[This question paper contains 4 printed pages.]

Your Roll No...............

C

Sr. No. of Question Paper : 1381

Unique Paper Code : 32351301

Name of the Paper : BMATH 305 – Theory of Real Functions

Name of the Course : **CBCS (LOCF) B.Sc. (H) Mathematics**

Semester : III

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory.

1. (a) Let $A \subseteq \mathbb{R}$ and $c \in \mathbb{R}$ be a cluster point of A and $f : A \to \mathbb{R}$, then define limit of function f at c.

Use $\varepsilon - \delta$ definition to show that $\lim_{x \to 1} \frac{x}{x+1} = \frac{1}{2}$.

(6)

P.T.O.

(b) Let $f: A \to \mathbb{R}$, $A \subseteq \mathbb{R}$ and $c \in \mathbb{R}$ be a cluster point of A. Then show that $\lim_{x \to c} f(x) = L$ if and only if for every sequence $\langle x_n \rangle$ in A that converges to c such that $x_n \neq c$, $\forall n \in \mathbb{R}$, the sequence $\langle f(x_n) \rangle$ converges to L. (6)

(c) Show that $\lim_{x \to 0} \sin\left(\frac{1}{x^2}\right)$ does not exist in $\mathbb{R}$ but $\lim_{x \to 0} x^2 \sin\left(\frac{1}{x^2}\right) = 0$. (6)

2. (a) Let $A \subseteq \mathbb{R}$, $f: A \to \mathbb{R}$, $g: A \to \mathbb{R}$ and $c \in \mathbb{R}$ be a cluster point of A. Show that if f is bounded on a neighborhood of c and $\lim_{x \to c} g(x) = 0$, then $\lim_{x \to c} (fg)(x) = 0$. (6)

(b) Let $f(x) = e^{1/x}$ for $x \neq 0$, then find $\lim_{x \to 0} f(x)$ and $\lim_{x \to 0^+} f(x)$. (6)

(c) Let $f: \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} 2x & \text{if } x \text{ is rational} \\ x + 3 & \text{if } x \text{ is irrational}. \end{cases}$$

Find all the points at which f is continuous. (6)

3. (a) Let $A \subseteq \mathbb{R}$ and let f and g be real valued functions on A. Show that if f and g are continuous on A then their product f g is continuous on A. Also, give examples of two functions f and g such that both are discontinuous at a point $c \in A$ but their product is continuous at c. (7½)

(b) State and prove Boundedness Theorem for continuous functions on a closed and bounded interval. (7½)

(c) State Maximum-Minimum Theorem. Let $I = [a,b]$ and $f : I \to \mathbb{R}$ be a continuous function such that $f(x) > 0$ for each x in I. Prove that there exists a number $\alpha > 0$ such that $f(x) \geq \alpha$ for all x in I. (7½)

4. (a) Let $A \subseteq \mathbb{R}$ and $f : A \to \mathbb{R}$ such that $f(x) \geq 0$ for all $x \in A$. Show that if f is continuous at $c \in A$, then $\sqrt{f}$ is continuous at c. (6)

(b) Show that every uniformly continuous function on $A \subseteq R$ is continuous on A. Is the converse true? Justify your answer. (6)

(c) Show that the function $f(x) = \frac{1}{x^2}, x \neq 0$ is uniformly continuous on $[a, \infty)$, for $a > 0$ but not uniformly continuous on $(0, \infty)$. (6)

5. (a) Let $I \subseteq \mathbb{R}$ be an interval, let $c \in I$, and let $f : I \to \mathbb{R}$ and $g : I \to \mathbb{R}$ be functions that are differentiable at c. Prove that if $g(c) \neq 0$, the function f/g is differentiable at n, and

$$\left(\frac{f}{g}\right)'(c) = \frac{f'(c)g(c) - f(c)g'(c)}{(g(c))^2}.$$ (6)

(b) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = |x| + |x + 1|$, $x \in \mathbb{R}$. Is f differentiable everywhere in $\mathbb{R}$? Find the derivative of f at the points where it is differentiable. (6)

(c) State Mean Value Theorem. If $f: [a, b] \to \mathbb{R}$ satisfies the conditions of Mean Value Theorem and $f'(x) = 0$ for all $x \in (a,b)$. Then prove that f is constant on [a, b]. (6)

6. (a) Let I be an open interval and let $f : I \to \mathbb{R}$ have a second derivative on I. Then show that f is a convex function on I if and only if $f''(x) \geq 0$ for all $x \in I$. (6)

(b) Find the points of relative extrema of the functions $f(x) = |x^2 - 1|$, for $-4 \leq x \leq 4$. (6)

(c) Use Taylor's Theorem with n = 2 to approximate

$\sqrt[3]{1+x}$,, $x > -1$. (6)

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[This question paper contains 8 printed pages.]

Your Roll No..............

C

Sr. No. of Question Paper : 1409

Unique Paper Code : 32351302

Name of the Paper : BMATH306 – Group Theory-I

Name of the Course : **B.Sc. (Hons) Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question from **Q2** to **Q6**.
4. In the question paper, given notations have their usual meaning unless until stated otherwise.

P.T.O.

1. Give short answers to the following questions. Attempt any six.

(i) What is the total no of rotations and total no of reflections in the dihedral group D_3? Describe them (rotations and reflections) in pictures or words. What can you say about the total no of rotations and total no of reflections in the dihedral group D_n?

(ii) Give one non- trivial, proper subgroup of GL(2, R). Is GL(2, R) a group under addition of matrices? Answer in few lines.

(iii) Let G be a group with the property that for any a, b, c in G,

ab = ca implies b = c. Prove that G is Abelian.

(iv) Give an example of a cyclic group of order 5. Show that a group of order 5 is cyclic.

(v) Prove that a cyclic group is Abelian. Is the converse true?

(vi) Find all subgroups of Z_{15}.

(vii) Prove that 1 and -1 are the only two generators of (Z,+). Give short answer in few lines.

(viii) "Z_n, $n \in N$, is always cyclic whereas U(n), $n \in N$; $n \geq 2$ may or may not be cyclic". Prove or disprove the statement in a few lines. (6×2=12)

2. (a) Let $G = \{a + b\sqrt{2} \mid$ a and b are rational nos not both zero$\}$

Prove that G is a group under ordinary multiplication. Is it Abelian or Non-Abelian? Justify your answer.

(b) Prove that a group of composite order has a non-trivial, proper subgroup.

(c) Prove that order of a cyclic group is equal to the order of its generator. (2×6.5=13)

3. (a) Prove that every permutation of a finite set can be written as a cycle or as a product of distinct cycles. (6)

(b) (i) In S_4, write a cyclic subgroup of order 4 and a non-cyclic subgroup of order 4.

(ii) Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 4 & 8 & 6 & 7 & 5 & 1 & 3 \end{bmatrix}$ and

$$\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 4 & 5 & 1 & 8 & 3 & 2 & 6 \end{bmatrix}$$

Write α, β and αβ as product of 2-cycles. (3+3=6)

(c) (i) Let $|a| = 24$. How many left cosets of $H = \langle a^4 \rangle$ in $G = \langle a \rangle$ are there? Write each of them.

(ii) State Fermat's Little theorem. Also compute 5^{25} mod 7 and 11^{17} mod 7. (3+3=6)

4. (a) (i) Let H and K be two subgroups of a finite group. Prove that

$HK \leq G$ if G is Abelian.

(ii) Give an example of a group G and its two subgroups H and K ($H \neq K$) such that HK is not a subgroup of G. (3+3.5=6.5)

(b) (i) Let G be a group and let Z (G) be the centre of G. If G/Z (G) is cyclic, prove that G is Abelian.

(ii) Let $|G| = pq$, p and q are primes. Prove that $|Z(G)| = 1$ or pq.

(4+2.5=6.5)

(c) (i) Prove that a subgroup of index 2 is normal.

(ii) Let $G = U(32)$, $H = U_8(32)$. Write all the elements of the factor group G/H. Also find order of $3H$ in G/H.

(3+3.5=6.5)

5. (a) Show that the mapping from $\mathbb{R}$ under addition to $GL(2, \mathbb{R})$ that takes x to $\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ is a group homomorphism. Also, find the kernel of the homomorphism.

(b) Let ϕ be a homomorphism from a group G to a group $\overline{G}$. Show that if $\overline{K}$ is a subgroup of $\overline{G}$, then $\phi^{-1}(\bar{K}) = \{k \in G : \phi(k) \in \bar{K}\}$ is a subgroup of G.

(c) If H and K are two normal subgroups of a group G such that $H \subseteq X$, then prove that

$$G/K \approx \frac{G/H}{K/H}.$$

(2×6=12)

6. (a) Show that the mapping ϕ from $\mathbb{C}^*$ to $\mathbb{C}^*$ given by $\phi(z) = z^4$ is a homomorphism. Also find the set of all the elements that are mapped to 2.

(b) Prove that every group is isomorphic to a group of permutations.

(c) Let G be the group of non-zero complex numbers under multiplication and N be the set of complex numbers of absolute value 1.

Show that G/N is isomorphic to the group of all the positive real numbers under multiplication.

(2×6.5=13)

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[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : **1427** C

Unique Paper Code : 32351303

Name of the Paper : BMATH 307 – Multivariate Calculus

Name of the Course : **B.Sc. (H) Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** sections are compulsory
3. Attempt any **Five** questions from each section. All questions carry equal marks

SECTION I

1. Let $f(x,y)=\dfrac{xy\left(x^2-y^2\right)x}{x^2+y^2}$ if $(x, y) \neq (0,0)$

$= 0$ otherwise

Show that $f(0, y) = -y$ and $f(x, 0) = x$ for all x and y.

P.T.O.

$$\vec{F}(x,y)=(3y-4x)\hat{i}+(4x-y)\hat{j}$$

when an object moves once counterclockwise around the ellipse $4x^2 + y^2 = 4$.

4. Use Stoke's theorem to evaluate the surface integral

$$\iint_S \left(\text{curl } \vec{F}.N\right) dS$$

where $F = x\,i + y^2 j + z\,e^{xy}\,k$ and S is that part of surface $z = 1 - x^2 - 2y^2$ with $z \geq 0$.

5. Use divergence theorem to evaluate the integral $\iint_S \vec{F}.N\, dS$ where $\vec{F}(x,y,z)=(\cos yz)\hat{i}+e^{xz}\hat{j}+3z^2\hat{k}$, where S is hemisphere surface $z=\sqrt{4-x^2-y^2}$ together with the disk $x^2 + y^2 \leq 4$, in x-yplane.

6. Evaluate the line integral $\int_C \vec{F}.d\vec{R}$

Where $\vec{F}(x, y) = [(2x - x^2 y)\, e^{-xy} + \tan^{-1} y]\,\hat{i} + \left[\frac{x}{y^2+1} - x^3 e^{-xy}\right]\hat{j}$ and C is the ellipse $9x^2 + 4y^2 = 36$.

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[This question paper contains 8 printed pages.]

Your Roll No...............

C

Sr. No. of Question Paper : 1013

Unique Paper Code : 32351501

Name of the Paper : BMATH511 – Metric Spaces

Name of the Course : **B.Sc. (Hons.) Mathematics** CBCS (LOCF)

Semester : V

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.

1. (a) Let (X, d) be a metric space. Define the mapping $d^*: X \times X \to \mathbb{R}$ by

$$d^*(x,y) = \frac{d(x,y)}{1+d(x,y)};\ \forall x, y \in X.$$

P.T.O.

