D_4)$. (7.5)

(b) Prove that the order of an element in a direct product of a finite number of finite groups is the lcm of the orders of the components of the element, i.e., $|(g_1, g_2, \ldots, g_n)| = \text{lcm}(|g_1|, |g_2|, \ldots, |g_n|)$. Also, find the number of elements of order 7 in $\mathbb{Z}_{49} \oplus \mathbb{Z}_7$. (7.5)

(c) Without doing any calculations in $Aut(\mathbb{Z}_{105})$, determine how many elements of $Aut(\mathbb{Z}_{105})$ have order 6. (7.5)

6. (a) For any group G, prove that $G/Z(G) \cong Inn(G)$. (7.5)

(b) Define the internal direct product of a collection of subgroups of a group G. Let $\mathbb{R}^*$ denote the group of all nonzero real numbers under multiplication. Let $\mathbb{R}^+$ denote the group of all positive real numbers under multiplication. Prove that $\mathbb{R}^*$ is the internal direct product of $\mathbb{R}^+$ and the subgroup $\{1, -1\}$. (7.5)

(c) The set G = {1,4,11,14,16,19,26,29,31,34,41,44} is a group under multiplication modulo 45. Write G as an external and an internal direct product of cyclic groups of prime-power order.

(7.5)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1140 **I**

Unique Paper Code : 2352012302

Name of the Paper : Riemann Integration

Name of the Course : **B.Sc. (H) Mathematics UGCF-2022**

Semester : III DSCC-8

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **all** questions by selecting **three** parts from each question.

3. **All** questions carry equal marks.

4. Use of Calculator is not allowed.

P.T.O.

1. (a) Find the upper and lower Darboux integrals foı $f(x) = x^2$ on the interval $[0, b]$ and show that fı

$$\int_0^b x^2 = \frac{b^3}{3}.$$

(b) Let f be a bounded function on [a, b]. If P and Q are partitions of [a, b] and $P \subseteq Q$, then prove that

$$L(f, P) \leq L(f, Q) \leq U(f, Q) \leq U(f, P)$$

(c) Let $f: [a, b] \to R$ be a bounded function on [a, b]. Prove that if f is integrable on [a, b], then for each $\in > 0$, there exists a partition P of [a, b] such that

$$U(f, P) - L(f, P) < \in$$

(d) Let $f(x) = 2x + 1$ over the interval [0,2]. Let $P = \left\{0, \frac{1}{2}, 1, \frac{3}{2}, 2\right\}$ be a partition of [0,2]. Compute $U(f, P)$, $L(f, P)$ and $U(f, P) - L(f, P)$.

2. (a) Let f be an integrable function on [a, b]. Show that –f is integrable on [a, b] and

$$\int_a^b (-f) = -\int_a^b f$$

(b) Let f: [0,2] → R be defined as

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ -1, & \text{if } x \text{ is irrational} \end{cases}$$

Calculate the upper and lower Darboux Integrals for f on the interval [0,2]. Is f integrable on [0,2]?

(c) Let f: [a, b] → R be a bounded function. Show that if f is integrable (Darboux) on [a, b], then it is Riemann integrable on [a, b].

(d) For a bounded function f on [a, b], define the Riemann Sum associated with a partition P. Hence, give Riemann's definition of integrability.

3. (a) Prove that every bounded piecewise monotonic function f on [a, b] is integrable.

(b) Show that if a function f is integrable on [a, b], then |f| is integrable on [a, b] and $\left|\int_a^b f\right| \le \int_a^b |f|$.

(c) If f is a continuous, non-negative function on [a, b] and if $\int_a^b f = 0$, then prove that f is identically 0 on [a, b]. Give an example of a discontinuous non-zero function f on [0, 1] for which $\int_0^1 f = 0$.

(d) State and prove Fundamental Theorem of Calculus I.

4. (a) If u and v are continuous functions on [a, b] that are differentiable on (a, b), and u′ and v′ are integrable, prove that $\int_a^b uv' + \int_a^b u'v = u(b)v(b) - u(a)v(a)$. Hence evaluate $\int_0^{\pi} x \cos x$.

(b) Use the Fundamental Theorem of Calculus to calculate $\lim_{x \to 0} \frac{1}{x} \int_0^x e^{t^2} dt$.

(c) Let f be an integrable function on [a, b]. For x in [a, b], let $F(x) = \int_a^x f(t)dt$. Then show that F is uniformly continuous on [a, b]. For f(x) =

$$\begin{cases} 0, & t < 0 \\ t, & 0 \le t \le 1, \\ 4, & t > 1 \end{cases}$$

(i) Determine the function $F(x) = \int_0^x f(t)dt$.

(ii) Where is F continuous?

(d) For $t \in [0,1]$, define $F(t) = \begin{cases} 0, & t < \frac{1}{2} \\ 1, & t \geq \frac{1}{2} \end{cases}$ and let $f(x) = x^2$, $x \in [0,1]$. Show that f is F-integrable and that $\int_0^1 f\,dF = f\left(\frac{1}{2}\right)$.

5. (a) Find the volume of the solid generated when the region enclosed by the curves $x = \sqrt{y}$ and $x = y/4$ is revolved about the x – axis.

(b) Use cylindrical shells to find the volume of the solid generated when the region under $y = x^2$ is revolved about the line $y = -1$.

(c) Find the exact arc length of the curve $x = \frac{1}{3}(y^2 + 2)^{3/2}$ from $y = 0$ to $y = 1$.

(d) Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about the y – axis.

6. (a) Discuss the convergence or divergence of the following improper integrals :

(i) $\int_0^1 \frac{1}{\sqrt{x}} dx$

(ii) $\int_{-\infty}^{+\infty} e^x \, dx$

(b) Find the value of r for which the integral $\int_0^1 x^{-r} \, dx$ exists or converges, and determine the value of the integral.

(c) Show that the improper integral $\int_1^{\infty} \frac{\sin x}{x^2} dx$ converges absolutely.

(d) Define the Gamma function $\Gamma(m)$. Prove that $\Gamma(m)$ converges if $m > 0$.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1160 **I**

Unique Paper Code : 2352012303

Name of the Paper : Discrete Mathematics (DSC-9)

Name of the Course : **B.Sc. (H) Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.

2. All questions are compulsory.

3. Questions 1 to 6 have three parts each. Attempt any **TWO** parts from each Question. Each part carries 7.5 marks.

4. Use of the Calculator is not allowed.

P.T.O.

1. (a) Suppose $\mathbb{N}_0 = \mathbb{N} \cup \{0\} = \{0,1,2, \ldots\}$. Define $m \leq n$ in $\mathbb{N}_0$, if and only if there exists $k \in \mathbb{N}_0$, such that $n = km$. Prove that $(\mathbb{N}_0, \leq)$ is a partially ordered set. Is $(\mathbb{N}_0, \leq)$ a chain, an antichain or none? Justify your answer. (7.5)

(b) Define when we say that the two sets have the same cardinality. Prove that the sets $(0,1)$ and (a, ∞) have the same cardinality. (7.5)

(c) Draw the Hasse diagrams for the following ordered sets:

(i) $(\wp(X), \subseteq)$, with $X = \{1,2,3\}$. Here $\wp(X)$ is the power set of X.

(ii) Dual of $\oplus\, M_3$, where $M_n = 1 \oplus \bar{n} \oplus 1$.

(iii) **2 x 3**

Here n denotes the chain obtained by giving the set $P = \{0,1,\ldots, n\text{-}1\}$, the order in which $0 < 1 < \ldots < n\text{-}1$ and $\bar{\mathbf{n}}$ for P regarded as an antichain. (3, 2.5, 2)

2. (a) Define maximal element of an ordered set. Give an example of an ordered set which has exactly one maximal element but does not have a greatest (or maximum) element. Give one example of an ordered set with exactly 3 maximal elements. (7.5)

(b) (i) State duality principle in ordered sets.

(ii) Define an order preserving map between two ordered sets and prove that the composite map of two order preserving maps is order preserving. (3, 4.5)

(c) Define bottom and top element in an ordered set. Give one example of an ordered set in which bottom and top both exist and one example in which none of them exist. (7.5)

3. (a) (i) Prove that in a lattice L, for any a, b, c, $d \in L$, $a \leq c$, $b \leq d$ implies that $a \vee b \leq c \vee d$.

(ii) Prove that a lattice L is a chain if and only if every nonempty subset of L is a sublattice of L. (3, 4.5)

(b) Prove that in any lattice L, the following holds

$$((x \wedge y) \vee (x \wedge z)) \wedge ((x \wedge y) \vee (y \wedge z)) = x \wedge y, \text{ for all } x, y, z \in L. \quad (7.5)$$

(c) Let L and K be lattices and $f: L \to K$ be a map. Show that the following are equivalent:

(i) f is order preserving.

(ii) $(\forall\, a, b \in L), f(a \wedge b) \leq f(a) \wedge f(b)$.

(7.5)

4. (a) Let L and M be two ordered sets and f be an order isomorphism from L onto M. Prove that if L is a lattice, then M is also a lattice, and f is a lattice isomorphism. (7.5)

(b) Prove that the direct product L x K of two distributive lattices L and K is also a distributive lattice. (7.5)

(c) (i) Prove that every sublattice of a modular lattice L is modular.

(8)

This question paper contains 4 printed pages]

Roll No. [ ]

S. No. of Question Paper : **1121**

Unique Paper Code : **2352013503**

Name of the Paper : **Partial Differential Equations**

Name of the Course : **Bachelor of Science (Honours Course) Mathematics**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the six questions are compulsory.

Attempt any *two* parts from each question.

Use of calculator is not allowed.

1. (*a*) Define the linear and non-linear partial differential equations with examples. Find the first order partial differential equation satisfied by the family of right circular cones whose axes coincide with the z-axis, and is given by :

$$x^2 + y^2 = (z - c)^2 \tan^2\alpha.$$

(*b*) Solve the following initial value problem using method of characteristics :

$$u_t + 2uu_x = v - x, \quad v_t - cv_x = 0 \quad \text{with} \quad u(x, 0) = x, \quad v(x, 0) = x.$$

(*c*) Solve the equation $u_x + xu_y = y$ with the Cauchy data $u(1, y) = 2y$.

P.T.O.

2. (a) Reduce the following equation into canonical form and then find the general solution :

$$u_x + 2xyu_y = x.$$

(b) Apply the method of separation of variables $u(x, y) = f(x)\,g(y)$ to solve the following equation :

$$u_x + 2u_y = 0, \quad u(0, y) = 3e^{-2y}.$$

(c) Find a complete integral of the equation by using Charpit's method :

$$p = (z + qy)^2.$$

3. (a) Show that the equation of motion of the vibrating string is :

$$u_{tt} = c^2 u_{xx}, \quad c^2 = T/\rho,$$

where T is the tension at the end point of the string and ρ is the density.

(b) Classify and determine the region in which the following equation is hyperbolic, parabolic, or elliptic, and transform the equation in the respective region to canonical form :

$$x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} = 0.$$

(c) Find the traffic density $\rho(x, t)$, satisfying :

$$\frac{\partial \rho}{\partial t} + x\sin(t)\frac{\partial \rho}{\partial x} = 0,$$

with the initial condition $\rho_0(x) = 1 + \frac{1}{1+x^2}$.

4. (a) Transform the following equation to the form $u_{\xi\eta} = cv$, c = constant;

$$3u_{xx} + 7u_{xy} + 2u_{yy} + u_y + u = 0,$$

by introducing the new variables $v = ue^{a\xi + b\eta}$, where a and b are undetermined coefficients.

(*b*) Define the homogeneous and non-homogeneous of partial differential equations with examples, and find the general solution of the equation :

$$u_{xxxx} - u_{yyyy} = 0.$$

(*c*) Find the general solution of the equation :

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y + e^{2x+y}.$$

5. (*a*) Determine the solution of the initial-value problem for the semi-infinite string with a fixed end :

$$\begin{aligned} &u_{tt} = c^2 u_{xx}, && 0 < x < \infty, t > 0 \\ &u(x, 0) = f(x), u_t(x, 0) = g(x), && 0 \le x < \infty \\ &u(0, t) = 0, && 0 \le t < \infty \end{aligned}$$

(*b*) Determine the solution of the initial-value problem for the semi-infinite string with free end :

$$\begin{aligned} &u_{tt} = 9u_{xx}, && 0 < x < \infty, t > 0 \\ &u(x, 0) = 0, u_t(x, 0) = x^3, && 0 \le x < \infty \\ &u_x(0, t) = 0, && 0 \le t < \infty \end{aligned}$$

(*c*) Determine the solution of the initial-value problem with non-homogeneous boundary conditions :

$$\begin{aligned} &u_{tt} = c^2 u_{xx}, && 0 < x < \infty, t > 0 \\ &u(x, 0) = \sin x, u_t(x, 0) = x^2, && 0 \le x < \infty \\ &u(0, t) = x, && 0 \le t < \infty \end{aligned}$$

P.T.O.

6. (*a*) Determine the solution of the Cauchy problem for non-homogeneous wave equation :

$$u_{tt} - c^2 u_{xx} - \sin x = 0, \quad u(x, 0) = \cos x, \quad u_t(x, 0) = 1 + x.$$

(*b*) Determine the solution of the initial-value problem :

$$\begin{aligned} &u_{tt} = 16u_{xx}, && 0 < x < \infty, t > 0 \\ &u(x, 0) = 1 + x, u_t(x, 0) = x^3, && 0 \le x < \infty \\ &u_x(0, t) = \cos x, && 0 \le t < \infty \end{aligned}$$

(*c*) Determine the solution of the initial-value problem :

$$\begin{aligned} &u_{tt} - c^2 u_{xx} = 0 && 0 < x < \infty, t > 0 \\ &u(x, 0) = \log(1 + x^2), u_t(x, 0) = 2, && 0 \le x < \infty \end{aligned}$$

(9)

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1064 **I**

Unique Paper Code : 2352013501

Name of the Paper : Metric Spaces

Name of the Course : **B.Sc. (Hons.) Mathematics (DSC-13)**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.

2. Attempt all question by selecting two parts from each question.

3. Part of the questions to be attempted together.

4. All questions carry equal marks.

5. Use of Calculator not allowed.

P.T.O.

1\. (a) Let (X, d) be a metric space. Define the function $d': X \times X \to \mathbf{R}$ by $d' = \frac{|x-y|}{1+|x-y|}$ Show that d' is a metric on X. Besides, $d'(x, y) < 1$ for all $x, y \in$ X. (7.5)

(b) Let $X = C[a,b]$ be the space of all continuous functions on $[a,b]$. Define $d(x,y) = \int_a^b |f(x) - g(x)|\, dx$, then check whether this metric imply pointwise Convergence or not. (7.5)

(c) Define Cauchy Sequence and Complete metric space. Let X be any non-empty set and d be defined by

$$d(x,y) = \begin{cases} 0, & x = y \\ 1, & x \neq y \end{cases}$$

then show that (X, d) is a Complete metric space. (7.5)

2. (a) Let (X, d) be a metric space. Then show that

(i) $\emptyset$ and X are open sets in (X, d);

(ii) the union of an arbitrary family of open sets is open;

(iii) the intersection of any finite family of open sets is open. (7.5)

(b) Let A be a subset of a metric space (X, d). Then prove that

(i) A° is the largest open subset of A. (3.5)

(ii) A is open if and only if $A = A^\circ$. (4)

(c) (i) Let (X, d) be a metric space and $F \subseteq X$. Then show that a point x_0 is a limit point of F if and only if it is possible to select from the set F a sequence of distinct

$x_1,\ x_2, \ldots, x_n, \ldots$ such that $\lim_{n\to\infty} d(x_n, x_o) = 0.$

(4.5)

(ii) Let $A \subseteq [0,\ 1]$ and $F = \{f \in C[0,\ 1]: f(t) = 0,\ \forall\, t \in A\}$. Show that F is a closed subset of C[0, 1] equipped with the uniform metric. (3)

3. (a) Let $(X,\ d)$ be a metric space and $F \subseteq X$. Then show that the following statements are equivalent:

(i) $x \in \bar{F}$;

(ii) $S(x,\varepsilon) \cap F \neq \emptyset$ for every open ball $S(x,\varepsilon)$ centred at x;

(iii) There exists an infinite sequence $\{x_n\}$ of points (not necessarily distinct) of F such that

$$\lim_{n\to\infty} x_n = x.$$ (7.5)

(b) State and prove Cantor's intersection theorem. (7.5)

(c) (i) If Y is a nonempty subset of a metric space (X, d), and (Y, d_y) is complete, then show that Y is a closed in X. (3.5)

(ii) Let (X, d) be a complete metric space and Y a closed subset of X. Then show that (Y, d_y) is a complete space. (4)

4. (a) Prove that a mapping $f: X \to Y$ is continuous on X if and only if $f^{-1}(G)$ is open in X for all open subsets G of Y. (7.5)

(b) Let $T: X \to X$ be a contraction mapping of the complete metric space (X, d). Then show that T has a unique fixed point. (7.5)

(c) Show that the metric spaces (X, d) and (X, ρ) where $\rho(x,y)= \dfrac{d(x,y)}{1+d(x,y)}$ are equivalent. (7.5)

5. (a) Let (X, d) be a metric space. Then show that the following statements are equivalent:

(i) (X, d) is disconnected;

(ii) there exists a continuous mapping of (X, d) onto the discrete two element space (X_o, d_o). (7.5)

(b) Let $I= [1,1]$and let $f: I \to I$ be continuous. Then show that there exists a point $c \in I$ such that $f(c) = c$. Discuss the result if $I = [-1,1]$. Discuss the result if $I = [-1,1)$ and $I = [-1, \infty)$. (4+3.5)

(c) (i) If C is a connected subset of a disconnected metric space $X = A \cup B$, where A, B are nonempty and $\bar{A} \cap B = \emptyset = A \cap \bar{B}$, then show that either $C \subseteq A$ or $C \subseteq B$.