Show that (X, d^*) is a metric space and $d^*(x, y) < 1$, for every $x, y \in X$. (6)

(b) Let $\langle x_n \rangle_{n\geq 1}$ be a sequence of real numbers defined by $x_1 = a$, $x_2 = b$ and $x_{n+2} = \frac{1}{2}(x_{n+1} + x_n)$ for $n = 1, 2, \cdots$. Prove that $\langle x_n \rangle_{n\geq 1}$ is a Cauchy sequence in $\mathbb{R}$ with usual metric. (6)

(c) Define a complete metric space. Is the metric space $(\mathbb{Z}, d)$ of integers, with usual metric d, a complete metric space? Justify. (6)

2. (a) (i) Let (X, d) be a metric space. Show that for every pair of distinct points x and y of X, there exist disjoint open sets U and V such that $x \in U$, $y \in V$. (2)

(ii) Give an example of the following :

(a) A set in a metric space which is neither a closed ball nor an open set. (1)

(b) A metric space in which the interior of the intersection of an arbitrary family of the subsets may not be equal to the intersection of the interiors of the members of the family. (2)

(c) A metric space in which every singleton is an open set. (1)

(b) Let (X, d) be a metric space. Let A be a subset of X. Define closure of A and show that it is the smallest closed superset of A. (6)

(c) Let (X, d) be a complete metric space. Let $\langle F_n \rangle$ be a nested sequence of non-empty closed subsets of X such that $d(F_n) \to 0$. Show that $\bigcap_{n=1}^{\infty} F_n$ is a singleton. Does it hold if (X, d) is incomplete? Justify.

(6)

3. (a) Let (X, d_X) and (Y, d_Y) be metric spaces and $f: X \to Y$ be a function. Prove that f is continuous on X if and only if $f(\bar{A}) \subseteq \overline{f(A)}$ for all subsets A of X.

(6)

(b) Let A and B be non-empty disjoint closed subsets of a metric space (X, d). Show that there is a continuous real valued function f on X such that $f(x) = 0, \forall x \in A$, $f(x) = 1, \forall x \in B$ and $0 \leq f(x) \leq 1$, $\forall x \in X$. Further show that there exist disjoint open subsets G, H of X such that $A \subseteq G$ and $B \subseteq H$.

(6)

(c) Define a dense subset of a metric space (X, d). Let $A \subseteq X$. Show that A is dense in X if and only if A^C has empty interior. Give an example of a metric space that has only one dense subset.

(6)

4. (a) Show that the metrics d_1, d_2 and d_∞ defined on $\mathbb{R}^n$ by

$$d_1(x,y) = \sum_{i=1}^{n} |x_i - y_i|,$$

$$d_2(x,y) = \left(\sum_{i=1}^{n} (x_i - y_i)^2\right)^{1/2} \text{ and}$$

$$d_\infty(x,y) = \max\{|x_i - y_i| : 1 \le i \le n\}$$

are equivalent where $x = (x_1, x_2, \cdots, x_n)$ and $y = (y_1, y_2, \cdots, y_n)$.

(6.5)

(b) Show that the function $f: \mathbb{R} \to (-1, 1)$ defined by

$f(x) = \frac{x}{1+|x|}$ is a homeomorphism but not an isometry.

(6.5)

(c) (i) Let (X, d) be a complete metric space. Let T: X → X be a mapping such that $d(Tx,Ty) < d(x, y)$, ∀ x, y ∈ X. Does T always have a fixed point? Justify. (4)

(ii) Let X be any non-empty set and T: X → X be a mapping such that T^n (where n is a natural number, $n > 1$) has a unique fixed point $x_0 \in X$. Show that x_0 is also a unique fixed point of T. (2.5)

5. (a) Let (ℝ, d) be the space of real numbers with usual metric. Prove that a connected subset of E must be an interval. Give an example of two connected subsets of E, such that their union is disconnected. (4+2.5)

(b) Let (X, d) be a metric space such that every two points of X are contained in some connected subset of X. Show that (X, d) is connected.

(6.5)

(c) Let (X, d) be a metric space. Then prove that (X, d) is disconnected if and only if there exists a continuous mapping of (X, d) onto the discrete two element space (X_0, d_0). (6.5)

6. (a) Prove that homeomorphism preserves compactness. Hence or otherwise show that

$$S(0,1) = \{z \in \mathbb{C} : |z| < 1\} \text{ and}$$

$$S[0,1] = \{z \in \mathbb{C} : |z| < 1\}$$

are not homeomorphic. (4+2.5)

(b) Let (X, d) be a metric space and $A \subseteq X$ such that every sequence in A has a subsequence converging in A. Show that for any $B \subseteq X$, there is a point $p \in A$ such that $d(p,B) = d(A,B)$. If B be a closed subset of X such that $A \cap B = \phi$, show that $d(A,B) > 0$. (4.5+2)

(c) Let f be a continuous real-valued function on a compact metric space (X, d_X), then show that f is bounded and attains its bounds. Does the result hold when X is not compact? Justify.

(4+2.5)

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[This question paper contains 4 printed pages.]

Your Roll No...............

D

Sr. No. of Question Paper : 1027

Unique Paper Code : 2352011101

Name of the Paper : DSC-1 : Algebra

Name of the Course : **B.Sc. (H) Mathematics, UGCF-2022**

Semester : I

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory and carry equal marks.
3. Attempt any **two** parts from each question.

1. (a) Prove that one root of $x^3 + px^2 + qx + r = 0$ is negative of another root if and only if $r = pq$. (7.5)

   (b) Solve $x^4 - 2x^3 - 21x^2 + 22x + 40 = 0$, whose roots are in arithmetical progression. (7.5)

(c) Find all the integral roots of

$$x^7 + 2x^5 + 4x^4 - 8x^2 - 32 = 0.$$ (7.5)

2. (a) Find the polar representation of the complex number

$$z = \text{Sin } a + i(1 + \text{Cos } a),\ a \in [0, 2\pi)$$ (7.5)

(b) Find $|z|$ and arg z for $z = \dfrac{\left(2\sqrt{3}+2i\right)^8}{(1-i)^6} + \dfrac{(1+i)^6}{\left(2\sqrt{3}-2i\right)^8}$

(7.5)

(c) Find the geometric image for the complex number z such that

$$|z + 1 + i| < 3 \text{ and } 0 < \arg z < \frac{\pi}{6}$$ (7.5)

3. (a) Let $U_n = \{\epsilon_0, \epsilon_1, \epsilon_2, \ldots \epsilon_{n-1}\}$ be the set of n^{th} roots of unity, where $\epsilon_k = \cos(2k\pi/n) + i \sin(2k\pi/n)$, $k \in \{0, 1, 2, \ldots, n-1\}$. Prove the following :

(i) Prove that $\epsilon_j.\varepsilon_k \in U_n$ for all $j, k \in \{0, 1, 2, \ldots, n-1\}$

(ii) $\epsilon_j^{-1} \in U_n$ for all $j \in \{0, 1, 2, \ldots, n-1\}$

(4+3.5)

(b) Let a be an integer, show that there exists an integer k such that

$$a^2 = 3k \text{ or } a^2 = 3k + 1. \qquad (7.5)$$

(c) (i) Prove that gcd(n, n+1) = 1 for every natural number n. Find integers x and y such that n.x + (n+1).y = 1.

(ii) Let a, b and c be three natural numbers such that gcd(a, c) = 1 and b divides c. Prove that gcd(a, b) = 1. (4+3.5)

4. (a) Let $n > 1$ be a fixed natural number. Let a, b, c be three integers such that $ac \equiv bc \pmod{n}$ and gcd(c, n) = 1. Prove that $a \equiv b \pmod{n}$. (7.5)

(b) Solve the congruence, $7x \equiv 8 \pmod{11}$. (7.5)

(c) Solve the following pair of congruences:

$2x + 3y \equiv 1 \pmod{6}$

$x + 3y \equiv 5 \pmod{6}$ (7.5)

5. (a) Let G be the set of all 2×2 real matrices with non-zero determinant. Show that G is a group under the operation of matrix multiplication. Further show that it is not an Abelian Group. (7.5)

(b) Let G be a group such that for any x, y, z in the group, xy = zx implies y = z (called left-fight cancellation property). Show that G is Abelian. Give an Example of a non-abelian group in which left-right cancellation property does not hold.

(7.5)

(c) Show that the set $G = \{1, 5, 7, 11\}$ is a group under multiplication modulo 12 with the help of the Cayley table. (7.5)

6. (a) Show that for any integer n, the set $H_n = \{n.x \mid x \in Z\}$ is a subgroup of the group Z of integers under the operation of addition. Further show that $H_2 \cup H_3$ is not a subgroup of Z.

(5.5+2)

(b) Let G be a group. Show that $|aba^{-1}| = |b|$ for all a and b in G ($|x|$ denotes the order of an element x in G). (7.5)

(c) Show that the group $Z_n = \{0, 1, 2, \ldots, n-1\}$ is cyclic under the operation of addition modulo n. How many generators Z_n have? Further, describe all the subgroups of Z_{40}. (2+1+4.5)

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[This question paper contains 4 printed pages.]

Your Roll No..............

D

Sr. No. of Question Paper : 1046

Unique Paper Code : 2352011102

Name of the Paper : DSC-2: Elementary Real Analysis

Name of the Course : **B.Sc. (H) Mathematics (UGCF-2022)**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **three** parts from each question.
3. **All** questions carry equal marks.

1. (a) If $a \in \mathbb{R}$ is such that $0 \le a < \epsilon$ for any $\epsilon > 0$, then show that $a = 0$.

(b) Find all values of x that satisfy $|x - 1| > |x + 1|$. Sketch the graph of this inequality.

(c) Find the supremum and infimum, if they exist, of the following sets:

(i) $\left\{\cos\frac{n\pi}{2} : n \in \mathbb{N}\right\}$

(ii) $\left\{\frac{x+2}{3} : x > 3\right\}$

P.T.O.

(d) Show that $\text{Sup}\left\{1-\frac{1}{n}: n \in \mathbb{N}\right\}=1$.

2. (a) Let S be a non-empty subset of $\mathbb{R}$ that is bounded. Prove that

$$\text{Inf } S = -\text{Sup } \{-s: s \in S\}$$

(b) State and prove the Archimedean Property of real numbers.

(c) If $S=\left\{\frac{1}{n}-\frac{1}{m}: n, m \in \mathbb{N}\right\}$, find *Inf* S and Sup S.

(d) Define a convergent sequence. Show that the limit of a convergent sequence is unique.

3. (a) Using the definition of limit, show that

$$\lim_{n\to\infty} \frac{2n+3}{3n-7}=\frac{2}{3}$$

(b) Show that $\lim_{n\to\infty}\left(n^{1/n}\right)=1$.

(c) State and prove the Sandwich Theorem for sequences.

(d) Show that every increasing sequence which is bounded above is convergent.

4. (a) Let $x_1 = 1$ and $x_{n+1} = \sqrt{2+x_n}$ for all $n \geq 1$. Prove that $\langle x_n \rangle$ converges and find its limit.

(b) Prove that every Cauchy sequence is bounded.

(c) Show that the sequence $\langle x_n \rangle$ where

$$x_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}, \text{ for all } n \in \mathbb{N},$$

does not converge.

(d) Find the limit superior and limit inferior of the following sequences:

(i) $x_n = (-2)^n \left(1 + \frac{1}{n}\right)$, for all $n \in \mathbb{N}$

(ii) $x_n = (-1)^n \left(\frac{1}{n}\right)$, for all $n \in \mathbb{N}$

5. (a) Show that the geometric series $\sum_{k=1}^{\infty} ar^{k-1}$ converges if and only if $|r| < 1$.

(b) Find the sum of the following series, if it converges,

$$\sum \frac{1}{(n+a)(n+a+1)}, \ : a > 0$$

(c) Find the rational number which is the sum of the series represented by the repeating decimal $0.\overline{15}$.

(d) Check the convergence of the following series:

(i) $\sum \frac{1}{\log n}$, $n \geq 2$

(ii) $\sum \tan^{-1}\left(\frac{1}{n}\right)$

6. (a) State the Ratio Test (limit form) for positive series. Using this test or otherwise, check the convergence of the following series:

(i) $\sum\left(\frac{n!}{n^n}\right)$

(ii) $\sum\left(\frac{n!}{e^n}\right)$

(b) Check the convergence of the following series:

(i) $\sum_{n=2}^{\infty}\left(\frac{\log n}{n^2}\right)$

(ii) $\sum\left(\frac{n^{n^2}}{(n+1)^{n^2}}\right)$

(c) Define absolute convergence of a series. Show that every absolutely convergent series is convergent. Is the converse true? Justify your answer.

(d) Check the following series for absolute or conditional convergence :

(i) $\sum(-1)^{n+1}\left(\frac{n}{n^2+1}\right)$

(ii) $\sum_{n=2}^{\infty}(-1)^n\left(\frac{1}{n^2+(-1)^n}\right)$

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[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1136 **D**

Unique Paper Code : 2352571101

Name of the Paper : DSC: Topics in Calculus

Name of the Course : **B.A. / B.Sc. (Prog.) with Mathematics as Non-Major/ Minor**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **Two** parts from each question.
3. **All** questions carry equal marks.

1. (a) If $f(x)=\begin{cases} x\dfrac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}}, & x\neq 0 \\ 0 & x=0 \end{cases}$. Show that f is continuous but not differentiable at x = 0.

(b) State Leibnitz theorem for finding the n^{th} differential coefficient of the product of two functions.

If $y = (\sin^{-1}x)^2$, prove that

$(1 - x^2)xy_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0.$

(c) State Euler's theorem on homogeneous functions.

If $u = \tan^{-1}\frac{(x^3 + y^3)}{(x - y)}$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$

and $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \sin 4u - \sin 2u$.

2. (a) Show that the function f defined as $f(x) = |x - 1| + |x + 1| \ \forall \ x \in R$ is not derivable at the points $x = -1$ and $x = 1$ and is derivable at every other point.

(b) If $y = e^{m \sin^{-1}x}$, show that

$(1 - x^2)xy_{n+2} - (2n + 1)y_{n+1} - (n^2 + m^2)y_n = 0$ and hence find $y_n(0)$.

(c) If $u = \sin^{-1}\frac{(x + y)}{\left(\sqrt{x} + \sqrt{y}\right)}$, prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u.$$

3. (a) State Cauchy's mean value theorem. Verify the Cauchy's mean value theorem for (i) $f(x) = e^x$, $g(x) = e^{-x}$ in $[0,1]$ and (ii) $f(x) = \sin x$, $g(x) = \cos x$ in $\left[-\frac{\pi}{2}, 0\right]$.

(b) State Taylor's theorem with Lagrange's form of remainder. Show that $\sin x$ lies between $x - \frac{x^3}{6}$ and $x - \frac{x^3}{6} + \frac{x^5}{120}$ $\forall x \in R$.

(c) Determine the values of p and q for which

$$\lim_{x \to 0} \frac{x(1 + p\cos x) - q\sin x}{x^3}$$ exists and equals 1.

4. (a) State Taylor's theorem with Cauchy's form of remainder. Use Taylor's theorem to prove that

$$1 + \frac{x}{2} - \frac{x^3}{8} < \sqrt{1+x} < 1 + \frac{x}{2} \text{ if } x > 0.$$

(b) State Lagrange's mean value theorem. Verify Lagrange's mean value theorem for the function $f(x) = \sqrt{x^2 - 4}$ in the interval $[2, 4]$.

(c) Evaluate $\lim_{x \to 0} \frac{(1+x)^{1/x} - e}{x}$.

5. (a) Find the asymptote of the curve

$$x^3 + 3x^2y - 4y^3 - x + y + 3 = 0.$$

(b) Trace the curve $y(x^2 + a^2) = a^3$.

(c) Find the reduction formula for $\int \cos^n x \, dx, n \geq 2$.

Hence find $\int \cos^4 x \, dx$.

6. (a) Determine the position and nature of double points on the curve

$$x^4 - 2y^3 - 3y^2 - 2x^2 + 1 = 0.$$

(b) Trace the curve $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$.

(c) Prove that $\int_0^{\pi/2} \sin^{2n} x \, dx = \frac{2n!}{2^n (n!)^2} \cdot \frac{\pi}{2}$.

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[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1267 **D**

Unique Paper Code : 2352011103

Name of the Paper : DSC-3: Probability and Statistics

Name of the Course : **B.Sc. (H) Mathematics**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.
4. **All** questions carry equal marks.
5. Use of non-programmable scientific calculators and statistical tables is permitted.

1. (a) A sample of 26 offshore oil workers took part in a simulated escape exercise, resulting in the following data on time (sec) to complete the escape. Construct a stem-and-leaf display and comment on any interesting features of the display.

389	356	359	363	375	424	325	394	402	373
373	370	364	366	364	325	339	393	392	369
374	359	356	403	334	397				

(b) The following data gives the sample of total nitrogen loads (kg N/day) from a particular Chesapeake Bay location. Calculate the median, upper fourth (third quartile) and lower fourth (first quartile).

9.69	13.16	17.09	18.12	23.70	24.07	24.29	26.43	30.75	31.54

(c) Data was collected for the sale of homes for homes in a particular city. The following data gives the sale amounts of homes (in 1000's of $) that were sold in the previous month.

590	815	575	608	350	1285	408	540	555	679

(i) Calculate the sample variance and standard deviation.

(ii) The evaluator realized that he had by mistake forgotten to multiply each observation by 5 while collecting data. What would be the resulting values of the sample variance and standard deviation after the correction? Answer without reperforming the calculations.

2. (a) A computer consulting firm presently has bids out on three projects. Let A_k = {awarded project k}, for k = 1,2,3, and suppose that $P(A_1) = .22$, $P(A_2) = .25$, $P(A_3) = .28$, $P(A_1 \cap A_2) = .11$, $P(A_1 \cap A_3) = .05$, $P(A_2 \cap A_3) = .07$, $P(A_1 \cap A_2 \cap A_3) = .01$. Compute the $P(A_1' \cap A_2')$ and $P(A_1 \cup A_2 \cup A_3)$.

(b) State Baye's Theorem. An individual has three different email accounts. 70%, of her messages come into account A1, whereas 20% come into account A2 and the remaining 10% into account A3. Of the messages into account A1, only 1% are spam, whereas the corresponding percentages for accounts A2 and A3 are 2% and 5%, respectively. A randomly selected mail is found to be a spam. What is the probability that it came in account A2?

(c) Components of a certain type are shipped to a supplier in batches of ten. Suppose that 50% of all such batches contain no defective components, 30% contain one defective component, and 20% contain two defective components. Two components from a batch are randomly selected and tested. What are the probabilities associated with 0,1, and 2 defective components being in the batch under the condition that one of the two tested components is defective?

3. (a) Starting at a fixed time, the gender of each new born child is observed at a certain hospital until a boy (B) is born. Let p = P(boy is born) = P(B) and assume that the successive births are independent. Let the random variable

X = the number of births upto and including that of the first boy

(i) Find the probability mass function (pmf) of X.

(ii) Determine the cumulative mass function (cmf) of X.

(b) Let the random variable be defined as :

X = 1 if a randomly selected vehicle passes an emission test and X = 0 otherwise

Assume that the probability mass function (pmf) of this variable is: $p(1) = p$ and $p(0) = 1 - p$.

(i) Name the random variable.

(ii) Compute $E(X^2)$

(iii) Show that $V(X) = p(1 - p)$

(c) For any random variable X, let $E(X) = 5$ and $E[X(X - 1)] = 27.5$. Compute :

(i) $E(X^2)$

(ii) V(X)

(iii) V(2X + 3)

4. (a) The error involved in making a certain measurement is a continuous random variable with probability density function as follows

$$f(x) = \begin{cases} .09375(4 - x^2) & -2 \leq x \leq 2 \\ 0 & otherwise \end{cases}$$

(i) Obtain the cumulative density function F(x) of X

(ii) Compute $P(-1 < X < 1)$

(iii) Compute E[X]

(iv) Compute V[X]

(b) Suppose that 25% of all students at a large public University receive financial aid. Let X be the number of students in a random sample of size 50 who receive financial aid. Using normal approximations find the approximate probabilities that

(i) at most 10 students receive aid

(ii) between 5 and 15 (inclusive) of the selected students receive aid.

(c) Define exponential distribution. Find the mean and standard deviation of an exponentially distributed random variable X. Are they equal?

5. (a) If a publisher of non technical books takes great pains to ensure that its books are free of typographical errors, so that the probability of any given page containing at least one such error is 0.005 and errors are independent from page to page, what is the probability that one of its 400 page novels will contain

(i) Exactly one page with errors?

(ii) At most three page with errors?

(b) An insurance company offers its policyholders a number of different premium payment options. For a randomly selected policyholder, let the random variable be defined as :

X = the number of months between successive payments with the cumulative density function(cdf) as follows :

$$F(x) = \begin{cases} 0 & x < 1 \\ .30 & 1 \le x < 3 \\ .40 & 3 \le x < 4 \\ .45 & 4 \le x < 6 \\ .60 & 6 \le x < 12 \\ 1 & 12 \le x \end{cases}$$

(i) Determine the probability mass function of X?

(ii) Using just the cdf, compute $P(3 \leq X \leq 6)$.

(c) The weight distribution of parcels sent in a certain manner is normal with mean value of 12 lb and standard deviation 3.5 lb. The parcel service wishes to establish a weight value c beyond which there will be a surcharge. What value of c is such that 99% of all parcels are under the surcharge weight?

6. (a) The Turbine Oil Oxidation Test (TOST) and Rotating Bomb Oxidation Test (RBOT) are two different procedures for evaluating the oxidation stability of steam turbine oils. The following table gives these observations on x = TOST time (in hours) and y = RBOT time (in minutes) for 10 oil specimens

X	4200	3600	3750	3675	4050	2770	4870	4500	3450	2700
y	370	340	375	310	350	200	400	375	285	225

(i) Calculate the value of the sample correlation coefficient. Based on this value, how would you describe the nature of relationship between the two variables?

(ii) If RBOT is also measured in hours, what happens to the value of r? Why?

(b) The following table gives the data on x = rainfull volume (m^3) and y = runoff volume (m^3) for a particular location.

x	5	12	14	17	23	30	40	47	55	67
y	4	10	13	15	15	25	27	46	38	46

(i) Determine the equation of the estimated regression line using the principle of least square.

(ii) Estimate the runoff volume when the rainfull volume is 70.

(c) The amount of a particular impurity in a batch of a certain chemical product is a random variable with mean 4.0 g and standard deviation 1.5 g. If $\overline{X}$ is the sample mean impurity for a random sample of 50 batches.

(i) Where is the sampling distribution of $\overline{X}$ centered?

(ii) What is the standard deviation of the $\overline{X}$ distribution?

(iii) What is the probability that the sample average amount of impurity $\overline{X}$ is between 3.5 and 3.8 g?

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[This question paper contains 8 printed pages.]

Your Roll No...............

C

Sr. No. of Question Paper : 1132

Unique Paper Code : 32357501

Name of the Paper : DSE-I Numerical Analysis (LOCF)

Name of the Course : B.Sc. (Hons.) Mathematics

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. All **six** questions are compulsory.

3. Attempt any two parts from each question.

4. Use of non-programmable scientific calculator is allowed.

P.T.O.