(ii) If Y is a connected set in a metric space (X, d) then show that any set Z such that is $Y \subseteq Z \subseteq \bar{Y}$ connected. (4+3.5)

6. (a) Let (X, d) be a metric space. Then show that the following statements are equivalent:

(i) every infinite set in (X, d) has at least one limit point in X;

(ii) every infinite sequence in (X, d) contains a convergent subsequence. (7.5)

(b) If f is a one-to-one continuous mapping of a compact metric space (X, d_x) onto a metric space (Y, d_y), then show that f^{-1} is continuous on Y and, hence, f is a homeomorphism of (X, d_x) onto (Y, d_y). (7.5)

(c) Let A be a compact subset of a metric space (X, d). Show that for any $B \subseteq X$, there is a point $p \in A$ such that $d(p, B) = d(A, B)$.. (7.5)

(2000)

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[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1102 **I**

Unique Paper Code : 2352013502

Name of the Paper : Ring Theory

Name of the Course : **B.Sc. (H) Mathematics**

Semester : V – DSC-14

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **All** questions are compulsory and are of **15** marks each.

3. Attempt any **six** parts from Question **1**. Each part is of **2.5** marks.

4. Attempt any **two** parts from each of the Questions **2** to **6**. Each part is of **7.5** marks.

5. Use of calculator is not allowed.

P.T.O.

1. (i) Find all the units of $\mathbb{Z}_7[x]$.

(ii) Check whether $\mathbb{Q} \oplus \mathbb{Q}$ is an integral domain or not.

(iii) Give an example of a subring S of a ring R which is not an ideal of R.

(iv) Prove that a ring homomorphism carries an idempotent to an idempotent.

(v) Let ϕ be a ring homomorphism from a ring R to a ring S. If R has unity 1, $S \neq \{0\}$ and ϕ is onto then prove that $\phi(1)$ is the unity of S.

(vi) Let $f(x) = 2x^5 + 14x^2 - 21x + 7$. Is f(x) an irreducible polynomial over $\mathbb{Q}$? Justify your answer.

(vii) Let D be an intergral domain. Suppose that $p, q \in D$ and $q \neq 0$. Show that if p is not a unit, then $\langle pq \rangle$ is a proper subset of $\langle q \rangle$.

(viii) Explain why $3x^2 + 6$ is reducible over $\mathbb{Z}$.

2. (a) Prove that intersection of two subrings in a ring R is a subring of R. Is the union of two subrings necessarily a subring of R? Justify your answer.

(b) Find all the units, zero divisors and idempotent elements in $\mathbb{Z}_3 \oplus \mathbb{Z}_6$.

(c) Prove that $\mathbb{Z}_n$, the ring of integers modulo n, is a field if and only if n is a prime.

3. (a) Let R be a commutative ring with unity and let U(R) denote the set of units of R. Prove that U(R) is a group under multiplication. Also, find $U(\mathbb{Z}[i])$.

(b) Define the characteristic of a ring. Prove that the characteristic of an integral domain is either 0 or prime.

(c) Prove that in a commutative ring R with unity, an ideal A is a maximal ideal if and only if $\frac{R}{A}$ is a field.

4. (a) Prove that the ideal $\langle x \rangle$ is a prime ideal in $\mathbb{Z}[x]$ but not a maximal ideal in $\mathbb{Z}[x]$.

(b) Let ϕ be a ring homomorphism from a ring R onto a ring S. Prove that $\frac{R}{\text{Ker}\,\phi} \approx S$.

(c) Determine all ring homomorphisms from $\mathbb{Z}_4 \rightarrow \mathbb{Z}_{10}$.

5. (a) Let F be a field and let $I = \{a_0 + a_1x + \ldots + a_nx^n : a_0, a_1, \ldots, a_n \in F \text{ and } a_0 + a_1 + \ldots + a_n = 0\}$. Show that I is an ideal of F[x] and find a generator for I.

(b) Let $f(x) = 5x^4 + 3x^3 + 1$ and $g(x) = 3x^2 + 2x + 1 \in \mathbb{Z}_7|x|$. Determine the quotient and remainder obtained when f(x) is divided by g(x).

(c) Prove that the product of two primitive polynomials is a primitive polynomial.

6. (a) Show that $p(x) = x^3 + x + 1$ is an irreducible polynomial over $\mathbb{Z}_2$.

Let $M = \langle x^3 + x + 1 \rangle$ be an ideal of $\mathbb{Z}_2[x]$.

Show that $F = \frac{\mathbb{Z}_2[x]}{M}$ is a field of order 8. Exhibit all the 8 elements of F. Find the product of $x^2 + x + 1 + M$ and $x^2 + 1 + M$ and express it as a member of F.

(b) In a principal ideal domain, prove that the element is irreducible if and only if it is prime.

(c) Show that integral domain $\mathbb{Z}[t]$ is Euclidean Domain. Is $\mathbb{Z}[i]$ a Unique Factorization Domain? Justify.

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1227 **I**

Unique Paper Code : 2353010007

Name of the Paper : DSE-3(i) : Mathematical Data Science

Name of the Course : **B.Sc. (H) Mathematics**

Semester : V, DSE

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt all questions by selecting **three** parts from **Q1-Q4.** and two parts from **Q5.-Q6.**
3. Parts of the questions to be attempted together.
4. **All** questions carry equal marks. **All** parts of a question carry equal marks. Marks are indicated.
5. Use of non-programmable Scientific Calculator is allowed.

P.T.O.

(c) Define L_p distances mathematically and sketch these geometrically for $p = 1,2$ and ∞. (5)

(d) Consider two vectors $a = (1,2,-4,3,-6)$ and $b = (1,2,5,-2,1)$ in $\mathbb{R}^5$. Find Kullback-Liebler divergence. (5)

4. (a) Find the regression line for the given data

height (in):	66	68	60	70	65	61	74	73	75	67
weight (lbs):	160	170	110	178	155	120	223	215	235	164

(5)

(b) Explain polynomial regression, Consider the input data set with $n = 3$ points $\{(2,1), (3,6), (4,5)\}$. Find polynomial expansion of x generates with $p = 5$. (5)

(c) Define the Gradient in data sciences and find the value of the Gradient for

$f(x,y,z) = 3x^2 - 2y^3 - 2xe^z$ at $(3,-2,1)$. (5)

(d) Consider two-variable function $f = (x-5)^2 + (y+2)^2 - 2xy$. Starting with $(x, y) = (0,2)$, using the gradient descent algorithm for the function, perform 3 iterations and report the function value at the end of each step. (5)

5. (a) Find Singular Value Decomposition (SVD) of the matrix $A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}$. (7.5)

P.T.O.

(b) Use Lloyd's Algorithm for k-means clustering to divide the following data into two clusters with initial cluster centers $C_1 = (2,1)$ and $C_2 = (2,3)$

x	1	2	2	3	4	5
y	1	1	3	2	3	5

(7.5)

(c) Reduce the feature 2-dimensional data of the following into one dimensional using Principal Component Analysis (PCA)

Class → / Feature ↓	C_1	C_2	C_3	C_4
X_1	4	8	13	7
X_2	11	4	5	14

(7.5)

6. (a) Consider a matrix $A \in \mathbb{R}^{8\times4}$ with squared singular values $\sigma_1^2 = 10$, $\sigma_2^2 = 5$, $\sigma_3^2 = 2$ and $\sigma_4^2 = 1$.

(i) Find the rank of matrix A.

(ii) Find the value of $\|A - A_2\|_F^2$, where A_2 is the best rank-2 approximation of A. (7.5)

(b) Draw Voronoi diagram with three points A(–6,7), B(–6, –3) and C(2,5). (7.5)

(c) Consider a set of 1 - dimensional data points

$(x_1 = 0,\ y_1 = +1)$, $(x_2 = 1,\ y_1 = -1)$, $(x_3 = 2,\ y_1 = +1)$, $(x_4 = 4,\ y_1 = +1)$, $(x_5 = 6,\ y_1 = -1)$, $(x_6 = 7,\ y_1 = -1)$, $(x_7 = 8,\ y_1 = +1)$, $(x_8 = 9,\ y_1 = -1)$.

Predict –1 or +1 using a kNN (k-nearest neighbor) classifier with $k = 3$ on $x = 3$ and $x = -1$.

(7.5)

(4000)

(12)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1228 **I**

Unique Paper Code : 2353010008

Name of the Paper : Linear Programming and Applications (DSE)

Name of the Course : **B.Sc. (H) Mathematics**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.

1. (a) Solve the following linear programming problem by Graphical Method:

$$\begin{aligned} \text{Minimize} \quad & z = x_1 + x_2 \\ \text{subject to} \quad & x_1 + x_2 \geq 2 \\ & 5x_1 + 9x_2 \leq 45 \\ & x_2 \leq 4 \\ & x_1 \geq 0,\ x_2 \geq 0. \end{aligned}$$

P.T.O.

(b) Examine the convexity of the set $S = \{(x_1, x_2): x_2^2 \geq 4x_1\}$. Justify your conclusion. Further prove or disprove that intersection of two convex sets is a convex set.

(c) Solve the following problem by simplex method:

$$\begin{aligned} \text{Minimize} \quad & x_1 + 2x_2 - 4x_3 \\ \text{subject to} \quad & x_1 + x_2 + 2x_3 \leq 9 \\ & x_1 + x_2 - x_3 \leq 2 \\ & -x_1 + x_2 + x_3 \leq 4 \\ & x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0. \end{aligned}$$

2. (a) Consider the following problem:

$$\begin{aligned} \text{Minimize} \quad & z = cx \\ \text{subject to} \quad & Ax \geq b, \\ & x \geq 0, \end{aligned}$$

Let $(x_B, 0)$ be a basic feasible solution corresponding to basis B such that $z_j - c_j \leq 0$ for all non- basic variables x_j. Prove that $(x_B, 0)$ is an optimal basic feasible solution.

(b) Find all the basic feasible solutions for the following system of equations:

$$x_1 + 2x_2 + x_3 = 4$$
$$2x_1 + x_2 + 5x_3 = 5.$$

Classify the obtained solutions as degenerate or non-degenerate.

(c) Reduce the feasible solution, $x_1 = 2, x_2 = 3, x_3 = 1$ of the following system to two different basic feasible solutions:

$$2x_1 + x_2 + 4x_3 = 11$$
$$3x_1 + x_2 + 5x_3 = 14.$$

3. (a) Solve the following problem by Big-M method

Maximize $z = 6x_1 + 4x_2$

subject to
$$2x_1 + 3x_2 \leq 30$$
$$3x_1 + 2x_2 \leq 24$$
$$x_1 + x_2 \geq 3$$
$$x_1, x_2 \geq 0$$

Is the solution unique? If not, give two different solutions.

(b) Solve the following problem by Two-phase method

$$\text{Minimize } z = -5x_1 + 4x_2 - 3x_3$$
$$\text{subject to } \quad 2x_1 + x_2 - 6x_3 = 20$$
$$6x_1 + 5x_2 + 10x_3 \le 76$$
$$8x_1 - 3x_2 + 6x_3 \le 50$$
$$x_1, x_2, x_3 \ge 0$$

(c) Solve the following system of simultaneous equations using simplex method:

$$x_1 + x_2 = 1$$
$$2\, x_1 + x_2 = 3$$

4 (a) (i) Prove that if either the primal or the dual problem has an unbounded objective function

value, then the other problem has no feasible solution.

(ii) Show that the dual of the dual is primal for the following primal problem

$$\text{Minimize } z = x_1 + 2x_2 - 3x_3$$
$$\text{subject to } \quad 4x_1 + 5x_2 - 6x_3 = 7$$
$$8x_1 - 9x_2 + 10x_3 \leq 11$$
$$x_1, x_2 \geq 0 \;, x_3 \text{ unrestricted.}$$

(b) Obtain the dual of the following primal problem:

$$\text{Minimize } z = 3x_1 - 2x_2 + 4x_3$$
$$\text{subject to } \quad 3x_1 + 5x_2 + 4x_3 \geq 7$$
$$6x_1 + x_2 + 3x_3 \geq 4$$
$$7x_1 - 2x_2 - x_3 \leq 10$$
$$x_1 - 2x_2 + 5x_3 \geq 3$$
$$x_1, x_2, x_3 \geq 0.$$

(c) Solve the following problem using Complementary Slackness theorem

$$\text{Minimize } z = 2x_1 + 9x_2 + x_3$$
$$\text{subject to} \quad x_1 + 4x_2 + 2x_3 \geq 5$$
$$3x_1 + x_2 + 2x_3 \geq 4$$
$$x_1, x_2, x_3 \geq 0.$$

5. (a) For the following cost minimization Transportation Problem, find the initial basic feasible solution by using North West Corner Rule, Least Cost Method and Vogel's Approximation Method. Compare the three solutions in terms of cost:

Machine	I	II	III	IV	V	Supply
A	2	11	10	3	7	4
B	1	4	7	2	1	8
C	3	9	4	8	12	9
Demand	3	3	4	5	6	

(b) Solve the following cost minimization Transportation Problem:

	I	II	III	IV	Availability
A	19	30	50	10	7
B	70	30	40	60	9
C	40	8	70	20	18
Requirements	5	8	7	14	

(c) Find the optimal solution of the Assignment Problem with the following cost matrix :

	U	V	X	Y	Z
I	2	9	2	7	1
II	6	8	7	6	1
III	4	6	5	3	1
IV	4	2	7	3	1
V	5	3	9	5	1

6. (a) (i) Define Saddle Point and Mixed Strategy for a "Two-Person Zero Sum" game.

(ii) Use maxmin and minmax Principle to find the saddle point, if it exists, for the following pay-off matrix

$$\begin{bmatrix} 3 & -1 & 5 \\ 6 & 4 & 0 \\ 10 & 8 & 6 \end{bmatrix}$$

(b) Convert the following game problem into a pair of linear programming problems for players A and B and provide the optimal strategies of the two players and value of the game.

Player B

Player A $\begin{bmatrix} 2 & 3 & 0 & -2 \\ 1 & 2 & 3 & 2 \end{bmatrix}$

(c) Find the inverse of the matrix using Simplex Method:

$$\begin{bmatrix} 3 & -2 \\ 1 & 2 \end{bmatrix}$$

(13)

This question paper contains 6 printed pages]

Roll No. | | | | | | | | | | |

S. No. of Question Paper : **1229**

Unique Paper Code : **2353010009**

Name of the Paper : **Mathematical Statstics**

Type of the Paper : **DSE**

Name of the Course : **B.Sc. (H) Mathematics**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

All questions carry equal marks.

Use of non-programmable scientific calculators and statistical tables is permitted.

Part A

1. (*a*) A large but sparsely populated country has two small hospitals, one at the south end of the country and the other at the north end. The south hospital's emergency room has 4 beds, whereas the north hospital's emergency room has only 3 beds. Let X denote the number of south beds occupied at a particular time on a given day, and let Y denote the number of north beds occupied at the same time on the same day. Suppose that these two rvs are independent, that the pmf of X puts probability masses. .1, .2, .3, .2 and .2 on the x values 0, 1, 2, 3, and 4, respectively, and that pmf of Y distributes probabilities .1, .3, .4, and .2 on the y values 0, 1, 2, and 3, respectively.

P.T.O.

(i) Display the joint pmf of X and Y in a joint probability table.

(ii) Compute P(X $\leq$ 1 and Y $\leq$ 1) and verify that this equals the product of P(X $\leq$ 1) and P(Y $\leq$ 1).

(iii) Express the event that the total number of beds occupied at the two hospitals combined is at most 1 in terms of X and Y, and then calculate this probability.

(iv) What is the probability that at least one of the two hospitals has no beds occupied ?

(b) An instructor has given a short quiz consisting of two parts. For a randomly selected student, let X = the number of points earned on the first part and Y = the number of points earned on the second part. Suppose that the joint pmf of X and Y is given in the accompanying table.

		y			
	$p(x, y)$	0	5	10	15
	0	.02	.06	.02	.10
x	5	.04	.15	.20	.10
	10	.01	.15	.14	.01

(i) Compute the covariance for X and Y.

(ii) Compute ρ for X and Y.

(c) A concert has three pieces of music to be played before intermission. The time taken to play each piece has a normal distribution. Assume that the three times are independent of each other. The mean times are 15, 30, and 20 min, respectively, and the standard deviations are 1, 2, and 1.5 min, respectively. What is the probability that this part of the concert takes at most one hour ? Are there reasons to question the independence assumption ? Explain.