1. (a) Define fixed point of a function and construct an algorithm to implement the fixed point iteration scheme to find a fixed point of a function. Find the fixed point of $f(x) = 2x(1 - x)$. (6)

(b) Perform four iterations of Newton's Raphson method to find the positive square root of 18. Take initial approximation $x_0=4$. (6)

(c) Find the root of the equation $x^3 - 2x - 6 = 0$ in the interval (2, 3) by the method of false position. Perform three iterations. (6)

2. (a) Define the order of convergence of an iterative method for finding an approximation to the root of $g(x) = 0$. Find the order of convergence of Newton's iterative formula. (6.5)

(b) Find a root of the equation $x^3 - 4x - 8 = 0$ in the interval (2, 3) using the Bisection method till fourth iteration. (6.5)

(c) Perform three iterations of secant method to determine the location of the approximate root of the equation $x^3 + x^2 - 3x - 3 = 0$ on the interval (1, 2). Given the exact value of the root is $x = \sqrt{3}$, compute the absolute error in the approximations just obtained. (6.5)

3. (a) Using scaled partial pivoting during the factor step, find matrices L, U and P such that LU = PA

where $A = \begin{pmatrix} 1 & 1 & 2 \\ -1 & 0 & 2 \\ 3 & 2 & -1 \end{pmatrix}$ (6.5)

(b) Set up the SOR method with w=0.7 to solve the system of equations:

$$3x_1 - x_2 + x_3 = 4$$

$$2x_1 - 6x_2 + 3x_3 = -13$$

$$-9x_1 + 7x_2 - 20x_3 = 7$$

Take the initial approximation as $X^{(0)} = (0, 0, 0)$ and do three iterations. (6.5)

(c) Set up the Gauss-Jacobi iteration scheme to solve the system of equations:

$$10x_1 + x_2 + 4x_3 = 31$$

$$x_1 + 10x_2 - 5x_3 = -23$$

$$3x_1 - 2x_2 + 10x_3 = 38$$

Take the initial approximation as $X^{(0)} = (1, 1, 0)$ and do three iterations. (6.5)

4. (a) Obtain the piecewise linear interpolating polynomials for the function f(x) defined by the data:

x	1	2	4	8
f(x)	3	7	21	73

(6)

(b) Calculate the Newton second order divided difference $\frac{1}{x^2}$ of based on the points x_0, x_1, x_2.

(6)

(c) Obtain the Lagrange form of the interpolating polynomial for the following data:

x	1	2	5
$f(x)$	-11	-23	1

(6)

5. (a) Find the highest degree of the polynomial for which the second order backward difference approximation for the first derivative

$$f'(x_0) \approx \frac{3f(x_0) - 4f(x_0 - h) + f(x_0 - 2h)}{2h}$$

provides the exact value of the derivative irrespective of h. (6)

(b) Derive second-order forward difference approximation to the first derivative of a function f given by

$$f'(x_0) \approx \frac{-3f(x_0) + 4f(x_0 + h) - f(x_0 + 2h)}{2h}.$$

(6)

(c) Approximate the derivative of f(x) = Sin x at $x_0 = \pi$ using the second order central difference formula taking h = ½, ¼ and 1/8 and then extrapolate from these values using Richardson extrapolation. (6)

6. (a) Using the Simpson's rule, approximate the value of the integral $\int_2^5 \ln x\, dx$. Verify that the theoretical error bound holds. (6.5)

(b) Apply Euler's method to approximate the solution of initial value problem $\frac{dx}{dt} = \frac{e^t}{x}$, $0 \le t \le 2$, x(0) = 1 and N = 4.

Given that the exact solution is $x(t) = \sqrt{2e^t - 1}$, compute the absolute error at each step. (6.5)

(c) Apply the optimal RK2 method to approximate the solution of the initial value problem $\frac{dx}{dt} = 1 + \frac{x}{t}$, $1 \le t \le 2$, $x(1) = 1$ taking the step size as $h = 0.5$. (6.5)

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[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1133 **C**

Unique Paper Code : 32357502

Name of the Paper : DSE-1 Mathematical Modelling and Graph Theory

Name of the Course : **B.Sc. (H) Mathematics – CBCS (LOCF)**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **three** parts from each question.

1. (a) (i) Determine whether x = 0 is an ordinary point, a regular singular point or an irregular singular point of the differential equation

$$xy'' + x^3y' + (e^x - 1)y = 0. \qquad (6)$$

(ii) Find the Laplace transform of the function $f(t) = 1 + \cosh 5t$.

P.T.O.

(iii) Find the inverse Laplace transform of the function $F(s) = \frac{1}{s+5}$.

(b) Use Laplace transforms to solve the initial value problem :

$x'' - x' - 2x = 0;\ x(0) = 0,\ x'(0) = 2$ (6)

(c) Find two linearly independent Frobenius series solutions of

$$2xy'' - y' - y - 0$$ (6)

(d) Find general solutions in powers of x of the differential equation. State the recurrence relation and the guaranteed radius of convergence.

$$5y'' - 2xy' + 10y = 0$$ (6)

2. (a) Using Monte Carlo simulation write an algorithm to compute volume of the surface $x^2 + y^2 + z^2 \leq 1$ that lies in the first octant $x > 0,\ y > 0,\ z > 0$. (6)

(b) Use Simplex method to solve the given linear programming problem (6)

Maximize : $3x_1 + x_2$

subject to $2x_1 + x_2 \leq 6$

$x_1 + 3x_2 \leq 9$

$x_1, x_2 \geq 0.$

(c) Consider a small harbor with unloading facilities for ships, where only one ship can be unloaded at any time. The unicading time required for a ship depends on the type and the amount of cargo. Below is given a situation with 5 ships:

	Ship 1	Ship 2	Ship 3	Ship 4	Ship 5
Time between successive ships	10	20	45	50	75
Unload time	70	35	40	80	90

Draw the timeline diagram depicting clearly the situation for each ship. Also determine length of longest queue and total time in which docking facilities are idle. (6)

(d) Use Linear Congruence method to generate 15 random real numbers with multiplier 2, increment 5, modulus 13 and seed 1. Is there cycling? If yes, then give the period of cycling. (6)

3. (a) (i) Determine the number of edges of C_{16}, Q_3 and $K_{9,10}$. (3)

(ii) State and prove Handshaking Lemma. (3)

(b) Prove that a bipartite graph with odd number of vertices is not Hamiltonian. (6)

(c) Determine whether the given four cubes having four colours, can be stacked in a manner so that each side of the stack formed will have all the four colours exactly once. (6)

cube 1 cube 2 cube 3 cube 4

(d) By finding an Eulerian trail in K_5, arrange a set of fifteen dominoes [0 – 0 to 4 – 4] in a ring.

(6)

4. (a) Use the factorization :

$$s^4 + 4a^4 = (s^2 - 2as + 2a^2)(s^2 + 2as + 2a^2)$$

and apply inverse Laplace transform to show that :

$$L^{-1}\left\{\frac{s^3}{s^4 + 4a^4}\right\} = \cosh at \cos at$$ (7)

(b) Fit the model to the data using Chebyshev's criterion to minimize the largest deviation, given the model y = cx and data set below :

y	1	2	3
x	2	5	8

(7)

(c) Solve the initial value problem using the Laplace transform

$$x'' + 4x' + 13x = te^{-t};\ x(0) = 0,\ x'(0) = 0$$ (7)

(d) Name the five Platonic graphs. What is the degree of each vertex in each of these five graphs? Draw any two platonic graphs.

(7)

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1231 **C**

Unique Paper Code : 32357505

Name of the Paper : DSE-2 Discrete Mathematics

Name of the Course : **B.Sc. (H) Mathematics**

Semester : V (under CBCS (LOCF) Scheme)

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. All the given **eight** questions are compulsory to attempt.

3. Do any **two** parts from each of the given **eight** questions.

4. Marks for each part are indicated on the right in brackets.

P.T.O.

SECTION I

1. (a) Let N_0 be the set of non-negative integers. Define a relation $\leq$ on N_0 as: For m, n $\in N_0$, $m \leq n$ if m divides n, that is, if there exists $k \leq N_0$: $n = km$. Then show that $\leq$ is an order relation on N_0. (2½)

(b) If '1', '2', '3' denote chains of one, two, three elements respectively and $\bar{3}$ denotes anti chain of three elements, then draw the Hasse diagram for the dual of $L \oplus K$ when $L = \bar{3}$ and $K = 1 \oplus (2 \times 2)$. (2½)

(c) Define maximum and a maximal element of a partially ordered set P. Give an example each for both definitions. (2½)

2. (a) Let P and Q be finite ordered sets and let $\psi: P \to Q$ be a bijective map. Then show that the following are equivalent :

(i) $x < y$ in P iff $\psi(x) < \psi(y)$ in Q

(ii) $x \prec y$ in P iff $\psi(x) \prec \psi(y)$ in Q (3)

(b) Define upper bound and lower bound of a subset S of a partially ordered set P. Construct an example of a partially ordered set P and its subset S and give the set of all upper bounds and lower bounds of S. (3)

(c) Let P and Q be ordered sets. Then show that the ordered sets P and Q are order isomorphic iff there exist order preserving maps $\varphi: P \to Q$ and $\psi: Q \to P$ such that :

$\varphi o \psi = id_Q$ and $\psi o \varphi = id_P$ where $id_S : S \to S$ denotes the identity map on S given by: $id_S(x) = x$, $\forall\, x \in S$. (3)

SECTION II

3. (a) Let $D_{60} = \{1, 2, 4, 5, 6, 12, 20, 30, 60\}$ be an ordered subset of $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$, $\mathbb{N}$ being the set of natural numbers. If '≤' is defined on D_{60} by $m \leq n$ if and only if m divides n then show that D_{60} does not form a lattice. Also Draw the diagram of D_{60} and find elements a, b, c, d $\in D_{60}$ such that $a \vee b$ and $c \wedge d$ do not exist in D_{60}. (5½)

(b) Define sublattice of a lattice. Prove that every chain of a lattice L is a lattice and also a sublattice of L. (5½)

(c) Define modular lattice. Prove that a homomorphic image of modular lattice is modular. (5½)

4. (a) Let L be a lattice. For any a, b, c ∈ L, show that the following inequalities hold :

(i) $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$

(ii) $a \geq c \Rightarrow a \wedge (b \vee c) \geq (a \wedge b) \vee c$

(5)

(b) Let $(L, \wedge, \vee)$ be an algebraic lattice. If we define

$$a \leq b :\Leftrightarrow a \vee b = b$$

then show that $(L, \leq)$ is a lattice ordered set.

(5)

(c) Let L_1 and L_2 be distributive lattices. Prove that the product $L_1 \times L_2$ is a distributive lattice.

(5)

SECTION III

5. (a) A voting-machine for three voters has YES-NO switches. Current is in the circuit precisely when YES has a majority. Draw the corresponding contact diagram and the switching/circuit diagram.

(5½)

(b) Show that a Boolean Algebra is relatively complemented. (5½)

(c) Simplify the polynomial :

$$f = x'yz + x'yz' + x'y'z + xy'z' + xy'z$$

using Quine's McCluskey method. (5½)

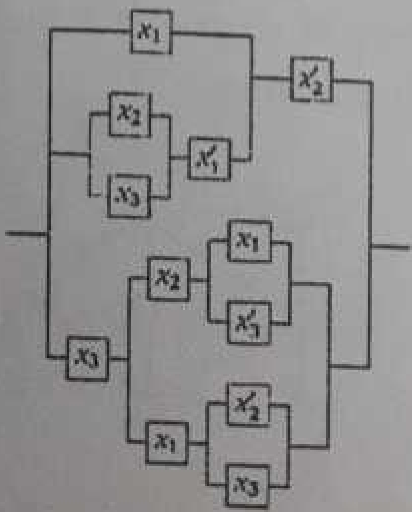
6. (a) Define a system of normal forms. Find conjunctive normal form for $p = y'z' + x'yz$. (5)

(b) Simplify the Boolean expression :

$$f = w'xy'z + w'xyz + w'xyz' + wxy'z + wxyz + wxyz' + wx'y'z + wx'yz$$

using Karnaugh Diagram. (5)

(c) Find the symbolic gate representation of the contact diagram : (5)



SECTION IV

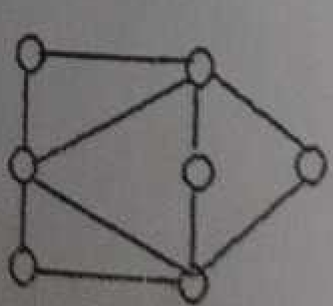
7. (a) (i) Show that the sum of the degrees of the vertices of a pseudograph is an even number equal to twice the number of edges.