2. (*a*) A system consisting of two components will continue to operate only as long as both components function. Suppose the joint pdf of the lifetimes (months) of the two components in a system is given by $f(x, y) = c\,[10 - (x + y)]$, for $x > 0$, $y > 0$, $x + y < 10$. Find the value of the constant c. If the first component functions for exactly 3 months, what is the probability that the second functions for more than 2 months.

(*b*) Define the bivariate normal distribution. For a few years, the SAT consisted of three components : writing, critical reading, and mathematics. Let W = SAT Writing score and X = SAT Critical Reading score for a randomly selected student. According to the College Board, in 2012 W had mean 488 and standard deviation 114, while X had mean 496 and standard deviation 114. Suppose X and W have a bivariate normal distribution with Corr(X, W) = .5. If an English department plans to use X + W, a student's total score on the nonmath sections of the SAT to help determine admission. Then determine the distribution of X + W.

(*c*) (*i*) Suppose that the lifetimes of two components are independent of each other and that the first lifetime, X_1, has an exponential distribtuion with parameter $\lambda_1 = 1/1000$ whereas the second, X_2, has an exponential distribution with parameter $\lambda_2 = 1/1200$. Compute the probability that the sum of their lifetimes is at most 3000 h.

(*ii*) A surveyor wishes to lay out a square region with each side having length L. However, because of measurement error, he instead lays out a rectangle in which the north-south sides both have length X and the east-west sides both have length Y. Suppose that X and Y are independent and that each one is uniformly distributed on the interval [L – A, L + A] (where 0 < A < L). What is the expected area of the resulting rectangle ?

3. (a) A particular brand of dishwasher soap is sold in three sizes : 25, 40, and 65 oz. 20% of all purchasers select a 25-oz box, 50% select a 40-oz box, and the remaining 30% choose a 65-oz box. Let X_1 and X_2 denote the package sizes selected by two independently selected purchasers. Determine the sampling distribution of $\overline{X}$, calculate $E(\overline{X})$, and compare to m

(b) State the Central Limit Theorem.

When a batch of a certain chemical product is prepared, the amount of a particular impurity in the batch is a random variable with mean value 4.0 g and standard deviation 1.5 g. If 50 batches are indepedently prepared, what is the (approximate) probability that the sample average amount of impurity X is between 3.5 and 3.8 g ? Justify why is the Central Limit Theorem applicable here ?

(c) (i) Define chi-squared distribution with v degrees of freedom.

(ii) Show that if $X_1, X_2, \ldots\ldots\ldots, X_n$ is a random sample from a $N(\mu, \sigma)$ distribution, then $(n - 1)S^2/\sigma^2 \sim \chi^2_{n-1}$. Further deduce that $E(S^2) = \sigma^2$ and $V(S^2) = 2\sigma^4/(n-1)$.

4. (a) (i) Show that the sample variance S^2 is an unbiased estimator of σ^2 for any population distribution.

(ii) Define minimum variance unbiased estimator (MVUE).

(b) (i) The traditional criteria for "strong" passwords and the emerging advice is to use longer passphrases by concatenating several everyday words. Suppose that 10 students at a certain university are randomly selected, and it is found that the three of them use passphrases for their email accounts. Let p = P(passphrase); i.e., p is the proportion of all students at the university using a passphrase on their email accounts. Find the maximum likelihood estimate of the parameter p.

(ii) Suppose X has the pdf $f(x; \theta) = \theta x^{\theta-1}$ for $0 \leq x \leq 1$. Obtain the Fisher information $I(\theta)$.

(*c*) State the Neyman factorization theorem. Use it to show that the sufficient statistic for μ is $T = \sum X_i$, by considering a random sample $X_1, X_2, \ldots, X_n$ from a Poissson distribution with parameter μ, describing the numbers of errors in *n* batches of tax returns where each batch consists of many returns.

5. (*a*) Assume that the helium porosity (in percentage) of coal samples taken from any of coal samples taken from any particular seam is normally distributed with true standard deviation 75.

(*i*) Compute a 95% confidence interval (CI) for the true average porosity of certain seam if the average porosity for 20 specimens from the seam was 4.85.

$[Z_{0.05} = 1.96]$

(*ii*) Compute a 98% confidence interval (CI) for true average porosity of another seam based on 16 specimens with a sample average porosity of 4.56.

$[Z_{0.01} = 2.33]$

(*b*) For healthy individuals the level of prothrombin in the blood is approximately normal distributed with mean 20 mg/dL and standard deviation 4 mg/dL. Low levels indicate low clotting ability. In studying the effect of gallstones on prothrombin, the level of each patient in a sample is measured to see if there is a deficiency. Let μ be the true average level of prothrombin for gallstone partients (and assume $\sigma = 4$).

(*i*) What are the appropriate null and alternative hypotheses ?

(*ii*) Let $\overline{X}$ denote the sample average level of prothrombin in a sample of $n = 20$ randomly selected gallstone patients. Consider the test procedure with test statistic $\overline{X}$ and rejection region $\bar{x} \leq 17.92$. What is the probability distribution of the test statistic when H_0 is true ? What is the probability of a type I error for the test procedure ?

(*c*) On the label, Pepperidge Farm bagels are said to weigh four ounces each (113 g). A random sample of six bagels resulted in the following weights (in grams) :

117.6, 109.5, 111.6, 109.2, 119.1, 110.8.

Based on this sample, is there any reason to doubt that the population mean is at least 113 *g* at 0.05 level of significance ? [$t_{0.05,5}$ = 2.015]

6. (*a*) A plan for an executive traveler's club has been developed by an airline on the premise that 5% of its current customers would qualify for membership. A random sample of 500 customers yielded 40 who would qualify. Using this data, test at level 0.01 the null hypothesis that the company's premise is correct against the alternative that it is not correct. [$z_{0.01}$ = 2.576]

(*b*) The accompanying data on x = current density (mA/cm^2) and y = rate of deposition (mm/min) appeared in the article "Plating of 60/40 Tin/Lead Solder for Head Termination Metallurgy". Do you agree with the claim by the article's author that a linear relationship was obtained from the tin-lead rate of deposition as a function of current density ? Explain your reasoning.

X	20	40	60	80
Y	0.24	1.20	1.71	2.22

(*c*) A statistics department at a large university maintains a tutoring center for students in its introductory service courses. The center has been staffed with the expectation that 40% of its clients would be from the business statistics course, 30% from engineering statistics, 20% from the statistics course for social science students, and the other 10% from the course for agriculture students. A random sample of n = 120 clients revealed 52, 38, 21, and 9 from the four courses. Does this data suggest that the percentages on which staffing was based are not correct ? State and test the relevant hypotheses using α = 0.05. [$\chi^2_{0.05,3}$ = 7.815]

(14)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5970 **I**

Unique Paper Code : 32357502

Name of the Paper : DSE – I Mathematical Modeling and Graph Theory

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any **three** parts of each question.

3. **All** questions are compulsory.

4. Use of non-programmable scientific calculators is permitted.

P.T.O.

1. (a) (i) Is $x = 0$ an ordinary point of the differential equation

$$xy'' + (\sin x)y' + x^2y = 0?$$

Justify your answer.

(ii) Find $\mathcal{L}(\cos^2 2t)$.

(iii) Find $\mathcal{L}^{-1}\left(\frac{5-2s}{s^2+7s+10}\right)$. (6)

(b) Use Laplace transform to solve the initial value problem

$$y'' + 3y' - 2y = e^{-t}, \quad y(0) = 0; \quad y'(0) = 0.$$ (6)

(c) Find two linearly independent Frobenius series solutions of the differential equation

$$2xy'' - y' - y = 0.$$ (6)

(d) Consider a spring-mass dashpot system with mass $m = 1$, the viscous damping $c = 0$ in mks units and the spring modulus $k = 9$ with an imposed external force $f(t) = 6\cos 3t$. Let $y(t)$ denote the displacement of the mass from its equilibrium position. Find $y(t)$. (6)

2. (a) Explain Middle Square Method to generate random numbers. Use it to generate 15 random numbers using 3043 as the seed. Was cycling observed? (6)

(b) Using Monte Carlo Simulation write an algorithm to find the volume of an ellipsoid

$$\frac{x^2}{2} + \frac{y^2}{4} + \frac{z^2}{8} \leq 16,$$

that lies in the first octant $x > 0$, $y > 0$, $z > 0$. (6)

(c) Solve graphically, the following linear programming problem : (6)

Minimize $5x_1 + 7x_2$

subject to $2x_1 + 3x_2 \geq 6$

$3x_1 - x_2 \leq 15$

$-x_1 + x_2 \leq 4$

$2x_1 + 5x_2 \leq 27$

$x_1, x_2 \geq 0.$

(d) Use Simplex method to solve the given linear programming problem : (6)

Maximize $6x_1 + 5x_2$

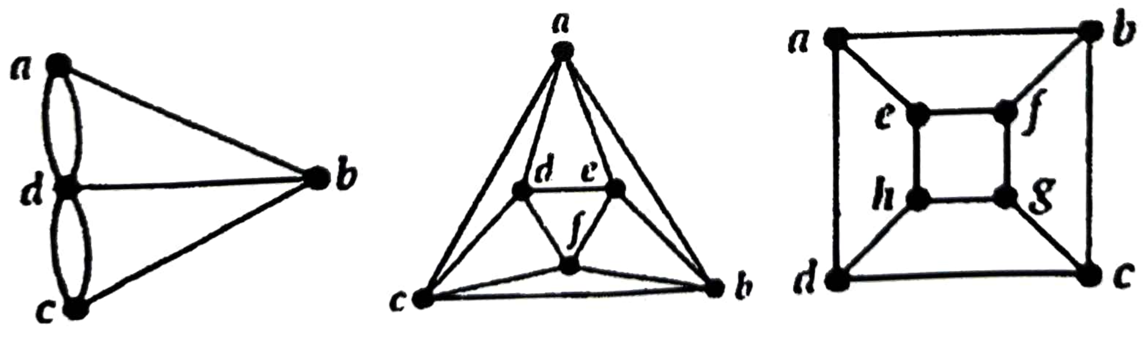
subject to $x_1 + x_2 \geq 6$

$2x_1 + x_2 \geq 9$

$x_1, x_2 \geq 0.$

3. (a) State handshaking lemma and use it to prove that, in any graph the number of vertices with odd degree is even. Verify the result for Petersen graph. (6)

(b) Which of the following graphs are Eulerian and/or Hamiltonian. Write down an Eulerian trail or Hamiltonian cycle where possible for the following graphs. (6)

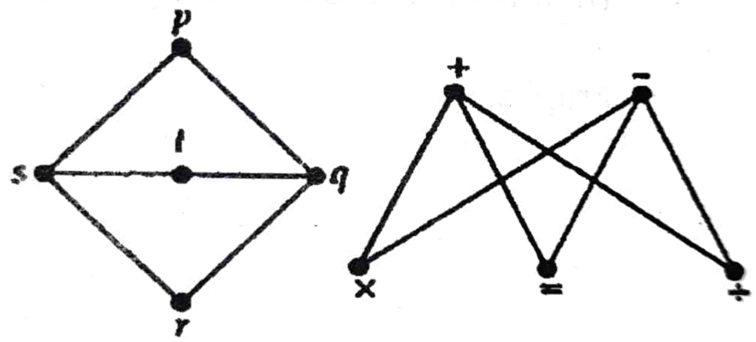


P.T.O.

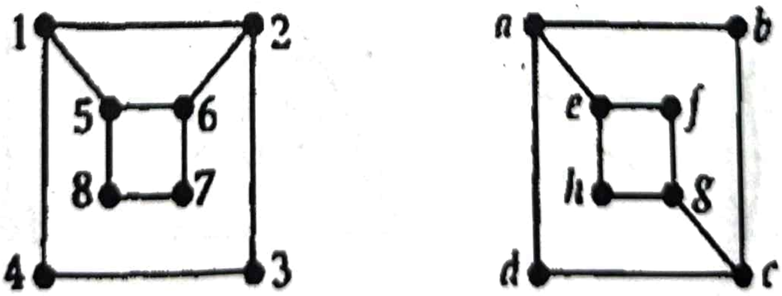
(c) Write a Gray code of 3-digit binary code. How is it connected to the graph of a cube? (6)

(d) Verify and explain whether the following graphs are isomorphic : (6)

(i)



(ii)



4. (a) Prove that a graph is bipartite if and only if every cycle of a graph has even number of edges. (7)

(b) Find the general solution in powers of x for the differential equation

$(x^2 - 3)y'' + 2xy' = 0.$ (7)

(c) Find two linearly independent solutions of the differential equation

$y'' + x^2y' + x^2y = 0.$ (7)

(d) Solve the given linear programming problem graphically.

Maximize $25x_1 + 30x_2$

subject to $20x_1 + 30x_2 \leq 690$

$5x_1 + 4x_2 \leq 120$

$x_1, x_2 \geq 0.$

Determine the sensitivity of the optimal solution to a change in c_2 using the objective function $25x_1 + c_2x_2$. (7)

This question paper contains 3 printed pages]

Roll No.

S. No. of Question Paper : **1565**

Unique Paper Code : **2352571101**

Name of the Paper : **DSC : Topics in Calculus**

Name of the Course : **B.A./B.Sc. (Prog.) with Mathematics as Non-Major/Minor**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all questions by selecting *two* parts from each question.

All questions carry equal marks.

1. (*a*) Show that the function $f(x) = |x| + |x - 1|$ is not differentiable at $x = 0$ and $x = 1$.

   (*b*) Find the *n*th differential coefficients of $\sin^5 x \cos^3 x$.

   (*c*) If $u = \sin^{-1} \dfrac{x^2 + y^2}{x + y}$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \tan u$.

2. (*a*) Examine the continuity of the function $f(x) = \dfrac{\sqrt{1 - \cos 2x}}{x}$, $x \neq 0$; $f(0) = 0$, $x = 0$.

   In case of discontiunity, state the kind of discontinuity.

   (*b*) If $y = \left[x + \sqrt{1 + x^2}\right]^m$, find $y_n(0)$.

   (*c*) If $= e^{xyz}$, show that $\dfrac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2h^2z^2)e^{xyz}$.

P.T.O.

3. (*a*) Evaluate : $\lim_{x \to 0} \left(\frac{\tan x}{x} \right)^{\frac{1}{x}}$.

(*b*) Show that

$$\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots \ldots \ldots \ldots \ldots \ldots + \frac{(-1)^{n-2} x^{n-1}}{n-1} + \frac{(-1)^{n-1} x^n}{n(1+\theta x)^n}, \quad 0 < \theta < 1.$$

(*c*) Verify whether Rolle's theorem is applicable to the function $f(x) = 2 + (x - 1)^{\frac{2}{3}}$ in the interval $[0, 2]$ or not.

4. (*a*) Expand $e^{\sin x}$ as far as the term containing x^4.

(*b*) If in Cauchy's mean value theorem, $f(x) = \frac{1}{x^2}$, and $g(x) = \frac{1}{x}$, then show that c is the harmonic mean of a and b.

(*c*) Find a, b, c so that $\lim_{x \to 0} \frac{ae^x - b\cos x + ce^{-x}}{x \sin x} = 2$.

5. (*a*) Find all the asymptotes of the curve :

$$x^3 + 2x^2y - xy^2 - 2y^3 + 3xy + 3y^2 + x + 1 = 0.$$

(*b*) Find the range of values of x in which the curve

$$y = 3x^4 - 4x^3 + 1$$

is concave upwards or downwards. Also find its points of inflexion.

(*c*) Find the reduction formula for

$$\int \sin^n x \, dx.$$

Hence, evaluate $\int_0^{\frac{\pi}{2}} \sin^7 x \, dx$.

6. (*a*) Determine the position and nature of the double points on the curve :

$$x^3 + x^2 + y^2 - x - 4y + 3 = 0.$$

(*b*) Trace the curve $y = x(x^2 - 1)$.

(*c*) Obtain a reduction formula for $\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x \, dx$. Hence evaluate :

$$\int_0^{\frac{\pi}{2}} \sin^4 x \cos^5 x \, dx.$$

This question paper contains 3 printed pages]

Roll No. [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

S. No. of Question Paper : **2564**

Unique Paper Code : **2352571101**

Name of the Paper : **DSC : Topics in Calculus**

Name of the Course : **B.A./B.Sc. (Prog.) with Mathematics as Non-Major/Minor**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting *two* parts from each question.

All questions carry equal marks.

1. (*a*) Show that the function $f(x) = \begin{cases} x \sin \dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous but not differentiable at $x = 0$.

(*b*) Find the *n*th differential coefficients of $\sin 3x \sin 2x$.

(*c*) If $u = x^3 + y^3 + z^3 + 3xyz$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 3u$.

2. (*a*) Test the continuity of a function at $x = 0$ which is defined as follows :

$$f(x) = \begin{cases} \sin \dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

P.T.O.

(*b*) If $y = e^{a \sin^{-1} x}\left(or\ x = \sin\left[\frac{(\log y)}{a}\right]\right)$,

prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$.

(*c*) If $u = \tan^{-1}\frac{xy}{\sqrt{1+x^2+y^2}}$, show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{1}{(1+x^2+y^2)^{\frac{3}{2}}}$.