(ii) A graph has five vertices of degree 4 and two vertices of degree 2. How many edges does it have? (5½)

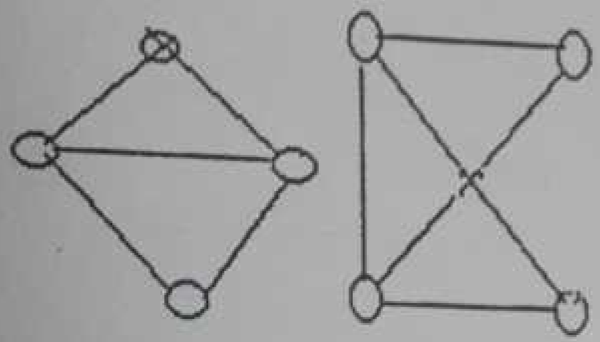
(b) (i) Define the degree sequence of a graph. Does there exist a graph with following degree sequence 6, 6, 5, 5, 4, 4, 4, 4, 3?

(ii) Show that the number of vertices of odd in a graph must be even. (5½)

(c) (i) What is a bipartite graph? Determine whether the graph given below is bipartite or not. Give the bipartition sets or explain why the graph is not bipartite.

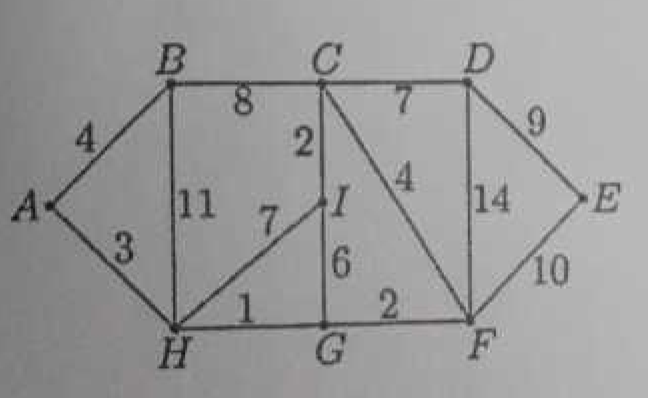


(ii) Define isomorphism of graphs. Also label the following graphs so as to show an isomorphism. (5½)



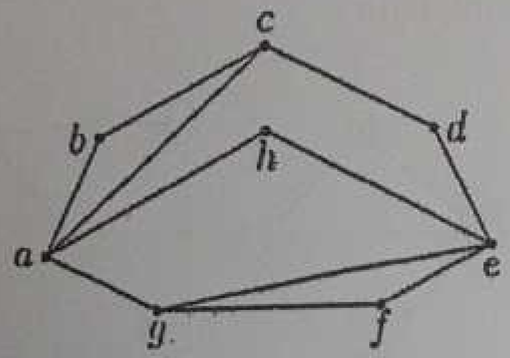
8. (a) Construct a Gray Code of length 3 using the concept of Hamiltonian Cycles. (5½)

(b) Apply Dijkstra's algorithm to find a shortest path from A to all other vertices in the weighted graph shown. (5½)



(c) (i) Does there exist a graph G with 28 edges and 12 vertices each of degree 3 or 6?

(ii) Define Eulerian circuit. Is the given graph Eulerian? Give reasons for your answer.



(5½)

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[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1233 C

Unique Paper Code : 32357507

Name of the Paper : DSE – 2 Probability Theory and Statistics

Name of the Course : **CBCS (LOCF) B.Sc. (H) Mathematics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **all** questions selecting any **two** parts from each questions no.'s **1** to **6**.

3. Use of scientific calculator is permitted.

P.T.O.

1. (i) If the random variable be the time in seconds between incoming telephone calls at a busy switchboard. Suppose that a reasonable probability model for X is given by the probability density function :

$$f_X(x) = \begin{cases} \frac{1}{4} e^{-\frac{x}{4}} & 0 < x < \infty \\ 0 & \text{elsewhere} \end{cases}$$

Show that f_X satisfies the properties of a probability density function. Also show that the probability that the time between successive phone call exceed 4 seconds is 0.3679. (6)

(ii) Let the random variables X_1 and X_2 have the joint pdf

$$f(x_1, x_2) = \begin{cases} 8x_1x_2 & \text{if } 0 < x_1 < x_2 < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find $E(X_1X_2^2)$, $E(X_2)$, $E(7X_1X_2^2 + 5X_2)$. (6)

(iii) Let the random variables X and Y have the joint pdf

$$f(x, y) = \begin{cases} e^{-y} & \text{if } 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases}$$

Find the marginal pdf of X and Y. (6)

2. (i) Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} kxe^{-\lambda x}, & x \geq 0, \lambda > 0 \\ 0, & \text{otherwise} \end{cases}$$

Determine the constant k, mean, variance and the cumulative distribution function of X. (6)

(ii) If a random variable X is uniformly distributed over the interval $[\alpha, \beta]$ then find the mean, variance and moment generating function of X. (6)

(iii) Let the random variables X and Y have the joint pdf

$$f(x, y)=\begin{cases}6y & \text{if } 0<y<x<1\\ 0, & \text{elsewhere}\end{cases}$$

Find the E(Y|x) and E[E(F|x)]. (6)

3. (i) Let (X, y) be a random vector such that the variance off is finite. Then show that Var[E(Y|X)] ≤ Var(Y). (6)

(ii) If X is a binomial variate with parameter n and p then prove that

$$\mu'_{r+1}=\left[np\mu'_r+pq\frac{d\mu'_r}{dp}\right],$$ where $\mu'_r=E\left[x^r\right]$ and r is a non-negative integer. (6)

(iii) Let the random variables X and Y have the linear conditional means $E(Y|x) = 4x + 3$ and $E(X|y) = \frac{1}{16}y - 3$. Find the mean of X, mean of Y, the correlation coefficient. (6)

4. (i) Let the random variables X_1 and X_2 have the joint pdf

$$f(x_1, x_2) = \begin{cases} x_1 + x_2 & \text{if } 0 < x_1 < 1,\ 0 < x_2 < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Show that X_1 and X_2 are not independent. (6.5)

(ii) State and prove the Chebyshev's Theorem. (6.5)

(iii) If the probability is 0.25 that an applicant for driver's license will pass the road test on the given try, what is the probability that an applicant will finally pass the test on the fourth try?

(6.5)

5. (i) Calculate the correlation coefficient for the following age (in years) of husband's (X) and wife's (Y) :

(6.5)

X	23	27	28	28	29	30	31	33	35	36
Y	18	20	22	27	21	29	27	29	28	29

(ii) If X and Y have a bivariate normal distribution, the conditional density of Y given $X = x$ is a normal distribution with the mean,

$$\mu_{Y|x} = \mu_2 + \rho\frac{\sigma_2}{\sigma_1}(x - \mu_1)$$

and the variance

$$\sigma^2_{Y|x} = \sigma^2_2(1 - \rho^2)$$

(6.5)

(iii) The joint density of X_1, X_2 and X_3 is given by

$$f(x_1, x_2, x_3) = \begin{cases} (x_1 + x_2)e^{-x_3} & \text{if } 0 < x_1 < 1,\ 0 < x_2 < 1,\ 0 < x_3 \\ 0, & \text{elsewhere} \end{cases}$$

Find the regression equation of X_2 on X_1 and X_3. (6.5)

6\. (i) Two fair dice are tossed 600 times. Let X denote the number of times a total of 7 occurs. Use Central limit theorem to find $P[95 \le X \le 115]$. (6.5)

(ii) To show how an exponential distribution might arise in practice. If random variable X has an exponential distribution with parameter θ then find its mean, variance and moment generating function. If X has exponential distribution with mean 2 then find $P[X < 1]$. (6.5)

(iii) If X is a random variable having a binomial distribution with parameter n and θ, then the moment generating function of $Z = \frac{X - n\theta}{\sqrt{n\theta(1-\theta)}}$ approaches that of the standard normal distribution when $n \to \infty$. (6.5)

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[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1589 **C**

Unique Paper Code : 42357502

Name of the Paper : Mechanics and Discrete Mathematics

Name of the Course : **B.Sc. (Math Sci)–II/B.Sc. (Phy Sci)–II/B.Sc. (Life Sci)–II/Applied Sciences – II**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory.
4. Marks are indicated.

1. (a) Three forces acting at a point are parallel to the sides of a triangle ABC, taken in order and have their magnitudes proportional to the cosines of the opposite angles. Show that the resultant is proportional to

$$(1 - 8 \cos A \cos B \cos C)^{1/2}$$ (8)

P.T.O.

(b) A uniform bar AB, 10 feet long is hinged at B and is supported in a vertical plane by a light string AC 10 feet above B. If AB weighs 20 lbs and AC = 15 feet, find the tension in AC and the reaction at B. (8)

(c) Find the mass centre of a cubical box with no lid, the sides and bottom being made of the same thin material. (8)

2. (a) A wheel of radius b rolls without slipping along a straight road. If the centre of the wheel has a uniform velocity v, find at any instant the velocity and acceleration of two points of the rim which are at a height h above the road. Examine the cases when $h = 0$ and $h = 2b$. (8)

(b) Derive the equation of path of a particle which is projected in a medium having no resistance. Show that the path is a parabola. Find its vertex and latus rectum. Also find its maximum horizontal range. (8)

(c) A particle is performing a simple harmonic motion of period T about a centre O, and it passes through a point P, where OP = b with velocity v in the direction OP. Prove that the time which elapses before its return to P is $\frac{T}{\pi}\tan^{-1}\left(\frac{vT}{2\pi b}\right)$. (8)

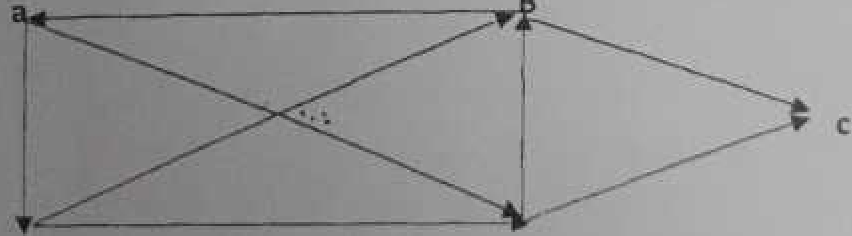
3. (a) How Many vertices and how many edges do $K_{m,n}$ have? For which value of m and n is $K_{m,n}$ regular? (7)

(b) Define complete Bipartite graph? For which values of n are C_n and W_n bipartite? (7)

(c) State Handshaking theorem. Also find number of edges in a graph with 10 vertices each of degree 4? (7)

4. (a) Define complete graph. Using principle of mathematical induction prove that number of edges in K_n is $\frac{n(n-1)}{2}$. (7)

(b) Define strongly connected and weakly connected graphs. Is the graph given below strongly connected? Is it weakly connected? (7)

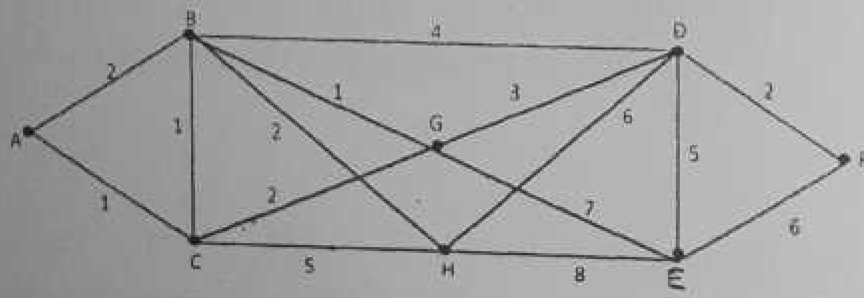


(c) Define the vertex connectivity and edge connectivity of a graph. Also, let G = (V, E) be a graph. Then show that

$$\kappa(G) \le \lambda(G) \le \min_{v \in V} \deg(v)$$

where $\kappa(G)$ and $\lambda(G)$ are vertex connectivity and edge connectivity of graph respectively. (7)

5. (a) Use the Dijkstra's Algorithm to find shortest path from A to F in the graph given below : (7.5)



(b) Construct a niche overlap graph for six species of birds, where the hermite thrush compete with the robin and with the blue jay, the robin also competes with the mocking bird, the mockingbird also competes with the blue jay, and the nuthatch also competes with the hairy woodpecker. Is there any pendant vertex? (7.5)

(c) Show that a connected graph with n vertices has at least $n-1$ edges. (7.5)

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[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1703 **C**

Unique Paper Code : 42344304

Name of the Paper : Operating System

Name of the Course : **B.Sc. Programme / B.Sc. Mathematical Science**

Semester : III [Admission of 2019-2021]

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Question no. **1** is compulsory.
3. Attempt any **FIVE** questions from Question no. **2** to **8**.