3. (*a*) State Taylor's theorem with Lagrange's form of remainder. Find the Taylor series expansion of $f(x) = \sin x$.

(*b*) Evaluate : $\lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$.

(*c*) Discuss the applicability of Rolle's theorem for $f(x) = \tan x$ in $[0, \pi]$.

4. (*a*) If in Cauchy's mean value theorem, $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{\sqrt{x}}$, then show that c is the geometric mean of a and b.

(*b*) Verify Rolle's theorem for :

(*i*) $f(x) = (x^2 + 2x - 3)e^x$ $\quad x \in [-3, 1]$

(*ii*) $f(x) = 10x - x^2$ $\quad x \in [0, 10]$.

(*c*) With the help of Maclaurin's theorem give the expansion of $f(x) = \cos x$ in ascending power of x.

5. (*a*) Find all the asymptotes of the curve :

$$x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 = 1.$$

(*b*) Find the range of values of x in which the curve :

$$y = x^4 - 6x^3 + 12x^2 + 5x + 7.$$

is concave upwards or downwards. Also find its points of inflexion.

(*c*) Find the reduction formula for

$$\int_0^{\frac{\pi}{2}} \cos^n x \, dx .$$

Hence, evaluate $\int_0^{\frac{\pi}{2}} \cos^6 x \, dx$.

6. (*a*) Determine the position and nature of the double points on the curve :

$$y^2 = (x - 2)^2 (x - 1).$$

(*b*) Trace the curve $y = x^3$.

(*c*) Obtain a reduction formula for $\int \sin^m x \cos^n x \, dx$. Hence evaluate $\int_0^{\frac{\pi}{2}} \sin^3 x \cos^2 x \, dx$.

(17)

[This question paper contains 8 printed pages.]

Your Roll No..............

I

Sr. No. of Question Paper : 2597

Unique Paper Code : 2352201102

Name of the Paper : Elements of Discrete Mathematics (DSC-2)

Name of the Course : **B. A. (Prog.)**

Semester : I

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.

2. Attempt all questions by selecting two parts from each question.

3. Parts of question to be attempted together.

4. Use of Calculator not allowed.

P.T.O.

1. (a) (i) Compute the truth table of the statement

$$\sim (p \Rightarrow q) \Rightarrow p$$

Is it a tautology?

(ii) If A = {3, 7, 2}, find $P(A)$, the Power set of A. (7.5)

(b) Show that the relation "is less than" in the set of real numbers is transitive and anti- symmetric, but not reflexive. (7.5)

(c) Show that the relation

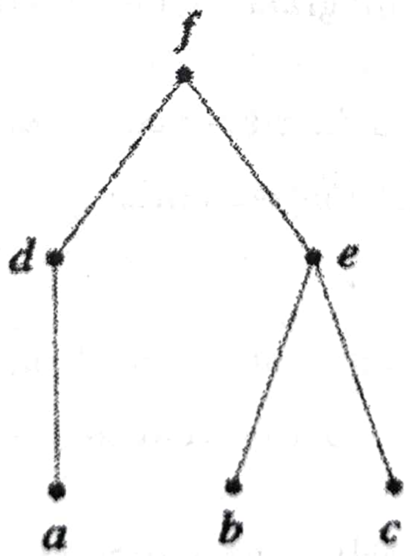
$$R = \{(a, b) : a, b \in \mathbb{Z}, \text{ and } a - b \text{ is divisible by } 3\}.$$

where $\mathbb{Z}$ is the set of integer, is an equivalence relation. (7.5)

2. (a) Show, by mathematical induction, that for all $n \geq 1$,

$$1+2+3.......+n = \frac{n(n+1)}{2}$$ (7.5)

(b) Find the greatest element, least element, all maximal and minimal elements of the Poset whose Hasse diagram is given by:



(7.5)

(c) Let $A = B = C = \mathbb{R}$, and let $A \to B, g : B \to C$ be defined by

$f(a) = a + 1$ and $g(b) = b^2 + 2.$

Find $(g^\circ f)(x)$ and $(f^\circ g)(x)$. (7.5)

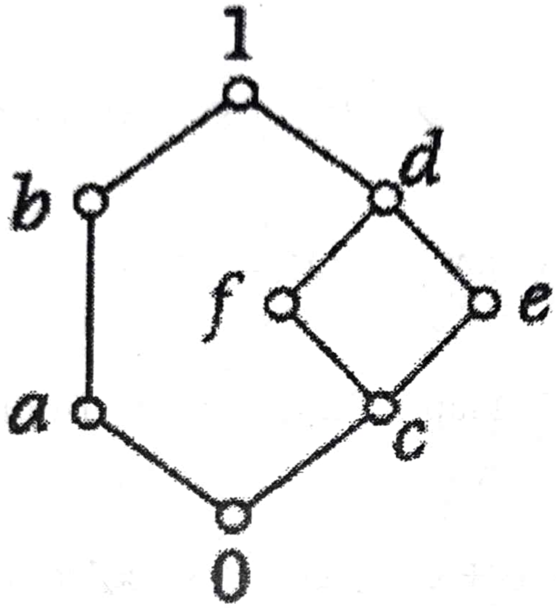
3. (a) Show that the relation of divisibility (a R b if and only if a/b) is a partial order on Z^+, set of positive integers. (7.5)

(b) Let A= {1, 2, 3, 4}, R = {(1, 1), (1, 2), (2, 2), (2, 4), (1, 3), (3, 3), (3, 4), (1, 4), (4, 4)}, draw the Hasse diagram of the relation R. (7.5)

(c) Define a distributive lattice. Show that every chain is a distributive lattice. (7.5)

4. (a) Is complement of an element in a lattice always unique? Justify your answer. (7.5)

(b) Is the following lattice distributive? Justify your answer.



(7.5)

(c) Let A be the set Z^+ of positive integers, and let $\leq$ be the usual partial order on A. Let A_1 be the set of positive even integers, and let $\leq_1$ be the usual partial order on A_1. Show that the function f: $A \to A_1$ given by:

$f(a) = 2a$ (7.5)

is an isomorphism from $(A, \leq)$ to $(A_1, \leq_1)$.

P.T.O.

5. (a) Define Disjunctive Normal Form. Convert the Boolean function

$$(x' + y' + z').\ (x' + y + z).\ (x + y + z)$$

in disjunctive normal form. (7.5)

(b) Use K-map to simplify the Boolean polynomial

$$x_1 x_2 x_3 + x_1' x_2 x_3' + (x_1 + x_2' x_3')(x_1 + x_2 + x_3)' x_3 (x_1' + x_2).$$ (7.5)

(c) For all x, y in a Boolean algebra, we have:

$(x \wedge y)' = x' \vee y'$ and $(x \vee y)' = x' \wedge y'$. (7.5)

6. (a) Use Quine Mc-Cluskey to find the minimal form of f

$f = x'yz + x'yz' + x'y'z + xy'z' + xy'z$ (7.5)

(b) Construct the circuit diagram corresponding to the Boolean function

(i) $f = (x_1 + x_2 + x_3)(x_1' + x_2)(x_1 x_3 + x_1' x_2)(x_2' + x_3)$

(ii) $g = x_1' x_2' x_3 + x_1' x_2 x_3 + x_1 x_2'$ (7.5)

(c) Determine the output of the symbolic representation in figure (i) and figure (ii)

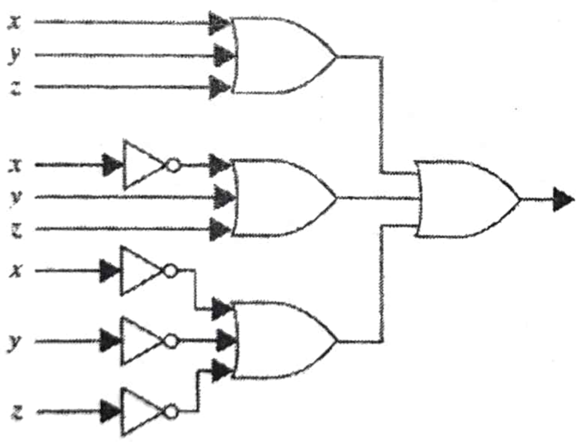


Figure (i)

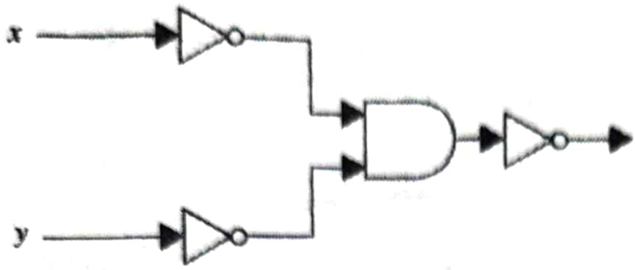


Figure (ii)

(7.5)

(18)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1476 **I**

Unique Paper Code : 2352572301

Name of the Paper : Differential Equations

Name of the Course : **B.A./B.Sc.(Programme) with Mathematics as Non-Major/ Minor**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.

1. (a) Find values of m so that the function $y = e^{mx}$ is a solution of the given differential equation

$$\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 12y = 0.$$

P.T.O.

(b) Define exact differential equation. Show that the given differential equation is exact and also solve the equation

$$(3x^2y^2 - y^3 + 2x)dx + (2x^3y - 3xy^2 + 1)dy = 0.$$

(c) Solve the initial-value problem that consist of the differential equation

$$x \cos y \, dx + (x^2 + 1) \sin y \, dy = 0$$

and the initial condition $y(1) = \frac{\pi}{2}$.

2. (a) Solve the homogeneous differential equation

$$(x^3 + 3xy^2)dx + (y^3 + 3x^2y)dy = 0.$$

(b) Solve the initial-value problem

$$(x^2 + 1)\frac{dy}{dx} + 4xy = x, \quad y(2) = 1.$$

(c) Find an integrating factor of the differential equation

$$(2x + \tan y)dx + (x - x^2 \tan y) dy = 0$$

and then solve.

3. (a) Solve initial value problem

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 12y = 0, \; y(0) = 3, \; y'(0) = 5.$$

(b) Find general solution of the differential equation using method of undetermined coefficients

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 16x - 12e^{2x}.$$

(c) Find general solution of the differential equation by the method of variation of parameters

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \frac{1}{1+e^x}.$$

4. (a) Find general solution of the following differential equation

$$x^2\frac{d^2y}{dx^2} - 6x\frac{dy}{dx} + 10y = 3x^4 + 6x^3$$

Given that $y = x^2$ and $y = x^5$ are linearly independent solutions of the corresponding homogeneous differential equation.

(b) Find general solution of the following Cauchy-Euler equation using the transformation $x = e^t$, assuming $x > 0$

$$x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 4y = 4x^2 - 6x^3, \quad y(2) = 4, \; y'(2) = -1.$$

(c) Find the general solution of the given linear system

$$\frac{dx}{dt} + \frac{dy}{dt} + 2y = \sin t$$

$$\frac{dx}{dt} + \frac{dy}{dt} - x - y = 0.$$

P.T.O.

5. (a) Find the general solution of the partial differential equation

$$x(y^2 + z)p - y(z + x^2)q = z(x^2 - y^2).$$

(b) Form partial differential equation by eliminating arbitrary constants a and b from

$$u = (x - a)^2 + (y - b)^2.$$

Also, find the order and degree of the following partial differential equation

$$u_x^2 + u_{xxx} = 2x\, u_x.$$

(c) Reduce the equation into canonical form and hence find its general solution

$$u_{xx} = (1 + y)^2\, u_{yy}.$$

6. (a) Eliminate the arbitrary function to find the partial differential equation from

$$x + y + z = f(x^2 + y^2 + z^2).$$

(b) Find the solution of the following partial differential equation by the method of separation of variables

$$u_x - u_y = u, \quad u(x, 0) = 4e^{-2x}.$$

(c) Classify the partial differential equation and reduce it to the canonical form

$$4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2.$$

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This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2222**

Unique Paper Code : **2342572301**

Name of the Paper : **Computer System Architecture**

Name of the Course : **B.A. (Prog.)/B.Sc. (P)/B.Sc. (Maths Sc.)**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Section A is compulsory.

Attempt any *four* questions from Section B.

Parts of a question should be attempted together.

SECTION–A

1. (*a*) Using De-Morgan's theorem show that : 3

(*i*) (A + B)´ . (A´ + B´)´ = 0

(*ii*) A + A´ B + A´B´ = 1.

(*b*) Represent $(-13)_{10}$ in 8-bit register by the following representations : 3

(*i*) Signed-magnitude representation

(*ii*) Signed-1's complement representation

(*iii*) Signed 2's complement representation.

(*c*) How is a memory implemented through a stack organization ? 3

(*d*) What is the size and function of the each of the register in a basic computer ? 3

(*e*) What are logic gates ? Which gates are also known as universal gates ? Draw their symbol and the truth table ? 3

P.T.O.

R 12/12/2024 (E)

(*f*) How many address lines and input-output lines are needed for the following memory units specified by number of words times number of bits per word ? 3

(*i*) 64K × 8

(*ii*) 16M × 32.

(*g*) Write all the microinstructions to execute ISZ instruction starting at timing signal T_4. 4

(*h*) List the excitation tables of D and T flip-flops. 4

(*i*) Convert the following into the indicated base : 4

(*i*) $(69.15)_{10} = (?)_2$

(*ii*) $(1938)_{10} = (?)_{16}$.

SECTION–B

2. (*a*) What is the overall goal of memory hierarchy in computer memory system ? Draw and explain the block diagram of memory hierarchy. 4

(*b*) Perform the arithmetic operations in binary using 2's complement representation : 5

(*i*) (+42) + (–13)

(*ii*) (–42) – (–13).

(*c*) Draw the flow-chart of the instruction cycle and write down the set of microinstructions starting at timing T_4 for the following instructions : 6

(*i*) ADD

(*ii*) BSA.

3. (*a*) Write any *four* characteristics of GPU. 4

(*b*) What is meant by instruction set completeness ? List the microoperations for the modified fetch phase of the interrupt cycle. 5

(*c*) Draw the logic diagram that implements the complement of the following function using NAND gates : 6

$$F(A, B, C, D) = \Sigma(0, 1, 2, 3, 4, 8, 9, 10, 11, 12).$$

4. (*a*) If the content of AC and E are as follows : 5

AC = A93, E = 1

Show the contents of AC and E after the execution of each of the following instructions :

(*i*) CMA

(*ii*) CME

(*iii*) CIR

(*iv*) CIL

(*v*) INC.

(*b*) What is the purpose of having addressing modes in a computer ? Describe any *four* addressing modes with help of suitable example. 5

(*c*) Draw the logic diagram and explain the working of a 4 – to – 1 line multiplexer. 5

5. (*a*) A 4-bit adder–subtractor circuit has the following values for the mode input M, and the data inputs A and B : 5

	M	A	B
i.	0	0111	0101
ii.	0	1001	1001
iii.	1	1100	1000
iv.	1	0101	1010
v.	1	0000	0001

In each case determine the values of the four SUM outputs, the carry C, and the overflow V.

(*b*) Explain with a suitable examples the role of BUN and BSA instrctions in an interrupt cycle. 5

(*c*) Convert the following function in *sum–of–product* form to product–of–sum form and draw the logic diagram using only *NOR* gates : 5

$$F = B'D' + B'C' + A'C'D$$

6. (*a*) Find the complement of the following function and draw the logic diagram : 4

$$F_1 = x' yz' + x'y'z$$

(*b*) Design a combinational circuit with three inputs and one output. The output is 1 when the binary value of the input is an odd number and is 0 otherwise. 5

(*c*) Write short notes on the following : 6

(*i*) Parallel processing using Pipeline

(*ii*) DMA

(*iii*) Multicore processors.

7. (*a*) Construct a 5–to–32 line decoder with four 3–to–8 line decoders with enable and one 2–to–4 line decoder. Use block diagrams only. 4

(*b*) With the help of a truth table explain the working of a Full adder and draw the logic diagram. 5

(*c*) Differentiate between the following : 6

(*i*) RISC and CISC

(*ii*) Programmed I/O and Interrupt-initiated I/O modes of transfer

(*iii*) Combinational and sequential circuits (with examples of each).

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[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 7686 **I**

Unique Paper Code : 62354343

Name of the Paper : Analytic Geometry and Applied Algebra

Name of the Course : **B.A. (Prog.) (CBCS-LOCF) (SOL)**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any **two** parts from each question.

3. Questions **1, 3, 5** are for **12** marks each, and questions **2, 4, 6** are for **13** marks each.

1. (a) Describe and draw the graph of the given equation showing the vertices, foci and asymptotes.

$$16x^2 - 25y^2 = 400.$$

(b) Identify and sketch the curve :

$$9x^2 + 4y^2 - 18x + 24y + 9 = 0.$$

(c) State the reflection property of parabolas. Find an equation of the parabola that has its focus at (1,1) and directrix $y = -2$.

2. (a) Describe and sketch the given curve. Label its focus, vertex and directrix

$$y = 4x^2 + 8x + 5.$$

(b) Find the equation of the ellipse with end points of major axes $(\pm 5, 0)$ and end points of minor axes $(0, \pm 2)$.

(c) Find an equation of a hyperbola which passes through the origin with $y = x - 2$ and $y = -x + 4$ as asymptotes.