1. (a) Which address binding scheme generates different logical and physical addresses? (1)

   (b) Name the scheduler responsible for transition of a process from ready to running state. (1)

P.T.O.

(c) Name the piece of code that loads operating system into the memory and starts its execution. (1)

(d) How many new processes are created in the following code? (2)

```
main()
{
int i;
for (i=0;i<3;i++)
        fork();
}
```

(e) Briefly describe any two advantages of Multiprocessor systems. (2)

(f) Which of the following instructions are privileged?

(i) Set value of timer

(ii) Read the clock (2)

(g) Assuming a 1-KB page size, what are the page numbers and offsets for the following address references (provided as decimal numbers):

(i) 2378

(ii) 9360 (2)

(h) What is the purpose of command interpreter? Why is it usually separated from the kernel? (2)

(i) Write the Linux commands for the following : (2+2)

(i) Count the number of occurrences of the word "hello" in a file named "file1.txt".

(ii) Sort the data in descending order of marks in a two-column file "file2.txt" containing names of students and marks obtained.

(j) Differentiate between : (2+2)

(i) Multi-programming and Multi-tasking Operating System

(ii) Renaming and copying a file

(k) Define the following terms : (1×4)

(i) Dispatch Latency

(ii) Seek Time

(iii) Rotational Latency

(iv) Response Time

2. Consider the following set of processes, with the length of CPU burst time given in milliseconds :

Process	**Arrival Time**	**Burst Time**
P_1	0	5
P_2	2	3
P_3	5	6
P_4	6	2

(i) Draw Gantt charts illustrating the execution of these processes using non-preemptive Shortest Job First (SJF) and Round Robin (Time Slice = 2) scheduling algorithms.

(ii) Calculate the turnaround time and waiting time of each process for each of the above-mentioned scheduling algorithms. (6+4)

3. (a) Consider a paging system with the page table stored in memory.

   (i) If a memory reference takes 200 nanoseconds, how long does a paged memory reference take?

   (ii) If we add TLBs and 80% of all page table references are found in the TLBs, what is the effective memory access time? Assume that the time taken to access a TLB is 20 nanoseconds. (2+4)

   (b) List the circumstances under which a CPU scheduling decision may take place. (4)

4. (a) Why is it important for operating system to differentiate between CPU-bound and I/O-bound processes/jobs? Name the scheduler that takes care of this requirement. (3+1)

   (b) Describe the Two-Level Directory structure with the help of a suitable diagram. (3)

(c) Name the system programs which perform the following tasks : (3)

(i) Create or modify the contents of file.

(ii) Compile a program written in high level language.

(iii) Load a program into the main memory.

5. (a) What is a Process control block? Describe the information contained in it. (1+3)

(b) Enumerate any four activities of operating system in regard to the following :

(i) Process Management

(ii) File Management (2+2)

(c) List any two reasons for a parent process to terminate execution of its children processes. (2)

6. (a) Write a shell script to find the greatest common divisor of two numbers. (4)

(b) Explain three different uses of 'cat' command. Illustrate with suitable examples. (3)

(c) Discuss the main advantage of Demand Paging. How is Effective Access Time computed for a demand-paged memory? (3)

7. (a) Given memory partitions of 100KB, 500KB, 200KB, 300KB and 600KB (in order), how would each of the Best-fit and Worst-fit algorithms place processes of size 212KB, 417KB, 120KB and 426KB (in order)? Which algorithm makes the most efficient use of memory? (6)

(b) Differentiate between external and internal fragmentation by taking suitable examples. (4)

8. (a) Consider a logical address space of eight pages of 1024 words each, mapped onto a physical memory of 32 frames.

(i) How many bits are there in the logical address?

(ii) How many bits are there in the physical address? (4)

(b) What are the main advantages of using Layered Approach over Simple Structure for Operating System design? What are the disadvantages of using the Layered Approach? (4)

(c) What do you understand by Swapping? List any two reasons why swapping is not supported on Mobile systems. (2)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1713 C

Unique Paper Code : 42354302

Name of the Paper : Algebra

Name of the Course : B.Sc.(Prog)/Mathematical Sciences

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. This question paper has **six** questions in all.

3. Attempt **any two parts** from each question.

4. All questions are **compulsory**

Unit I

1. (a) Define the inverse of an element and show that inverse of an element in a group is unique. (6)

(b) Let the element "α" belong to a group and $\alpha^{12} = e$. Express the inverse of each of the elements α, α^6, α^8 and α^{11} in the form α^k for some positive integer k. (6)

(c) Let $\sigma = (1,5,7)(2,5,3)(1,6)$. Then find σ^{98}. (6)

2. (a) Let G be a group and let H be a nonempty subset of G. If ab is in H whenever a and b are in H and a^{-1} is in H whenever a is in H, then H is a subgroup of G. (6)

(b) Let G be an Abelian group and H and K be subgroups of G. Then $HK = \{hk : h \in H, k \in K\}$ is a subgroup of G. (6)

(c) State and prove Lagrange's Theorem. Is the converse of this theorem true? (6)

3. (a) In a finite cyclic group, the order of an element divides the order of the group. (6)

(b) Find the inverse of $\left\{\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}\right\}$ in $(2, Z_{11})$ (6)

(c) Every permutation of a finite set can be written as a cycle or as a product of disjoint cycles. (6)

Unit II

4. (a) State the subring test. Check whether the set $\left\{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : a, b \in Z\right\}$ is a subring of the ring of all 2×2 matrices over Z,the set of integers. (6.5)

(b) Define a field. Prove that a finite commutative ring with unity having no zero divisors is a field. (6.5)

(c) Show that the set $[Q\sqrt{2}] = \{a + b\sqrt{2} \mid a, b \in Q\}$ forms a ring. Is it a field? If yes, justify your answer. (6.5)

Unit III

5. (a) Prove that intersection of two subspaces of a vector space V(F) is a subspace of V(F). Is the result true for the union of two subspaces? Justify your answer. (6.5)

(b) Define basis of a vector space over a field F. Prove that every element of a vector space uniquely expressible as a linear combination of elements of the basis. (6.5)

(c) Check whether the vectors (1,-1, 2), (-1,2,-4), (-1,-1,2) form a basis of R^3. (6.5)

6. (a) Let T: V->U be a Linear Transformation. Define null space N(T) and range R(T) of T Show that N(T) and R(T) are subspaces of V and U respectively. (6.5)

(b) Define Linear Transformation. Prove that there exists a Linear Transformation T: R^2-> R^3 such that T (1,1) = (1,0,2) and T(2,3) = (1,-1,4). Find T (8, 11). (6.5)

(c) Let T: R^3-> R^3 be a Linear Transformation defined by T (x, y, z) = (x-y+2z, 2x+y, -x- 2y+2z). Find the Range, Rank, Kernal and Nullity of T. Verify the Dimension Theorem. (6.5)

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[This question paper contains 6 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 2142 C

Unique Paper Code : 62354343

Name of the Paper : Analytic Geometry and Applied Algebra

Name of the Course : B.A. (Prog.)

Semester : III CBCS (LOCF)

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Attempt any two parts from each questions.
4. Each question carries 12.5 marks.

1. (a) Identify and sketch the curve :

$$y = 4x^2 + 8x + 5$$

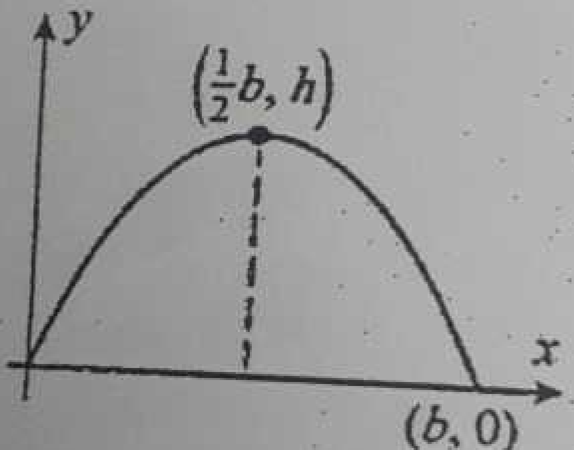
Also label the focus, vertex and directrix.

(b) Describe the graph of the curve:

$$x^2 + 9y^2 + 2x - 18y + 1 = 0$$

Find its foci, vertices and the ends of the minor axis.

(c) Find an equation for the parabolic arch with base b and height h, shown in the accompanying figure



2. (a) Find the equation for the parabola that has axis $y = 0$ and passes through $(3, 2)$ and $(2, -3)$.

(b) Find the equation for the ellipse that has foci $(1,2)$ and $(1,4)$ and minor axis of length 2.

(c) Describe the graph of the hyperbola :

$$x^2 - 4y^2 + 2x + 8y - 7 = 0$$

Also sketch its graph.

3. (a) If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular unit vectors, then prove that

$$\left\|\vec{a} + \vec{b} + \vec{c}\right\| = \sqrt{3}$$

(b) Express $\vec{v}$ as the sum of a vector parallel to $\vec{b}$ and a vector orthogonal to $\vec{b}$ where

$$\vec{v} = 3\hat{i} + \hat{j} + 2\hat{k} \quad \text{and} \quad \vec{b} = 2\hat{i} + \hat{k}$$

(c) (i) Using vectors, find the area of triangle with vertices P(2, 2, 0), Q(1, 4, –5) and R(7, 2, 9).

(ii) Use scalar triple product to determine whether the vectors :

$\vec{u} = \langle 5, -2, 1 \rangle$, $\vec{v} = \langle 4, -1, 2 \rangle$ and $\vec{w} = \langle 1, -1, 0 \rangle$ are co-planar.

4. (a) Consider the equation $x^2 - xy + y^2 + 12 = 0$. Rotate the coordinate axes to remove xy-terms. Then identify and sketch the curve.

(b) Let an $x'y'$-coordinate system be obtained by rotating an xy-coordinate system through an angle of $\theta = 45°$.

(i) Find the $x'y'$-coordinates of the point whose xy-coordinates are $\left(\sqrt{2}, \sqrt{2}\right)$.

(ii) Find an equation of the curve $x^2 + xy + 2y^2 + 6 = 0$ in $x'y'$-coordinates.