3. (a) Rotate the coordinate axes to remove the xy-term. Then identify the type of conic $x^2 + 4xy - 2y^2 - 6 = 0$.

(b) Let an $x'y'$-coordinate system be obtained by rotating an xy-coordinate system through an angle of $\theta = 60°$.

(i) Find the $x'y'$-coordinates of the point whose xy-coordinates are $(-2,6)$.

(ii) Find an equation of the curve $\sqrt{3}\,xy + y^2 = 6$ in x'y'-coordinates.

(c) Let v = 2i – j + 3k and b = i + 2j + 2k, find the vector component of v along b and the vector component of v orthogonal to b.

4. (a) (i) Determine whether (2,1,6), (4,7,9) and (8,5, –6) are the vertices of a right triangle.

(ii) Sketch the surface $z = 1 - y^2$ in 3-space.

(b) Find the component form of the vector v in 2-space that has length ||v|| = 3 and makes angle $\phi = \pi/4$ with positive x-axis. Find a vector u that is oppositely directed to v and has half the length of v.

(c) (i) Prove that $u.\ v = \frac{1}{4}\ ||u + v||^2 - \frac{1}{4}\ ||u - v||^2$.

(ii) Determine whether the vectors u = 6i + j + 3k and v = 4i – 6k make an acute angle, an obtuse angle or are orthogonal.

5. (a) (i) Find two unit vectors that are orthogonal to both u = –4i + 3j + k and v = 2i + 4k.

(ii) Using scalar triple product, determine whether the vectors u = 5i – 2j + k, v = 4i – j + k and w = i – j lie in the same plane.

(b) Show that the lines

L_1: $x = 2 - t,\ y = 2t,\ z = 1 + t,$

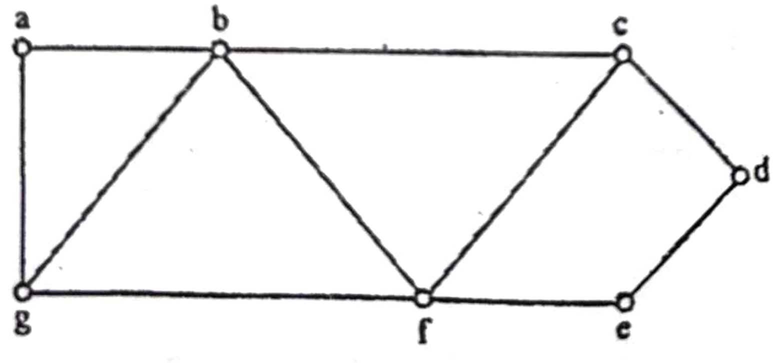
L_2: $x = 1 + 2t,\ y = 3 - 4t,\ z = 5 - 2t$

are parallel and find the distance between them.

(c) Find an equation of the sphere with center $(2, 1, -3)$ that is tangent to the plane $x - 3y + 2z - 4 = 0$.

6. (a) Prove that the Cayley table for any finite group $(G, +)$ of order n is a Latin square of order n based on G. Is there any Latin square which cannot be obtained from a group Cayley table, justify by an example.

(b) Define an edge cover for a graph. Also find a set of two vertices in the following figure whose removal disconnects the graph :



(c) Three pitchers of sizes 10-litres, 7-litres and 4-litres are given. Only 10-litres pitcher is full. Find a minimum sequence of pouring to get 2 litres in the 7-litres or 4-litres pitcher.

(2)

This question paper contains 3 printed pages]

Roll No.

S. No. of Question Paper : **2232**

Unique Paper Code : **2352572301**

Name of the Paper : **Differential Equations**

Name of the Course : **B.A./B.Sc. (Prog.) with Mathematics as Non-Major/Minor**

Type of the Paper : **DSC-B2**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting any *two* parts from each question.

All questions carry equal marks.

Use of simple Calculator is allowed.

1. (*a*) Find the differential equation from the relation $y = a \sin x + b \cos x + x \sin x$, where a and b are arbitrary constants. 7.5

(*b*) Solve the differential equation : 7.5

$$\frac{dy}{dx} + \left(\frac{2x+1}{x}\right) y = e^{-2x}.$$

(*c*) Show that $y_1 = e^x$ and $y_2 = xe^x$ are the solutions of the differential equation : 7.5

$$y'' - 2y' + y = 0.$$

Also, verify that y_1 and y_2 are linearly independent.

P.T.O.

2. (a) Solve the following homogeneous differential equation : 7.5

$$x\frac{dy}{dx} = x + y.$$

(b) Find the integrating factor of the following differential equation and solve it : 7.5

$$(x^2 + y^2 + x)dx + xy\, dy = 0.$$

(c) Find the orthogonal trajectories of the family of curves $y^2 = 2x^2 + c$, where c is a real parameter. 7.5

3. (a) Find the general solution of the differential equation by using the method of undetermined coefficients : 7.5

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10\sin x.$$

(b) Solve the initial value problem : 7.5

$$y'' + 3y' - 10y = 0,\ y(0) = 4,\ y'(0) = -9$$

(c) Find the general solution of the differential equation by using the method of variation of parameter : 7.5

$$\frac{d^2y}{dx^2} + y = \tan x.$$

4. (a) Find the general solution of the given differential equation : 7.5

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3.$$

(b) Write the general solution of the 10th-order homogeneous differential equation whose auxiliary equation has the following roots :

$$0,\ 4,\ 3,\ 3,\ 3,\ -3,\ 1 + 4i,\ 1 - 4i,\ 1 + i,\ 1 - i.$$

Verify that e^x and e^{2x} are solutions to the homogeneous differential equation :

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0.$$

Show that a linear combination of e^x and e^{2x} is also a solution. 7.5

(c) Find the general solution of the linear system : 7.5

$$\frac{dx}{dt}+\frac{dy}{dt}-2x-4y=e^{t},$$

$$\frac{dx}{dt}+\frac{dy}{dt}-y=e^{4t}.$$

5. (a) Eliminate the arbitrary function f from the question : 7.5

$$z = xy + f(x^2 + y^2),$$

and obtain the partial differential equation. Also, specify the order.

(b) Find the solution of the Cauchy problem for the first order PDE : 7.5

$$3\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0, \quad u(x,0)=\sin x.$$

(c) Reduce the equation : 7.5

$$u_x + u_y = u,$$

to canonical form and obtain the general solution.

6. (a) Use the Method of Characteristics to find the general solution of the equation : 7.5

$$x^2u_x + y^2u_y = -u^2.$$

(b) Apply the method of separation of variables $u(x, y) = X(x)Y(y)$ to solve : 7.5

$$u_x - u_y = -u, \qquad u(0, y) = 6e^{-2y}.$$

(c) Classify the partial differential equation as elliptic, parabolic or hyperbolic :

$$u_{xx} = x^2u_{yy},\ x \neq 0.$$

Also, reduce it to canonical form. 7.5

(22)

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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 2315

Unique Paper Code : 2352572301

Name of the Paper : **Differential Equations**

Name of the Course : B.A./B.Sc. (Prog.) with Mathematics as Non-Major/Minor

Types of the Paper : DSC-B2

Semester : III

Duration : 3 Hours | Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all questions by selecting any *two* parts from each question.

All questions carry equal marks.

Use of simple calculator is allowed.

1. (*a*) Find the differential equation of the family of curves $y = e^x (A \sin x + B \cos x)$, where A and B are arbitrary constants. 7.5

(*b*) Solve the initial value problem :

$$(x^2 + 1)\frac{dy}{dx} + 4xy = x, \; y(2) = 1.$$

7.5

(*c*) Solve the differential equation :

$$x(1 + y^2)\, dx + y\,(1 + x^2)\, dy = 0.$$

7.5

P.T.O.

2\. (*a*) In a certain bacteria culture, the rate of increase in the number of bacteria is proportional to the number present at any time. 7.5

(*i*) If the number triples in 5 hours, how many will be present in 10 hours ?

(*ii*) When will the number present be 10 times the number initially present ?

(*b*) Show that $y_1 = x^3$ and $y_2 = x^{-2}$ are solutions of the differential equation

$$x^2y'' - 6y = 0.$$

Also, verify that y_1 and y_2 are linearly independent. 7.5

(*c*) Find the integrating factor of the following differential equation and solve it

$$(y - xy)dx + x\,dy = 0.$$ 7.5

3\. (*a*) Find the general solution of the differential equation by using the method of undetermined coefficients 7.5

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2e^{2x}.$$

(*b*) Solve the initial value problem : 7.5

$$y'' - 4y' + 3y = 0,\ y(0) = 6,\ y'(0) = -7.$$

(*c*) Find the general solution of the differential equation by using the method of variation of parameter :

$$y'' + y' - 6y = \sin(2x).$$ 7.5

4. (*a*) Find the general solution of the given differential equation : 7.5

$$x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}-3y=4x.$$

(*b*) Write the general solution of the 10th-order homogeneous differential equation whose auxiliary equation has the following roots : 7.5

$$3,\ 3,\ 3,\ -3,\ 5+i,\ 5-i,\ 0,\ 0,\ 1+i,\ 1-i.$$

Verify that e^{2x} and e^{3x} are solutions to the homogeneous differential equation

$$\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0.$$

Show that a linear combination of e^{2x} and e^{3x} is also a solution.

(*c*) Find the general solution of the linear system : 7.5

$$\frac{dx}{dt}+\frac{dy}{dt}-x=-2t,$$

$$\frac{dx}{dt}+\frac{dy}{dt}-3x-y=t^2.$$

5. (*a*) Eliminate the arbitrary constants *a* and *b* from the equation :

$$2z=(ax+y)^2+b,$$

and obtain the partial differential equation. Also, specify the order. 7.5

(*b*) Find the solution of the Cauchy problem for the first order PDE :

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2xy,\ \text{with the condition } u=2 \text{ on } y=x^2.$$ 7.5

(*c*) Reduce the equation :

$$yu_x + u_y = x$$

to canonical form and obtain the general solution. 7.5

6. (*a*) Use the method of characteristics to find the general solution of the equation :

$$uxu_x + uyu_y = xy.$$ 7.5

(*b*) Apply the method of separation of variables $u(x, y) = X(x)\,Y(y)$ to solve

$$u_x - u_y = 0, \; u(0, y) = 4e^{3y}.$$ 7.5

(*c*) Classify the partial differential equation as elliptic, parabolic or hyperbolic :

$$yu_{xx} + 3yu_{xy} + 3u_x = 0, \; y \neq 0.$$

Also, reduce it to canonical form and obtain the general solution. 7.5

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[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 2386 I

Unique Paper Code : 2352202002

Name of the Paper : Theory of Equations and Symmetries

Name of the Course : **B.A. (Prog.)**

Semester : III – DSC-3 (A or B)

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of Calculator not allowed.

1. (a) Without actual division find the remainder when $x^4 - 3x^2 - x - 6$ is divided by $x + 3$.

P.T.O.

(b) Find all the integral roots of the equation $x^3 + x^2 - 3x + 9 = 0$.

(c) Find the nature of the roots of the equation $x^4 + 15x^2 + 7x - 11 = 0$, using Descartes's Rule of Signs.

2. (a) Find a necessary condition for the roots of the equation $ax^3 + bx^2 + cx + d = 0$ to be in arithmetic progression.

(b) If $z = \cos\theta + i \sin \theta$, then prove that $z^k + \frac{1}{z^k} = 2 \cos k\theta$ and $z^k - \frac{1}{z^k} = 2 \sin k\theta$, where k is any arbitrary integer.

(c) Find all the values of $\left(\sqrt{3} - i\right)^{\frac{2}{5}}$.

3. (a) Solve the following cubic equation by Cardon's Method

$$y^3 - 9y + 28 = 0.$$

(b) Find the equation whose roots are the reciprocals of the roots of the equation

$$x^4 - 3x^3 - 7x^2 + 5x - 2 = 0.$$

(c) Solve the following biquadratic equation by Descartes Method

$$x^4 - 3x^3 + 3x^2 - 3x + 2 = 0.$$

4. (a) Find the solution of equation $x^3 + x^2 - 16x + 20 = 0$ by Cardon's Method.

(b) Find the equation whose roots are the roots of the equation

$$x^4 - 5x^3 + 7x^2 - 17x + 11 = 0$$

each diminished by 4.

(c) Find the solution of equation $x^4 + 3x^2 + 2x + 12 = 0$ by Descartes Method.

5. (a) If a, p and y are the roots of the equation $x^3 + px^2 + qx + r = 0$. Find the value of $(\beta+\gamma)(\gamma+\alpha)(\alpha+\beta)$.

(b) Find the value of the symmetric function $\Sigma\alpha^2\beta\gamma$ of the roots of the biquadratic equation

$$x^4 + px^3 + qx^2 + rx + s - 0.$$

(c) If α, β and γ are the roots of the cubic equation $x^3 + qx + r = 0$, then calculate the symmetric function $\Sigma(\beta + \gamma - \alpha)^2$.

6. (a) Find the value of the symmetric function $\sum \frac{\beta}{\alpha}$ for the roots of the equation

$$x^3 + 2x^2 - 3x - 1 = 0.$$

(b) Find an equation whose roots are square of the roots of $x^4 + x^3 + 2x^2 + x + 1 = 0$.

(c) Use Newton's formula to show that for a cubic equation $x^3 + c_1x^2 + c_2x + c_3 = 0$,

$$s_4 = c_1^4 - 4c_1^2c_2 + 4c_1c_3 + 2c_2^2,$$

where, $s_k = \Sigma\alpha_1^k$, α_1, α_2, α_3 are the roots of the given cubic equation.

(2000)

This question paper contains 6 printed pages]

Roll No.

S. No. of Question Paper : **1198**

Unique Paper Code : **2343010012**

Name of the Paper : **Web Design and Development (Discipline-Specific Elective)**

Name of the Course : **B.A. (Prog.)/B.Sc. (P)/B.Sc. Maths. Sc.**

Semester : **V**

Duration : **3 Hours**

Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

This paper contains two sections—Section A and Section B.

Section A is compulsory. It carries **30** marks.

Attempt any *four* questions out of *six* from Section B.

Each question in Section B carries **15** marks.

Parts of a question must be answered together.

Section A

(Compulsory)

1. (*a*) Write an HTML tag to set a background image in the body of a webpage. Which attributes are used to control the size of the image on an HTML webpage ? Which attributes takes the text to be displayed in case the browser is unable to display an image specified in the '*src*' attribute of the image tag ? 3

P.T.O.

(*b*) Write a JavaScript code to check whether a number is a multiple of 5. 3

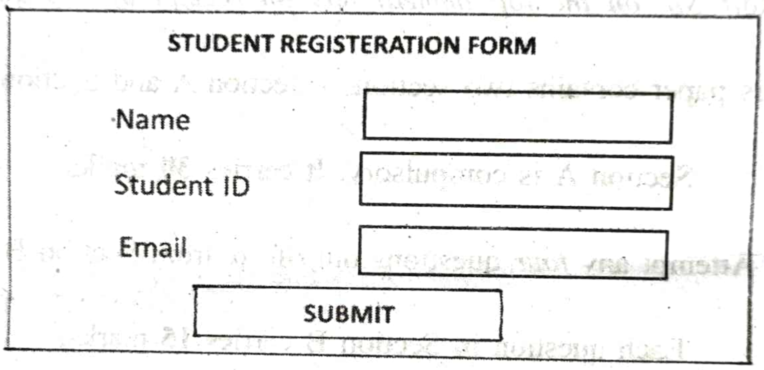
(*c*) Write a PHP script to print the sum of odd digits of a four-digit number. 4

(*d*) Differentiate between internal and external linking in HTML with the help of an example. 5

(*e*) Write HTML code to design the given HTML form with the following validations in JavaScript : 5

(*i*) **'Student ID'** is a 10-digit alphanumeric value

(*ii*) **'Name'** should be an alphabetical value (String)



(*f*) Write the output of the following codes with justification : 5

(*i*)

```
<?php  $a = "100";
 $b = "+100";
 if ($a == $b) echo "1";
 if ($a === $b) echo "2"; ?>
```

(*ii*)

```
<?php
$a = 1; $b = 0;
echo ($a AND $b);
echo ($a or $b);
echo ($a XOR $b);
echo !$a   ?>
```

(g) Answer the following : 5

(i) Give an example of a paired and unpaired tag in HTML.

(ii) Define WWW.

(iii) Give an example of a right and a left-associative operator in PHP.

(iv) List any *two* datatypes in JavaScript.

(v) What do you mean by 'DOCTYPE' in HTML ?