(c) Describe the surface whose equation is given as

$$x^2 + y^2 + z^2 + 2y - 6z + 5 = 0$$

5. (a) Find the distance from the point $P(2, 5, -3)$ to the plane

$$\vec{r}.\left(6\hat{i} - 3\hat{j} + 2\hat{k}\right) = 4$$

(b) Find the equation of the plane through the points $P_1(2, 1, 4)$, $P_2(0, 0, -3)$ that is perpendicular to the plane $4x + y + 3z = 2$.

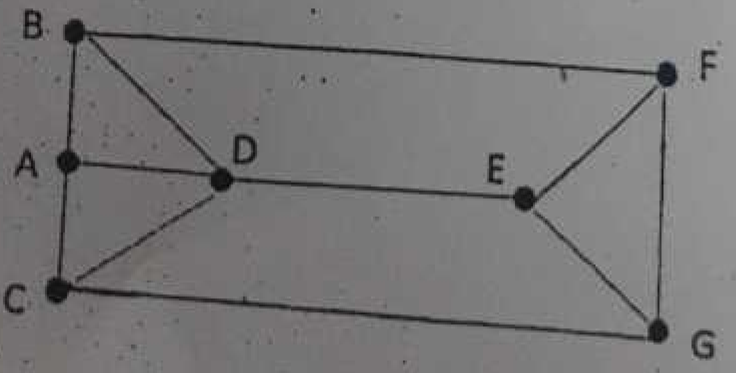
(c) Show that the lines L_1 and L_2 are parallel and find the distance between them

L_1: $x = 2 - t$, $y = 2t$, $z = 3 + t$

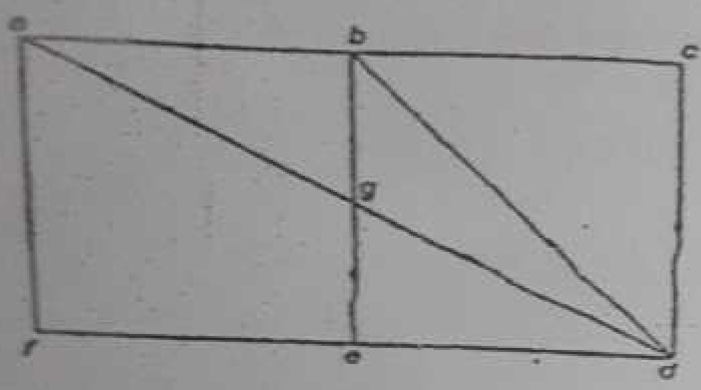
L_2: $x = -1 + 2t$, $y = 3 - 4t$, $z = 5 - 2t$

6. (a) Suppose a job placement agency wants to schedule interviews for candidates Ann, Judy and Carol with interviewers Al, Brian and Carl on Monday, Tuesday and Wednesday in such a way that each candidate gets interviewed by each interviewer. Solve this problem using a Latin Square.

(b) Find a vertex basis for the following graph:



(c) For the following graph, find a minimal edge cover and a maximal independent set of vertices.



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[This question paper contains 12 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 3016 **C**

Unique Paper Code : 62273506

Name of the Paper : Data Analysis

Name of the Course : **BA Prog. (CBCS)**

Semester : V

Duration : 3 Hours Maximum Marks : 65

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **All** questions carry equal marks (**13** marks each).

3. Attempt any **Five** questions.

4. Use of simple calculator is allowed.

5. Required Tables will be provided by the colleges.

6. Answers may be written either in English or Hindi; but the same medium should be used throughout the paper.

P.T.O.

छात्रों के लिए निर्देश

1. इस प्रश्न-पत्र के मिलते ही ऊपर दिए गए निर्धारित स्थान पर अपना अनुक्रमांक लिखिए ।
2. सभी प्रश्नों के अंक समान हैं (प्रत्येक के 13 अंक)।
3. किन्हीं पाँच प्रश्नों के उत्तर दीजिए ।
4. साधारण कैलकुलेटर के उपयोग की अनुमति है ।
5. आवश्यक तालिकाएं महाविद्यालयों द्वारा उपलब्ध कराई जाएंगी ।
6. इस प्रश्न-पत्र का उत्तर अंग्रेजी या हिंदी किसी एक भाषा में दीजिए, लेकिन सभी उत्तरों का माध्यम एक ही होना चाहिए ।

1. (i) How much time doing what activities do college students spend using their Mobile phones? A 2014 Ahamdabad University study showed college students spend an average of 9 hours a day using their mobile phones for the following activities : (6,2)

Mobile phone activity	Percentage
Banking	2%
Checking Data & Time	8%

Listening to Music	12%
Playing Games	4%
Reading	3%
Sending Emails	9%
Social Media	18%
Surfing the Internet	7%
Taking Photos	3%
Talking	6%
Texting	18%
YouTube	2%
Others	8%

(a) How can we construct a bar chart, a pie chart, and a Pareto chart of the above data by using Excel/R? Write syntax also.

(b) Which graphical method do you think is best for portraying these data?

(ii) What are the various methods of sampling? Explain any two. (5)

(i) कॉलेज के छात्र अपने मोबाइल फोन का उपयोग करने में कितना समय व्यतीत करते हैं? 2014 के अहमदाबाद विश्वविद्यालय

के एक अध्ययन से पता चला है कि कॉलेज के छात्र निम्नलिखित गतिविधियों के लिए अपने मोबाइल फोन का उपयोग करके दिन में औसतन 9 घंटे खर्च करते हैं :

Mobile phone activity	Percentage
Banking	2%
Checking Data & Time	8%
Listening to Music	12%
Playing Games	4%
Reading	3%
Sending Emails	9%
Social Media	18%
Surfing the Internet	7%
Taking Photos	3%
Talking	6%
Texting	18%
YouTube	2%
Others	8%

(अ) हम Excel/R का उपयोग करके उपरोक्त डेटा का एक बार चार्ट, एक पाई चार्ट और एक परेटो चार्ट कैसे बना सकते हैं? वाक्य रचना भी लिखें।

(ब) आपके विचार में इन आँकड़ों को चित्रित करने के लिए कौन-सी चित्रमय विधि सर्वोत्तम है?

(ii) प्रतिचयन की विभिन्न विधियाँ क्या हैं? किन्हीं दो को स्पष्ट कीजिए।

2. (i) The following data represents the per litre consumption of two wheelers : (6,2)

26	38	26	30	24	26	28	28	24	26	24	39
32	23	22	24	28	37	31	40	25	25	33	30

(a) How variance, standard deviation, range can be calculated by using either excel/R of the above data? Write syntax only.

(b) Graphically represent the skewness of the above data.

(ii) Highlight the differences among the mean, median, and mode. (5)

(i) निम्नलिखित डेटा दोपहिया वाहनों की प्रति लीटर खपत को दर्शाता है :

26	38	26	30	24	26	28	28	24	26	24	39
32	23	22	24	28	37	31	40	25	25	33	30

(अ) उपरोक्त डेटा के एक्सेल/आर का उपयोग करके भिन्नता, मानक विचलन, सीमा की गणना कैसे की जा सकती है? केवल सिंटैक्स लिखें।

(ब) उपरोक्त आँकड़ों की विषमता को आलेखीय रूप से निरूपित करें।

(ii) माध्य, माध्यिका और बहुलक के बीच के अंतरों पर प्रकाश डालिए।

3. (i) The following is a set of data from a sample of n = 7 : (6,2)

12	7	4	9	0	7	3

(a) Compute the first quartile (Q_1), Second Quartile (Q_2) and third quartile (Q_3) for the above data

(b) Compute the Z-score of the above data

(ii) Explain Chebyshev theorem. (5)

(i) निम्नलिखित n = 7 के नमूने से डेटा का एक सेट है :

12	7	4	9	0	7	3

(अ) उपरोक्त आंकड़ों के लिए प्रथम चतुर्थक (Q_1), द्वितीय चतुर्थक (Q_2) और तृतीय चतुर्थक (Q_3) की गणना करें।

(ब) उपरोक्त डेटा के Z-स्कोर की गणना करें।

(ii) चेबीशेव प्रमेय की व्याख्या करें।

4. (i) The Cereals lists the calories and sugar, in grams, in one serving of seven breakfast cereals : (6,2)

Cereals	Calories	Suger
Kellogg's All Bran	80	6
Kellogg's Corn Flakes	100	2
Wheaties	100	4
Nature's Path Organic Multigrain Flakes	110	4
Kellogg's Rice Krispies	130	4
Post Shredded Wheat Vanilla Almond	190	11
Kellogg's Mini Wheats	200	10

(a) Compute the covariance.

(b) Compute the coefficient of correlation.

(ii) Why is the sample mean an unbiased estimator of the population mean? Prove it with the help of given data set (5)

Administrative Assistant	Number of Errors
A	$X_1 = 3$
B	$X_2 = 2$
C	$X_3 = 1$
D	$X_4 = 4$

(i) अनाज कैलोरी और चीनी को ग्राम में, सात में से एक सर्विंग में सूचीबद्ध करता है नाश्ता अनाज :

Cereals	Calories	Suger
Kellogg's All Bran	80	6
Kellogg's Corn Flakes	100	2
Wheaties	100	4
Nature's Path Organic Multigrain Flakes	110	4
Kellogg's Rice Krispies	130	4
Post Shredded Wheat Vanilla Almond	190	11
Kellogg's Mini Wheats	200	10

(अ) सहप्रसरण की गणना कीजिए।

(ब) सहसंबंध के गुणांक की गणना करें।

(ii) नमूना माध्य जनसंख्या का निष्पक्ष अनुमानक माध्य क्यों है? दिए गए डेटा सेट की मदद से इसे साबित करें।

प्रशासनिक सहायक	त्रुटियों की संख्या
A	$X_1 = 3$
B	$X_2 = 2$
C	$X_3 = 1$
D	$X_4 = 4$

5. (i) What are the risks in decision making using hypothesis testing? (5)

(ii) The following is a series of annual sales (in $millions) over an 11-year period (2004 to 2014) : (6,2)

Year:	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
Sales:	13.0	17.0	19.0	20.0	20.5	20.5	20.5	20.0	19.0	17.0	13.0

(a) Construct a time-series plot.

(b) Does there appear to be any change in annual sales over time? Explain.

(i) परिकल्पना परीक्षण का उपयोग करके निर्णय लेने में जोखिम क्या हैं?

(ii) निम्नलिखित 11 साल की अवधि (2004 से 2014) में वार्षिक बिक्री ($ मिलियन में) की एक श्रृंखला है :

Year	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
Sales	13.0	17.0	19.0	20.0	20.5	20.5	20.5	20.0	19.0	17.0	13.0

(अ) एक समय-श्रृंखला प्लॉट का निर्माण करें।

(ब) क्या समय के साथ वार्षिक बिक्री में कोई बदलाव दिखाई देता है? समझाना।

6. (i) Determining the interval that includes a fixed proportion of the sample means by using symmetrically distributed around the population mean that will include 95% of the sample means, based on the sample of 25 boxes. (6)

(ii) Explain type 1 type II Error (4)

(iii) Explain graphically the empirical rule (3)

(i) अंतराल का निर्धारण जिसमें नमूना का एक निश्चित अनुपात शामिल है, जनसंख्या के चारों ओर सममित रूप से वितरित का उपयोग करके मतलब है कि 25 बक्से के नमूने के आधार पर नमूना साधनों का 95% शामिल होगा।

(ii) टाइप I टाइप II एरर की व्याख्या करें।

(iii) अनुभवजन्य नियम को आलेखीय रूप से समझाइए।

7. (i) A company that manufactures chocolate bars is particularly concerned that the mean weight of a chocolate bar is not greater than 6.03 ounces. A sample of 50 chocolate bars is selected; the sample mean is 6.034 ounces, and the sample standard deviation is 0.02 ounce. Using the $\alpha = 0.01$ level of significance, is there evidence that the population mean weight of the chocolate bars is greater than 6.03 ounces? Apply One-Tail Test for the mean on the given statement. (8)

(ii) Explain p – value approach. (5)

(i) चॉकलेट बार बनाने वाली कंपनी विशेष रूप से चिंतित है कि चॉकलेट बार का औसत वजन 6.03 ऑंस से अधिक नहीं है। 50 चॉकलेट बार का एक नमूना चुना गया हैय नमूना माध्य 6.034 औंस है, और नमूना मानक विचलन 0.02 औंस है। $\alpha = 0.01$ स्तर के महत्व का उपयोग करते हुए, क्या इस बात

का प्रमाण है कि जनसंख्या का मतलब चॉकलेट बार का वजन 6.03 औंस से अधिक है? दिए गए कथन पर माध्य के लिए वन-टेल टेस्ट लागू करें।

(ii) पी-मूल्य दृष्टिकोण की व्याख्या करें।

8. Write any **two** of the followings : (6.5,6.5)

(i) A Statistic and A parameter?