Section B

2. (a) What will be the output of the following : `"100"+200+300` in JavaScript ? Justify your answer. 2

(b) What is an event in JavaScript ? What is the purpose of '*onBlur*' and '*onFocus*' events in JavaScript ? 2

(c) Write a PHP code to find a minimum of three numbers. 3

(d) Differentiate between Pass-by-Value and Pass-by-Reference with the help of an example in PHP. 4

(e) What is the difference between GET and POST methods ? Explain with the help of an example. 4

3. (a) Write an HTML code to display the following list. 5

(1) Web Design and Development

- HTML
- CSS
- JavaScript
- PHP

(2) Programming Languages

(i) C++

(ii) Java

(iii) Python

(*b*) Write an event-driven program in JavaScript to take a number as input (in an input text box in an HTML form) and display its multiplication table at the click of a button. 5

(*c*) Write a PHP program to print the following pattern : 5

```
   *
  * *
 * * *
* * * *
```

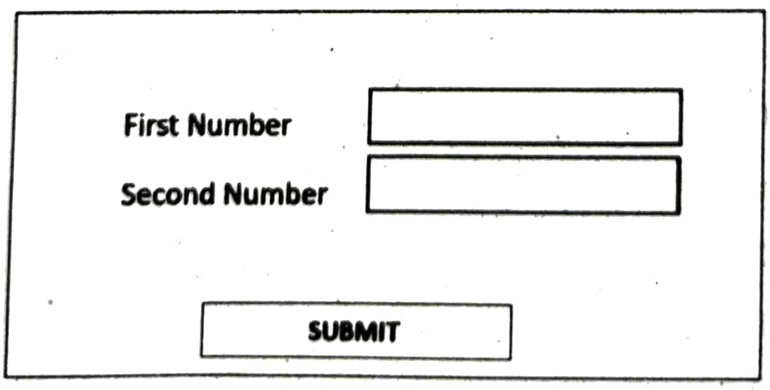
4. (*a*) Give an example of a dialog box in JavaScript. 2

(*b*) Write code in HTML to create the following table and display it on a webpage : 4

Station	Temperature		Humidity
	Max	Min	
USA	24	19	60%
Germany	5	-1	70%

(*c*) Write a program in PHP to calculate the factorial of a number using '*foreach*' construct. 4

(*d*) Write a JavaScript code that reads two numbers in an HTML form (as shown below), and when a 'Submit' button in the form is clicked, their sum is displayed in an alert box. 5



5. (*a*) Differentiate between the local and global scope of a variable with the help of an example in PHP. 5

(*b*) Write a JavaScript code to display whether a given number is a prime or not. 5

(*c*) How does a web browser communicate with a web server ? Explain with the help of a diagram. 5

6. (*a*) Write a program in PHP to print the sum of the first ten natural numbers. 5

(*b*) PHP connects to databases using connection objects. Write PHP commands for the following steps : 5

(*i*) Connect to the MySQL server with the server's name as **'localhost'**; the user's name as **'root'**; and the password as **'test@123'**.

(*ii*) Connect to a **'Company'** database.

(*iii*) Create an **'Employee'** table in the **'Company'** database, and insert the record for two employees with (**'Name'**, **'ID'**, **'Salary'**) into the table.

(*iv*) Display the details of all the employees.

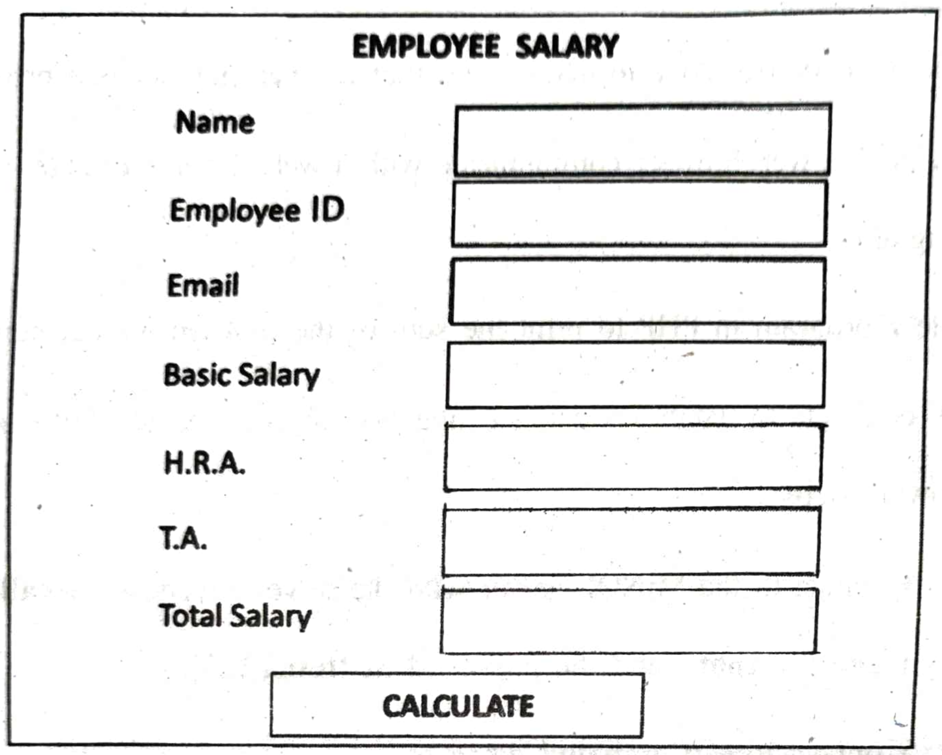
(*v*) Close the connection.

(*c*) Write JavaScript code that validates a username and password against the user name **'John Walt'** and password **'abc@123'**. 5

7. (*a*) Write a PHP function to swap any *two* numbers. 3

(*b*) Define associative arrays in PHP with the help of an example. 4

(*c*) Write an HTML code to design the following form : 8



Calculate Total Salary as **Basic Salary + H.R.A. + T.A.** on **"onclick()"** event handler using JavaScript for the **"CALCULATE"** button.

(26)

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1467 **I**

Unique Paper Code : 2352203501

Name of the Paper : Elements of Real Analysis

Name of the Course : **B.A. / B.Sc. (P) with Mathematics as Non-Major / Minor**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** the questions are compulsory.
3. Attempt any **two** parts from each question.
4. **All** questions carry equal marks.

1. (a) Define bounded and unbounded subsets of $\mathbb{R}$.

Show that the following sets are bounded above as well as bounded below :

(i) $S = \left\{ \frac{(-1)^n}{n} : n \in \mathbb{R} \right\}$

P.T.O.

(ii) $T = \{x \in \mathbb{R}: 1 < x < 2\}$.

Also find sup and inf of the given sets.

(b) Let S be a non-empty bounded subset of $\mathbb{R}$. Let $a > 0$ and let $aS = (as: s \in S\}$. Prove that $\sup(aS) = a \sup S$ and $\inf(aS) = a \inf S$.

(c) (i) Define absolute value of x.

Show that $|x - a| < \varepsilon$ if and only if $a - \varepsilon < x < a + \varepsilon$.

(ii) Find all $x \in \mathbb{R}$ that satisfy the inequality $\frac{1}{|2x-5|} > 5$.

2. (a) For any two real numbers a and b, Show that

(i) $\max\ (a, b) = \frac{1}{2}(a + b + |a - b|)$

(ii) $\min\ \{a, b\} = \frac{1}{2}(a + b - |a - b|)$.

(b) Using the definition of convergence, show that $\lim_{n\to\infty} \frac{1}{n} = 0$. Hence find the limit of the following :

(i) $\lim_{n\to\infty} \frac{8n^2+3}{5n^2-2n}$

(ii) $\lim_{n\to\infty} \frac{n-2n^2}{3n^2+1}$

(c) State and prove First Squeeze Principle.

3. (a) Let $\{a_n\} = \left\{\frac{2n+3}{3n-7}\right\}$. Prove that $\lim_{n\to\infty} a_n = \frac{2}{3}$. For arbitrary $\varepsilon > 0$, after how many terms are we guaranteed that a_n is within a distance ε of $\frac{2}{3}$?

(b) Prove that every convergent sequence is bounded. Is the converse true? Explain.

(c) State the Second Squeeze Principle and use it to prove that $\lim_{n\to\infty} \frac{10n-11}{7-2n} = -5$.

4. (a) State and prove Fundamental Theorem of Monotone Sequences.

(b) Prove that $\lim_{n\to\infty} n^{1/n} = 1$.

(c) Show that every convergent sequence is Cauchy. Use it to prove that $\left\{\frac{n}{n^2+1}\right\}$ is Cauchy.

P.T.O.

5. (a) State Limit Comparison Test for convergence of a positive term series. Use it to test for the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \tan\frac{1}{n}$.

(b) Prove that the series :

$$1+\frac{\alpha}{\beta}+\frac{\alpha(\alpha+1)}{\beta(\beta+1)}+\frac{\alpha(\alpha+1)(\alpha+2)}{\beta(\beta+1)(\beta+2)}+\cdots;(\alpha>0,\beta>0)$$

converges if $\beta > \alpha + 1$ and diverges if $\beta \leq \alpha + 1$.

(c) Let Σa_n be a convergent series of positive term series. Prove that Σa_n^2 is also convergent. Show by an example that the converse may not be true.

6. (a) Define absolute convergence and conditional convergence of an infinite series. Give one-one example of each with justification.

(b) State Cauchy's principle of convergence. Show that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ does not converge.

(c) State Raabe's test for the convergence of a positive term series. Use it to test the convergence of the series $\sum_{n=1}^{\infty} \frac{1.3.5...(2n-1)}{2.4.6...2n} \cdot \frac{1}{n}$.

7

[This question paper contains 16 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 2261 **I**

Unique Paper Code : 2342573501

Name of the Paper : Database Management Systems

Name of the Course : **B.A. (Prog.) /B.Sc.(P) /B.Sc. Maths. Sc. (NEP-UGCF)**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.

2. All parts of **Question 1 (Section A)** are compulsory.

3. Attempt **any four** questions from **Section B.**

4. All questions in **Section B** carry equal marks.

5. Attempt **all** parts of a question together.

P.T.O.

SECTION A

1. (a) Give a one-word answer for the following: (4)

   (i) Number of tuples in a relation

   (ii) An entity that doesn't have a primary key of its own

   (iii) Minimal superkey

   (iv) An attribute that can take multiple values

   (b) Consider the relation: (2)

   Laptop (Model, clockFrequency, RAM, SSDCapacity, price) Find the degree of the relation. Are all the attributes of the relation Laptop atomic?

   (c) Given relational schema **R(P, Q, R, S, T)** and the set of functional dependencies denoted by **FD = {P->QR, RS->T, Q->S, T-> P}**. Show the steps to compute the closure of **T** **(T^+)**. (3)

   (d) Consider the relation:

X	Y	Z
x1	y1	z1
x2	y1	z1
x1	y1	z2
x2	y1	z3

(i) List two functional dependencies that this relation state satisfies.

(ii) Assume that the value of attribute **z** changes from **z3** to **z2**. Now list one functional dependency that this relation instance satisfies. (3)

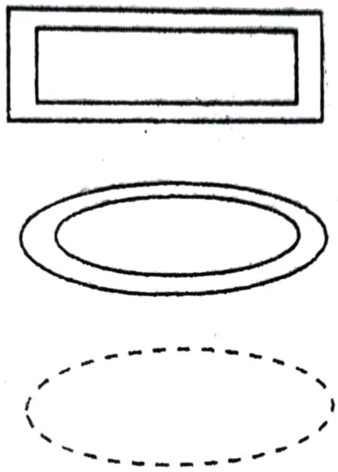
(f) Decompose the table into 1NF:

Roll No	Name	City	Phone No
1	Subodh	Delhi	11111111111,22222222222
2	Jitesh	Goa	33333333333
3	Sunil	Jaipur	44444444444,55555555555
4	Piyush	Rohtak	77777777777

(3)

P.T.O.

(g) What do these symbols represent in an ER Diagram? (3)



(h) Consider the following two relations, **M** and **N**:

M

X	Y	Z
5	3	6
10	7	9
15	2	7

N

P	Q	R
5	10	6
11	7	12
15	2	7

(3)

The attributes X, Y, and Z of relation M are union-compatible with the attributes P, Q, and R of relation N, respectively. What will be the result of the relational algebra expressions:

(i) $M \bowtie_{(M.X = N.P)} N$

(ii) $M \cap N$

(iii) $M \cup N$ (3)

(i) Consider a relation- R (V, W, X, Y, Z) with functional dependencies:

VW → XY
Y → V
WX → YZ

Find three possible candidate keys. (3)

(j) Consider the following relational schema:

Employee (empNo, name, office, age)

Books (isbn, title, authors, publisher)

Issued (empNo, isbn, date)

P.T.O.

Write an SQL query to perform the following operations:

(a) Find the names of employees who have issued a book published by **CS_Press.**

(b) Find the names of employees who have issued the maximum number of books.

(c) Find the titles of books issued by employees whose name ends with the letter **'m'.** (6)

SECTION B

2. (a) Differentiate between Entity set and Entity type with the help of suitable example(s). (2)

(b) Describe the three-schema architecture of database design with the help of a diagram. (3)

(c) A Company maintains the following tables for its Database:

3.

Table	Attributes
Executive	E_Id, Name, Address, Salary, Date_of_Joining,
Product	P_Id, Name, Price
Customer	C_Id, Name
Order	Invoice_No, Date_of_Purchase, P_Id, E_Id, C_Id, Quantity

Construct an ER diagram for the above database. An invoice is given by the executive. Each executive can write many invoices but each invoice is written by only one executive. The invoice is written for a single customer but each customer can have many invoices. Clearly specify cardinality ratios and participation constraints. State any assumptions that you make for drawing an ER diagram. (10)

3. (a) Suggest cardinality ratios of the given relationships based on common meaning. In each case, justify your answer.

Entity 1	Entity 2	Relationship name
Student	Course	Enrols
Player	Sports	Play
State	Chief Minister	Governs
Customer	Product	Purchases
Person	Mobile	Owns

(5)

P.T.O.

(b) Consider the following relation:

STUDENT_COURSE

Stud NO	CourseID	SNAME	PHONE	AGE	CourseName
1	C1	RAM	9716271721	20	DBMS
2	C2	JATIN	9898291281	19	HTML
3	C1	SUJIT	7898291981	18	DBMS
4	C3	SUMAN	9899307318	21	C++

Does the following operation result in any modification anomaly? Justify your answer. Insert into **STUDENT _COURSE**

values (4,'C4','Mohit', 9876546788,20,' CN')

(2)

(c) Write the relational algebra expression to perform the following operations on the relation schema given below:

EMPLOYEE

Empno	EName	Address	Salary	Gender	Dno

DEPARTMENT

Dnum	DName	Manager

(i) Retrieve the details of all employees who either work in department number 4 and earn at least 25,000, or work in department number 5 and earn at least 30,000.

(ii) Retrieve the first name, last name, and salary of all employees who work in department number 5.

(iii) Retrieve the names of managers for each department.

(iv) Display the name and the address of female employees. (8)

4. (a) Consider the two relations given below:

STU_DETAILS (RNO, SNAME, DOB, ADDR1, ADDR2, CITY, COUNTRY) MARK_DETAILS (RNO, MARKS1, MARKS2, MARKS3, TOTAL)

P.T.O.

Write SQL statements to perform the following:

(i) To display the sum of the MARKS1 of all the students.

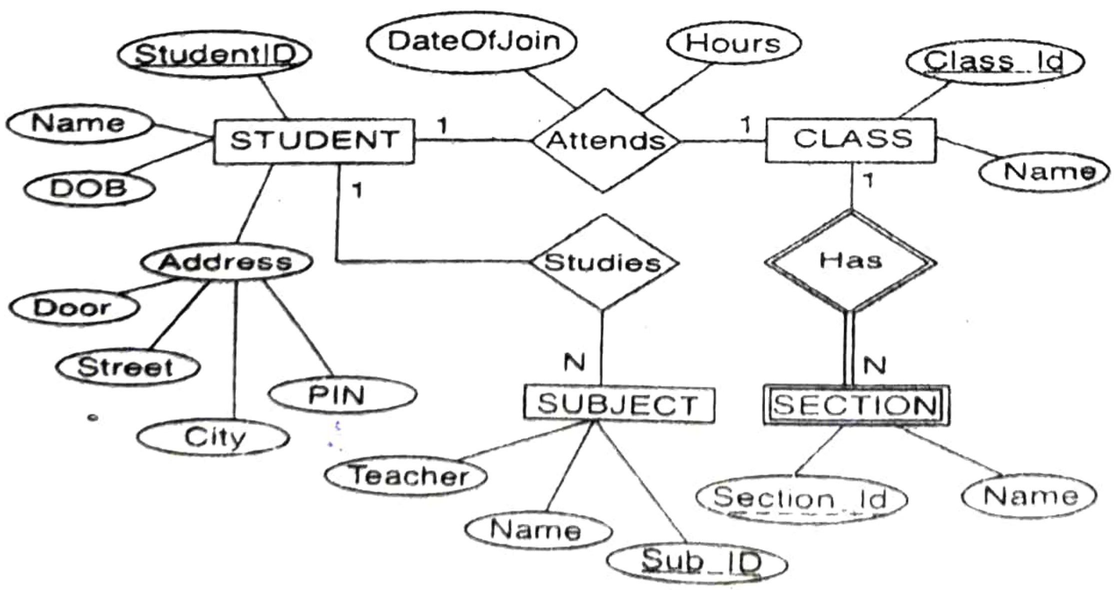
(ii) To display the names of all the students who scored more than 50 marks in MARKS 2.

(iii) Display record of all students whose total marks are greater than the total marks of the student whose name is 'AMAN'.

(iv) Display total marks of the youngest student.

(v) Using appropriate command, remove the column COUNTRY from STU_DETAILS. (10)

(b) Consider the following entity relationship diagram (ERD):



2261

5. (a)

(I

Map the ERD into relations. Specify the relations (tables) and identify the primary key and foreign key of each relation. (5)

5. (a) Consider a relation **R (A, B, C, D, E, F, G, H, I, J)** and the set of functional dependencies(10)

F = { AB -> C, BD -> EF, AD->GH, G -> I, H -> J}.

Find all possible candidate keys of R. Also, give an example of a super key of **R.**

Check whether **R** is in 2NF. If not, decompose the relation **R** into a set of 2NF relations. Check whether **R** is in 3NF. If not, Decompose the relation **R** into a set of 3NF relations. (5)

(b) **Consider the relation Employee:**

Employee (empNo, empName, street, empCity, salary)

(i) Rename the attribute **empNo** of the Employee table to **EID.**

(ii) Add an attribute for the phone number.

(iii) Change the city of the employee whose **empName** is **'Anil'.**

(iv) Delete the record of all employees who live in **Mumbai.**

(v) Insert a new employee

('E009', 'XYZ', 'North', 'Delhi',25000) (5)

6. (a) Consider the Relation PERSON (5)

PERSON_ID	NAME	AGE	ADDRESS
1	Sanjay	35	Patel Nagar
2	Sharad	30	Mayur Vihar
3	Vibhu	36	North Campus

Check if any of the constraints gets violated, if the following operations are performed on the above tables. Consider each operation independent of others and justify your answers.

1. **Insert <1, 'Vipin', 20, 'Mayur Vihar'>**

2. **Insert<'null' 'Anurag', 25, 'Patparganj'>**

3. **Insert<' abc', 'Suman', 25, 'IP college'>**

4. **Insert <10, 'Anu', 25, 'Patparganj'>**

5. **Insert <7, 'ajay', '25', 'Patparganj '>**

(b) Consider the supplier database **(6)**

Supplier

SNO	SNAME	STATUS	CITY
S1	Prentice Hall	30	Calcutta
S2	McGraw Hill	30	Mumbai
S3	Wiley	20	Chennai
S4	Pearson	40	Delhi
S5	Galgotia	10	Delhi

SupplyPart

SNO	PNO	Quantity
S1	P1	300
S1	P2	200
S2	P1	100
S2	P2	400
S3	P2	200
S4	P2	200

What is the output on the execution of following SQL queries?

(a) **SELECT SNO FROM** Supplier

WHERE STATUS > 20 **AND** CITY **LIKE** 'C%';

(b) **SELECT SNO, STATUS**

FROM Supplier

WHERE CITY = 'Delhi'

ORDER BY STATUS;

7.

(c) **SELECT DISTINCT SNAME**

FROM Supplier S, SupplyPart P

WHERE Quantity **BETWEEN** 100 **AND** 200 AND S.SNO=P.SNO;

(c) Consider the relation: **(4)**

P	Q	R	S	T
P1	Q1	50	S1	8
P2	Q2	30	S3	3
P3	Q3	50	S3	4
P1	Q1	50	S2	4
P1	Q3	50	S1	8

Which of the following functional dependencies does not hold for the above relation? Justify.

(i) Q → R

(ii) PQ→S

(iii) PQ→T

7. (a) Consider a file having 20,000 STUDENT records of fixed length. Each record has the (5) following fields: ID# (7 bytes), name (25 bytes), address (50 bytes), dob (5 bytes) and course (12 bytes).). An additional byte is used as a deletion marker. This file is stored on a disk of block size B:512 bytes. Compute the following: (5)

(i) record length (R)

(ii) blocking factor (bfr)

(iii) number of file blocks (b)

(iv) number of block accesses required during binary search on the data

(b) Define four desirable properties of a Transaction, that should be enforced by the Concurrency control? (4)

P.T.O.

(c) Consider the following interleaved transacti
and T2, executed concurrently. Assuming the
value X=5 and Y=10.

T1	T2
Read_item(X)	
X= X+10	
	Read_item(X)
write_item (X)	
	X=X+Y
	write_ item (X)
Read_item(X)	

Compute the value of X after the given schedul
executed. Is the value of X correct? If we rem
the interleaving between the transactions T1 and
what will be the value of X?

(28)

[This question paper contains 16 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 2281 **I**

Unique Paper Code : 2342573501

Name of the Paper : Database Management Systems

Name of the Course : **B.A. (Prog.) / B.Sc. (P) / B.Sc. Maths. Sc. (NEP-UGCF)**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All parts of **Question 1 (Section A)** are compulsory.
3. Attempt any **four** questions from **Section B**.
4. **All** questions in **Section B** carry equal marks.
5. Attempt **all** parts of a question together.

SECTION A

1. (a) Draw the Entity Relationship diagram (ERD) representation of the following complex attribute : (3)

P.T.O.

Address_phone({Phone(Area_code,Phone_number)}, Address(Street_address (Number,Street,Apartment_number),City,State,Zip))

(b) List three reasons that lead to the occurrence of NULL values in relations. (3)

(c) Consider relational schema R(A, B, C, D, E, F, G) and a set of functional dependencies

F = {AB->C, CD->E, EF->G, FG->E, DE->C, BC->A}.

Which amongst the following would form a candidate key for R?

(i) BDF

(ii) ACDF

(iii) ABDFG

(iv) BDFG (3)

(d) Consider the following tables R and S : (3)

R

X	Y	Z
10	a	7
25	b	8
30	a	9

S

A	B	C
25	c	7
30	d	8
25	c	7
10	a	7

The attributes X, Y, and Z of relation R are domain-compatible with the attributes A, B, and C of relation S, respectively. What will be the output produced on executing the following relational algebraic expressions?

(i) $R \bowtie_{R.X = S.A} S$

(ii) R X S

(iii) $R \cap S$

(e) Consider the following relation : (3)

P	Q	R	S	T
P1	Q1	50	S1	20
P2	Q2	60	S2	20
P3	Q1	70	S3	40

Which of the following functional dependencies are violated based on the above relation state? Justify your answer.

(i) Q->S

(ii) S->R

(iii) T->Q

(f) Explain the following operations on a file in database systems : (3)

(i) Open

(ii) Scan

(g) Differentiate between DELETE and DROP statements of SQL with the help of an example of each. (3)

(h) Consider the following relations : (3)

SUPPLIER		
Sid	Sname	Pnumber
101	A	P1
102	B	P2
103	C	P1

PART	
Pno	Pname
P1	X
P2	Y
P3	Z

Sid is the primary key of SUPPLIER and Pno is the primary key of PART. Pnumber is a foreign key in SUPPLIER relation. For each of the following operations, which database constraints may be violated? Justify your answer.

(i) Insert <102, 'W', 'P3'> in SUPPLIER relation.

(ii) Delete the tuple <'P2', 'Y'> from PART relation.

(iii) Update the value of Pname from 'X' to 10 in PART relation.

(i) Translate the following relational algebraic expressions defined on two relations R(a, b, c) and S(a, <u>d</u>, e, f), into their equivalent SQL statements : (3)

(i) $\pi_{c,a}(\sigma_{b=20 \text{ and } c=5}(R))$

(ii) $\pi_{b,d}(\sigma_{c=5}(R * S))$

(j) What is the role of data independence in three-schema architecture? Differentiate between physical data independence and logical data independence. (3)

SECTION B

2. (a) Consider a MAIL ORDER database in which employees take orders for parts from customers. The data requirements are summarized as follows : (10)

- The mail order company has employees, each identified by a unique employee number, first and last name, and zipcode.

- Each customer of the company is identified by a unique customer number, first and last name, and zipcode.

- Each part sold by the company is identified by a unique part number, a part name, price, and quantity in stock.

- Each order placed by a customer is taken by an employee and is given a unique order number. Each order contains specified quantities of one or more parts. Each order has a date of receipt as well as an expected ship date. The actual ship date is also recorded.

Design an entity-relationship diagram for the mail order database.

(i) Specify all entities and their attributes.

(ii) Specify primary key and foreign key.

(iii) Specify cardinality ratio and participation constraints on each relationship type.

(b) Consider the following relation : (3)

LIBRARY

AccountID	BookName	Days
212	Database-Korth	20
323	OS-Galvin	21
545	CSA-Morris Mano	14

AccountID is a primary key.

Identify whether the following statements violate the uniqueness constraint or the entity integrity constraint. Justify your answer:

(i) Insert into LIBRARY values(323, "Compiler Design", 7)

(ii) Insert into LIBRARY values(551, "Algorithms", 7)

(iii) Insert into LIBRARY values(NULL, "Hello World", 10)

(c) Write SQL statement to create a table EMPLOYEE_COPY to back up the EMPLOYEE table. (Assume that EMPLOYEE table already exists). (2)

3. (a) Consider the following relations for a database that keeps track of student enrollment in courses and the books adopted for each course: (10)

STUDENT (Ssn, Name, Major, Bdate)

COURSE (Course#, Cname, Dept)

ENROLL (Ssn, Course#, Quarter, Grade)

BOOK_ADOPTION (Course#, Quarter, Book_isbn)

TEXT (Book_isbn, Book_title, Publisher, Author)

Write relational algebra expressions for the following :

(i) List the number of courses taken by all students named John Smith in Winter 2009 (i.e., Quarter=W09).

(ii) Produce a list of textbooks (include Course#, Book_isbn, Book_tide) for courses offered by the 'CS' department that have used more than two books.

(iii) List department(s) that has adopted books published by 'Pearson Publishing'.

(iv) List the names of all those students who have "A" in their names.

(v) Display name of the youngest student.

(b) What will be the result of the following relational algebra queries on the relations M and N given below : (5)

(i) M $\bowtie$ $_{(M.X = N.P)}$ N

(ii) M $⟕$ $_{(M.Y = N.Q)}$ N

(iii) M $⟗$ $_{(M.X = N.P \text{ and } M.Y = N.Q)}$ N

(iv) M E N

(v) M $\cap$ N

M

X	Y	Z
5	3	6
10	7	9
5	2	7

N

P	Q	R
5	10	6
10	7	12
15	2	7

4. (a) Consider the following relation schema : (10)

Employee (Eid, cname. street, city)

Works (Eid, company-name, salary)

Company (company-name, city)

Manages (Eid, manager-name)

Write SQL queries for the following :

(i) Find the names, street address, and cities of residence for all employees who work for 'First Bank Corporation' and earn more than $10,000.

(ii) Find the names of all employees in the database who live in the same cities as the companies for which they work.

(iii) Find the names of all employees who live in the same cities and on the same streets as do their managers.

(iv) Find the names of all employees who do not work for 'First Bank Corporation'.

(v) Assume that the companies may be located in several cities. Find all companies located in every city in which 'Small Bank Corporation' is located.

(b) Suggest cardinality ratios of the given relationships based on common meaning. In each case, justify your answer. (5)

Entity 1	**Entity 1**	**Relationship name**
Teacher	Course	Teaches
Student	Book	Issues
Customer	Product	Purchase
Citizen	Aadhar Card	Has
Person	Bank Account	Owns

5. (a) Consider a relation R(A, B, C, D, E, F, G, H, I, J) and the set of functional dependencies (10)

$F = \{AB \rightarrow C, A \rightarrow DE, B \rightarrow F, F \rightarrow GH, D \rightarrow IJ\}$.

(i) Find two possible candidate keys of R. Also, give an example of a super key of

R.

(ii) Check whether R is in 2NF. If not, decompose the relation R into a set of 2NF relations.

(iii) Check whether R is in 3NF. If not, decompose the relation R into a set of 3NF relations.

(b) Consider the following database containing information of various branches of a company and staff at each branch. (5)

Branch (branchNo, street, city, postcode)

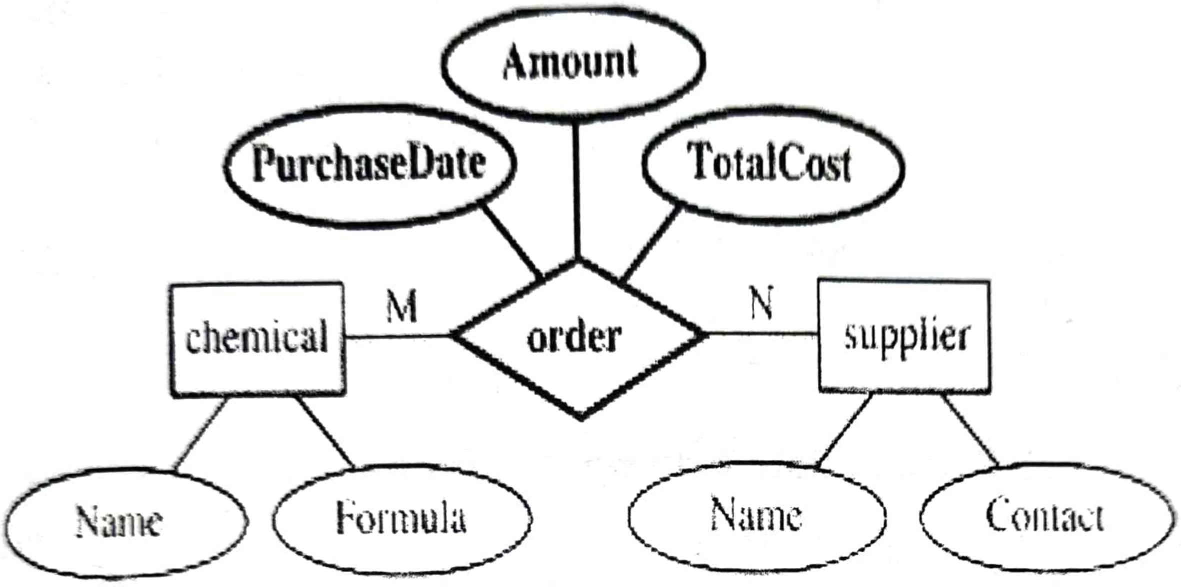
Staff(staffNo, fname, lname, position, salary,branchNo)

Write the following queries on the above mentioned database using SQL :

(i) Create tables Branch and Staff. Also define primary and foreign key.

(ii) Add an attribute phone number in Branch.

6. (a) Consider the following entity relationship diagram (ERD) : (6)



Map the Entity Relationship Diagram into relation tables. Specify the primary key and foreign key of each relation.

(b) Consider the relation CUSTOMER given below : (3)

Cust_Code	Cust_Name	Cust_City	Qty
1	A	AA	100
2	B	BB	600
3	C	BB	450

What will be the output of the following queries :

P.T.O.

(i) SELECT Cust_Code from CUSTOMER ORDER BY Qty DESC;

(ii) SELECT Cust_Name from CUSTOMER WHERE 'Cust_City= (SELECT Cust_city from CUSTOMER WHERE Cust_Code=2);

(iii) SELECT Qty+10 as INC_QTY FROM CUSTOMER;

(c) Differentiate between the following with example : (6)

(i) Alter and Update

(ii) DDL and DML

(iii) Where and Having

7. (a) Consider a disk with block size B = 512 bytes. A block pointer is P = 6 bytes long, and a record pointer is PR = 7 bytes long. A file has r = 30,000 EMPLOYEE records of fixed length. Each record has the following fields: Name (30 bytes),Ssn (9 bytes), Department_code (9 bytes), Address (40 bytes), Phone (10 bytes), Birth_date (8 bytes), Sex (1 byte), Job_code (4 bytes), and Salary (4 bytes, real number). An additional byte is used as

a deletion marker. Assuming an unspanned organization, compute the following : (5)

(i) Record length (R)

(ii) Blocking factor (bfr)

(iii) Number of file blocks (b)

(iv) Number of block accesses required during binary search on the data

(b) What is meant by the concurrent execution of database transactions in a multiuser system? Discuss why concurrency control is needed, and give two informal examples. (4)

(c) Consider the following interleaved transactions T1 and T2, executed concurrently. Assume that the initial value of X=5 and N=7. (6)

T1	T2
1. Read_item(X)	
2. X=X-N	
3. write_item (X)	
	4. Read_item(X)
	5.X=X+N
	6. write_ item (X)
7. Read-item(X)	

Compute the value of X after the given schedule is executed. Is the value of X correct? If we remove the interleaving between the transactions T1 and T2, what will be the value of X?

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[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 2291 **I**

Unique Paper Code : 2352203501

Name of the Paper : Elements of Real Analysis

Name of the Course : **B.A. / B.Sc. (P) with Mathematics as Non-Major / Minor**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** the questions are compulsory.
3. Attempt any **two** parts from each question.
4. **All** questions carry equal marks.

1. (a) Define supremum and infimum of non-empty subsets of $\mathbb{R}$. Find sup and inf of the following sets:

(i) $S = \left\{1 - \frac{(-1)^n}{n} : n \in \mathbb{N}\right\}$

(ii) $T = \left\{1, \frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \dots\right\}$

(iii) $U = \left\{\frac{4n+3}{n} : n \in \mathbb{N}\right\}$.

(b) Define the absolute value of x.

Find all $x \in \mathbb{R}$ that satisfy the inequality $|x + 1| < |x - 1|$.

(c) Suppose A and B are non-empty bounded above subsets of $\mathbb{R}$ and let

$$A + B = \{a + b : a \in A, \ b \in B\}.$$

Show that $\sup(A + B) = \sup A + \sup B$.

2. (a) (i) If $x, y \in \mathbb{R}$, then

$$||x| - |y|| \leq |x + y|.$$

(ii) Prove that in any ordered field,

$$\frac{x}{1+x} < \frac{y}{1+y}, \text{ for } 0 \leq x < y.$$

(b) Let $A \subseteq \mathbb{R}$. If $u = \sup A$, then show that there exists a sequence $\{a_n\}$ of elements of A such that $a_n \to u$.

(c) Prove that if, $|a| < 1$, $\lim_{n\to\infty} a^n = 0$.

(d) Prove that for any fixed real number c, $\lim_{n\to\infty} \frac{c^n}{n!} = 0$.

3. (a) Define the convergence of a sequence $\{x_n\}$ and show that $\lim_{n\to\infty} \frac{1}{n} = 0$.

(b) Prove that if $\{x_n\}$ and $\{y_n\}$ are convergent sequences, then the sequence $\{x_n + y_n\}$ is convergent. Also show that

$\lim_{n\to\infty} (x_n + y_n) = \lim_{n\to\infty} x_n + \lim_{n\to\infty} y_n$. Is the converse true? Explain.

(c) Prove that a monotone sequence converges if and only if it is bounded.

4. (a) Show that

(i) $\lim_{n\to\infty} \frac{n}{x^n} = 0$, if $x > 1$.

(ii) If $k \in \mathbb{N}$, then $\lim_{n\to\infty} \frac{n^k}{k^n} = 0$, for $k \geq 2$.

(b) Consider the sequence $\{x_n\}$ defined inductively by $x_1 = 1$ and $\forall\, n \in \mathbb{N}$ $x_{n+1} = \sqrt{x_n + 2}$.

Show that $\{x_n\}$ converges and find its limit.

(c) State the Second Squeeze Principle. Use it to prove that $\lim_{n\to\infty} \left(\frac{3n^2 - 4n}{n^2 + 5}\right) = 3$.

5. (a) State Cauchy's nth Root test for the convergence of a positive term series. Apply this to test for convergence the series

$$\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-1}+\cdots$$

(b) Test the convergence of the series $\sum_{n=2}^{\infty}\frac{1}{n\log n}$.

(c) Test the convergence of the series :

(i) $\frac{x}{\sqrt{5}}+\frac{x^3}{\sqrt{7}}+\frac{x^5}{\sqrt{9}}+\cdots \qquad x>0.$

(ii) $\frac{x^2}{2\sqrt{1}}+\frac{x^3}{3\sqrt{2}}+\frac{x^4}{4\sqrt{3}}+\cdots \qquad x>0.$

6. (a) Test the conditional and absolute convergence of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt{n}}$.

(b) Prove that the series

$$x-\frac{x^3}{3}+\frac{x^5}{5}-\cdots$$

is absolutely convergent if and only if $|x|<1$ but conditionally convergent for $|x|=1$.

(c) Test the convergence of the series :

(i) $\frac{1}{3.7}+\frac{1}{4.9}+\frac{1}{5.11}+\cdots$

(ii) $\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{5}}+\cdots$

(3000)

(30)

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 2475 **I**

Unique Paper Code : 2353200008

Name of the Paper : Elements of Number Theory (DSE)

Name of the Course : **B.A.(Programme)**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.

2. Both parts of a question to be attempted together.

4. All questions carry equal marks.

5. Use of Calculator not allowed

6. Attempt all questions by selecting **two** parts from each question.

P.T.O.

1. (a) Given an odd integer a, establish that a_2 + $(a + 2)^2 + (a + 4)^2 + 1$ is divisible by 12. (7.5)

(b) Find gcd(143, 227) by Euclidean Algorithm and hence, by using it, find integral values x and y for which

gcd(143, 227) = 143 x + 227 y. (7.5)

(c) A farmer purchased 100 head of livestock for a total cost of \$4000. Prices were as follow: calves, \$120 each; lambs, \$50 each; piglets, \$25 each. If the farmer obtained at least one animal of each type, how many of each did he buy? (7.5)

2. (a) Find the remainder obtained upon dividing the sum 1! +2!+3!+...+ 99! + 100! by 12. (7.5)

(b) Solve the following simultaneous congruences

$x \equiv 5 (mod\ 6),\ x \equiv 4 (mod\ 11),\ x \equiv 3 (mod\ 17)$

(7.5)

(c) Find the solutions of the system of congruences

$7x + 3y = 10 (mod\ 16),\ 2x+5y= 9 (mod\ 16)$

(7.5)

3. (a) State and prove Fermat's little theorem. (7.5)

(b) Using Wilson's theorem, prove that for any odd prime p.

$$1^2.3^2.5^2 \ldots (p-2)^2 \equiv (-1)^{\frac{p+1}{2}} \ (mod\ p)$$

(7.5)

(c) Prove that if n is a perfect square then $\tau(n)$ is an odd integer and if n is a square free integer then $\tau(n)=2^r$, where r is the number of prime divisors of n (7.5)

4. (a) Find the number of zeros with which the decimal representation of 50! terminates. (7.5)

(b) Prove that Euler's Phi function $\varphi(n)$ is an even integer for $n > 2$. and verify this statement for n=1001. (7.5)

(c) Prove that if the integer n has r distinct odd prime factors, then 2^r divides $\phi(n)$. (7.5)

5. (a) Employ Hill's Cipher to decrypt the message KJJ QQM, using the congruences $C_1 \equiv 2P_1 + 3P_2$ (*mod* 26), $C_1 \equiv 5P_1 + 8P_2$ (*mod* 26) (7.5)

P.T.O.

(b) Encrypt the message RETURN HOME using Caesar Cipher. (7.5)

(c) Given that n is an even perfect number, for instance $n = 2^{k-1}(2^k - 1)$ show that the integer $n = 1+2+3+\ldots+(2^k-1)$ and also that $\phi(n) = 2^{k-1}(2^{k-1}-1)$.

(7.5)

6. (a) Write Mersenne number M_7. By using primality test, verify that it is a prime number and hence show that $n = 2^6(2^7-1)$ is a perfect number. (7.5)

(b) Prove that the Fermat Number $F_5 = 2^{2^5} + 1$ is divisible by 641. (7.5)

(c) Find the Fibonacci numbers that divide both u_{24} and u_{36} (7.5)

(1000)

(25)

[This question paper contains 16 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1456 **I**

Unique Paper Code : 2342573501

Name of the Paper : Database Management Systems

Name of the Course : **B.A. (Prog.)/B.Sc.(P) /B.Sc. Maths. Sc. (NEP-UGCF)**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.

2. All parts of **Question 1 (Section A)** are compulsory.

3. Attempt **any four** questions from **Section B.**

4. All questions in **Section B** carry equal marks.

5. Attempt **all** parts of a question together.

P.T.O.

SECTION A

1 (a) Give three advantages of the DBMS approach over the traditional file system approach. (3)

(b) Draw the Entity Relationship diagram and provide (min, max) constraint for the below-mentioned statement:

"Each course must be taught by at least one teacher and can be shared by four teachers in total. Each teacher can teach many courses" (3)

(c) Differentiate between the following:

(i) Key and Super key

(ii) Inner Join and Outer Join (3)

(d) Classify the following commands as Data Definition Language (DDL) or Data Manipulation Language (DML):

(i) ALTER TABLE

(ii) INSERT INTO

(iii) CREATE TABLE (3)

(e) Consider the tables T1 and T2 as follows:

T1

P	Q	R
2	20	c
1	10	b
3	30	c
1	40	a

T2

A	B	C
1	10	a
2	20	b
3	10	a
4	30	a

The domain of A, B, C are compatible with P, Q, R, respectively.

Show the result for each of the following:

(i) $T1 ⟕_{(T1.R = T2.C)} T2$

(ii) $T1 ⋈_{(T1.Q = T2.B)} T2$ (3)

P.T.O.

(f) Consider the following relation:

A	B	C	Tuple#
10	B1	C1	1
10	B2	C2	2
11	B4	C1	3
12	B3	C4	4
13	B1	C1	5
14	B3	C4	6

Given the relation state, which of the following dependencies will not hold? Justify your answer.

(i) A → B

(ii) B → C

(iii) C → B (3)

(g) Identify the rules for each of the following Armstrong's axioms.

(i) $\{A \rightarrow B\} \mid= AC \rightarrow BC$

(ii) $\{A \rightarrow B, B \rightarrow C\} \mid= A \rightarrow C$

(iii) $\{A \rightarrow B, A \rightarrow C\} \mid= A \rightarrow BC$ (3)

(h) Consider the relation R (A, B, C, D, E) with the following dependencies:

$AB \rightarrow C, CD \rightarrow E, DE \rightarrow B$

Is AB a candidate key of this relation? If not, is ABD? Explain your answer. (3)

(i) Consider a data file **STUDENT** (Sid, Sname, CourseNo, Dob, Address). Create the primary index (on Sid) on the above file diagrammatically. Is the above index file is dense or sparse? Justify your answer. (3)

(j) Consider a relation R (A, B, C, D, E) and the following set of functional dependencies:

F: $\{ B \rightarrow C, BC \rightarrow D, E \rightarrow BD, E \rightarrow A \}$

Find closure of the following under F:

(i) B

(ii) BC

(iii) E (3)

SECTION B

2. (a) Explain the three-schema architecture of DBMS. Also draw the example. (5)

(b) Consider a real estate database. The database stores information about the real estate firms in Delhi and the houses that are available for sale. The data about the customers looking to buy the houses is also maintained. The data requirements are as follows:

(i) The data stored for a house includes its registration ID, address, owner name, and sale price.

(ii) The house for sale can be listed with one or more firms. Being "listed" with a firm means the house owner has a contract with an agent who works for that firm.

(iii) Agents are uniquely identified by their AgentID. Their names and addresses are also recorded.

(iv) The deal of a house can only be worked on by a single agent. An agent can simultaneously work on more than one house for sale.

(v) A house may also have one or more rooms. Each room is represented by its type (bed, bath, drawing, dining), type of heating (electric, water, none) and size. The room is to be modelled as a weak entity.

Draw an Entity Relationship diagram for the given real estate database. The diagram should necessarily include the following:

(i) Entities and their attributes

(ii) Relationships

P.T.O.

(iii) Cardinality and participation constraints (10)

3. (a) Consider an ordered file with the number of records r = 30000 stored on a disk with block size B=1024 bytes. Find the blocking factor for the file, the number of blocks needed for the file and number of block accesses needed by a binary search on this data file. (5)

(b) Consider a database that consists of the following relations.

SUPPLIER (Sno, Sname)

PART (Pno, Pname)

PROJECT (Jno, Jname)

SUPPLY (Sno, Pno, Jno)

The database records information about suppliers, parts, projects and includes a ternary relationship SUPPLY between suppliers, parts, and projects. Write relational algebraic expressions to perform the following:

(i) Retrieve the part numbers that are supplied to exactly two projects.

(ii) Retrieve the names of suppliers who supply more than two parts to project 'J1'.

(iii) Retrieve the total number of suppliers.

(iv) Retrieve the project names that are supplied by supplier 'S1' only.

(v) Retrieve the part number and total project for each supply. (10)

4 (a) How is OUTER UNION operation different from UNION operation? Explain with an example. (5)

(b) Specify the following queries in SQL on the database schema

CUSTOMER (Cust#, Cname, Address)

ORDER (Order#, Odate, Cust#, Ord_amt)

P.T.O.

ORDER_ITEM (Order#, Item#, Qty)

ITEM (Item#, Unit_price)

SHIPMENT (Order#, Warehouse#, Ship_date)

WAREHOUSE (Warehouse#, City)

(i) List the Order# and Ship_date for all orders shipped from Warehouse# W2.

(ii) List the Warehouse# from which the customer named "ABC" was supplied his orders along with the Order#.

(iii) Produce a listing Cname, No_of_orders, Avg_order_amt, where the middle column is the total number of orders ordered by the customer and the last column is the average order amount for that customer.

(iv) List the orders that were not shipped within 30 days of ordering.

(v) List the customer names who lives in a city starting with a letter D. (10)

5. (a) Consider the relation R= {P, Q, R, S, T, U, V, W, X, Y) following set of functional dependencies

F: { $PQ \rightarrow R, QS \rightarrow TU, PS \rightarrow VW, P \rightarrow X, W \rightarrow Y$ }

Find Primary Key and decompose R into 2NF and then 3NF. (6)

(b) Give SQL command to create a table T having attributes A,B,C,D where:

- A is a number (maximum 10 digits in length) and cannot contain null values.
- B is a character string (50 maximum characters in length)
- (A,B) form the primary key
- C and D are integer values.
- D has a default value of 6

P.T.O.

- D is a foreign key referring to attribute E from another table S of the database (assuming S is already created) (6)

(c) Prove that any relation schema with two attributes is in BCNF. (3)

6. (a) Give an example and draw the corresponding symbol used in an Entity Relationship diagram for the following:

(i) Complex attribute

(ii) Key attribute of an entity

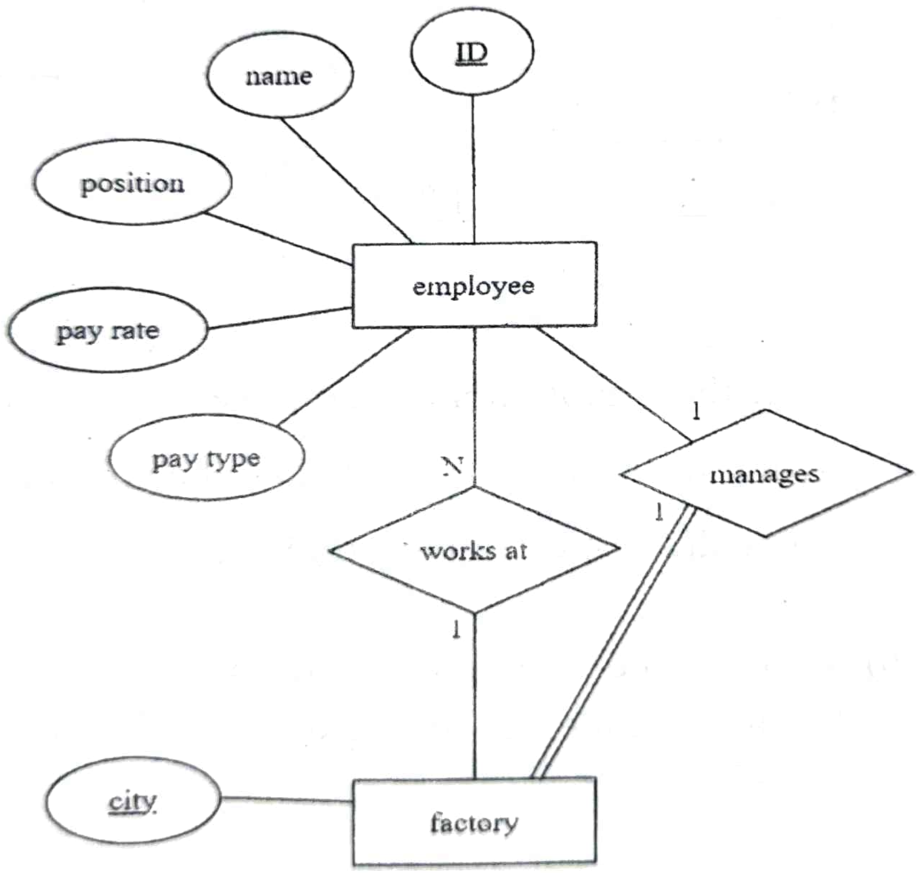
(iii) Derived attribute (6)

(b) Consider the relation R = (A, B, C, D, E, F) and the following set of functional dependencies.

$$\{C \rightarrow F,$$
$$E \rightarrow A,$$
$$EC \rightarrow D,$$
$$A \rightarrow B\}$$

Find the closure of E^+ and AC^+ (4)

(c) Map the following ER diagram into relational tables indicating primary Key and foreign key of each relation.



(5)

7. (a) Two transactions T1 and T2 are executing concurrently with initial values of P=50 and R= 5

T1	T2
read_item (P)	
P = P + 10	
	read_item (P)
	P = P + R
write_item (P)	
read_item (R)	
	write_item (P)

After the completion of the transaction T1 and T2 what will the value of P? Is this the correct value, if not then, specify the problem. (6)

(b) What is the significance of the Atomicity and Isolation properties amongst the ACID properties of transactions?. (4)

(c) Consider the following relations:

EMPLOYEE

Eno	Ename	Dno
1	Sakshi	11
2	Dhirti	22
3	Parth	33

DEPARTMENT

Dnum	Dname	Dloc
11	Research	Delhi
22	HR	Mumbai
33	Finance	Kolkata

Here Eno is the Primary key and Dno is the Foreign key of the EMPLOYEE relation. Dnum is the Primary key of Department relation.

For each of the following operations indicate, whether it results in constraint violation and if so, why?

(i) Insert <5, "Khushi", 10> in EMPLOYEE

(ii) Insert <4, "Samarth", 22> in EMPLOYEE

(iii) Delete <11, "Research", Delhi> from DEPARTMENT

(iv) Insert <10, "IT", Mumbai> in DEPARTMENT

(v) Modify <2, "Ayushi", 22> in EMPLOYEE

(5)