(ii) Six steps of Critical Value Approach to Hypothesis Testing

(iii) Box plot and Scatter plot

निम्नलिखित में से कोई दो लिखिए :

(i) एक आँकड़ा और एक पैरामीटर?

(ii) परिकल्पना परीक्षण के लिए महत्वपूर्ण मूल्य दृष्टिकोण के छह चरण

(iii) बॉक्स प्लॉट और स्कैटर प्लॉट

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[This question paper contains 6 printed pages.]

Your Roll No............

Sr. No. of Question Paper : 1143 D

Unique Paper Code : 2352201102

Name of the Paper : DSC: Elements of Discret Mathematics

Name of the Course : B.A. (Prog.)

Semester : I

Duration : 3 Hours Maximum Marks : 9(

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any **two** parts from each question.

3. **All** questions are compulsory. Marks are indicated.

1. (a) Determine the following :

(i) Compute the truth table of the statement $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$.

P.T.O.

(ii) If $p \Rightarrow q$ is false, then determine the truth value of $(\sim(p \wedge q)) \Rightarrow q$. Explain your answer.

(7.5)

(b) Let $A = \mathbb{Z}^+$ (the set of positive integers). Define the following relation R on A :

a R b if and only if $|a - b| \leq 2$.

Determine whether the relation R on A is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, or transitive. Is R an equivalence relation on A?

(7.5)

(c) Prove by mathematical induction that 3 divides $(n^3 - n)$ for every positive integer n. (7.5)

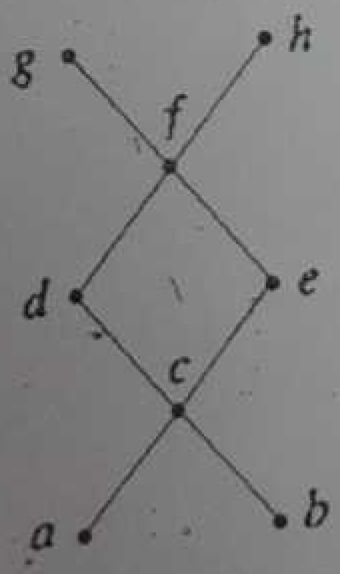
2. (a) For any positive integer n, let D_n denote the set of all positive integers which are divisors of n. Draw the Hasse diagram for D_{12} and D_{15} with the partial order $\leq$ of divisibility defined as $a \leq b$ if and only if a divides b.

(7.5)

(b) Consider $A = \{1,2,3,5,6,10,15,30\}$ and partial order $\leq$ of divisibility on the set A defined as $a \leq b$ if and only if a divides b. Let $B = P(S)$

where S = {e,f,g} be the poset with the partial order ≤′ defined as, U ≤′ V if and only if U ⊆ V ∀ U, V ∈ B. Show that (A, ≤) and (B, ≤′) are isomorphic posets. (7.5)

(c) Find all the maximal and minimal elements, all the lower and upper bounds along with greatest lower and least upper bound of the subset B = {c,d,e} in the following Hasse diagram. (7.5)



3. (a) Let (L, ∧, ∨) be an algebraic lattice. Define $l \leq m \Leftrightarrow l \wedge m = l$. Show that (L, ≤) is a lattice ordered set. (7.5)

P.T.O.

(b) If f is a homomorphism from a lattice L to another lattice M. Show that the homomorphic image of L, $f(L) = \{f(l) : l \in L\}$, is a sublattice of M.

(7.5)

(c) Define a sublattice of a lattice. Show that every non empty subset of a chain is a sublattice.

(7.5)

4. (a) Define a distributive lattice. Show that every chain is a distributive lattice.

(7.5)

(b) Let $(L_1, \leq_1)$ and $(L_2, \leq_2)$ be two ordered lattices. Define a relation $\leq$ on their Cartesian product $L = L_1 \times L_2$ by $(a_1, a_2) \leq (b_1, b_2)$ if and only if $a_1 \leq_1 b_1$ in L_1 and $a_2 \leq_2 b_2$ in L_2. Prove that $(L, \leq)$ is also a lattice.

(7.5)

(c) Justify with an example that complement of an element in a non-distributive lattice need not be unique.

(7.5)

5. (a) Construct circuits by using inverters, AND gates and OR gates to produce the output

$(x + y + z)\bar{x}\bar{y}\bar{z}$

(7.5)

(b) What is Disjunctive normal form and Conjunctive normal form? Find the DN form and CN form of the following Boolean function.

$f(x, y, z) = xy + xz + \bar{y}z$ (7.5)

(c) What is Karnaugh map? Use Karnaugh map diagram to find a minimal form of the function

$\bar{x}yzw + x\bar{y}z\bar{w} + \bar{x}\bar{y}z\bar{w} + xy\bar{z}\bar{w} + x\bar{y}\bar{z}\bar{w}$ (7.5)

6. (a) Let $f(x,y,z) = x\bar{y}z + xyz + \bar{y}\bar{z}$. Find the implicants, prime implicants and essential prime implicants of $f(x,y,z)$ (7.5)

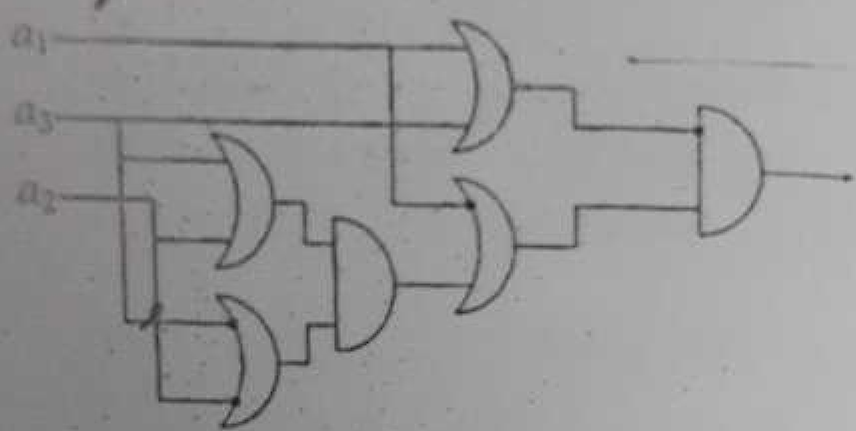
(b) Draw the switching circuit diagram for the following :–

(i) $p = x_1(x_2(x_3 + x_4) + x_3(x_5 + x_6))$

(ii) $p = x_1(x_2'(x_6 + x_3(x_4 + x_5')) + x_7(x_3 + x_6)x_8')$

(7.5)

(c) What is subjunction gate, NOR gate and NAND gate? Determine the Boolean polynomial of the circuit.



(7.5)

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[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1588 **C**

Unique Paper Code : 42357501

Name of the Paper : DSC – Differential Equations

Name of the Course : **B.Sc. (Math Sci)-II B.Sc. (Phy Sci)-II B.Sc. (Life Sci)-II Applied Sciences-II**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.

1. (a) Show that the differential equation

$(4x + 3y^2)dx + 2xydy = 0$ is not exact.

Find an integrating factor of form y^n, n being an integer. Multiply by the integrating factor and find the solution of the equation.

(b) Solve the initial value problem

$(2x + 3y + 1)dx + (4x + 6y + 1)dy = 0, \; y(-2) = 2.$

P.T.O.

(c) Solve

$$\left\{x\tan\left(\frac{y}{x}\right)+y\right\}dx-xdy=0.$$ (6+6)

2. (a) Find the orthogonal trajectories of $x^2 + y^2 = 2cx$, c being an arbitrary constant.

(b) Show that e^{2x} and e^{3x} are two linearly independent solutions on $-\infty < x < \infty$ of the differential equation

$$\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0.$$

Write the general solution. Find the solution that satisfies the conditions

$$y(0) = 2,\ y'(0) = 3.$$

(d) If $y = (x + 1)$ is a solution of

$$(x+1)^2\frac{d^2y}{dx^2}-3(x+1)\frac{dy}{dx}+3y=0,$$

find a linearly independent solution by reducing the order. Write the general solution. (6+6)

3. (a) Solve the initial value problem

$$\frac{d^3y}{dx^3}-6\frac{d^2y}{dx^2}+11\frac{dy}{dx}-6y=0,\ y(0)=0,\ y'(0)=0,$$
$y''(0) = 2.$

(b) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=\frac{1}{1+e^{2x}}$$

using the method of variation of parameters.

(c) Find the general solution of

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+4y=2x\ln(x).$$ (6+6)

4. (a) Show that $x=3e^{7t}$, $y=2e^{7t}$ and $x=e^{7t}$, $y=-2e^{-t}$ are two linearly independent solutions on every interval $a \le t \le b$ of the linear system

$$\frac{dx}{dt}=5x+3y,$$

$$\frac{dy}{dt}=4x+y.$$

Write the general solution.

(b) Solve the linear system

$$\frac{dx}{dt}+\frac{dy}{dt}+2y=\sin(t),$$

$$\frac{dx}{dt}+\frac{dy}{dt}-x-y=0.$$

(c) Solve the initial value problem

$$x^2\frac{d^2y}{dx^2}-2y=4x-8,\ y(1)=4,\ y'(1)=-1.$$ (6+6)

5. (a) Find the partial differential equation satisfied by the following surface

$$z = f(x-y).$$

(b) Find the general solution of the partial differential equation

$$u_x + yu_y = 0.$$

(c) Solve the following Cauchy problem

$$yu_x + xu_y = 0,\ u(0, y) = e^{-y^2}.$$ (6.5+6.5)

6. (a) Apply the method of separation of variables $u(x, y) = f(x) + g(y)$ to solve the following equation

$$u_x^{\,2} + u_y^{\,2} = 1.$$

(b) Find general solution of the following second order partial differential equation with constant coefficients

$$u_{xx} + 4u_{xy} + 4u_{yy} = 0.$$

(c) Classify the following equation and obtain general solution by reducing it to canonical form

$$yu_{xx} - xu_{yy} = 0,\ x > 0,\ y > 0.$$ (7+7)

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 2103 **C**

Unique Paper Code : 62357503

Name of the Paper : DSE – Statistics

Name of the Course : **B.A. (PROG)**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory and carry equal marks.

1. (a) If A and B are two events, such that $P(A^c) = \alpha$, $P(B^c) = \beta$, then show that

$$P(AB) > 1 - \alpha - \beta. \quad (6.5)$$

(b) Three groups of children contain 3 girls, 1 boy; 2 girls, 3 boys and 1 girl, 3 boys. One child is selected at random from each group, show that the

P.T.O.

probability of three selected children are 1 girl and 2 boys is 13/22. (6.5)

(c) Urn I has 2 white and 3 black balls, Urn II has 4 white and 1 black balls, and Urn III has 3 white and 4 black balls. An um is selected at random and a ball drawn at random to be white. Find the probability that the ball is from urn I. (6.5)

2. (a) Suppose X is a random variable and the pmf is given by

X	0	1	2	3	4
p(x)	0.08	0.15	0.45	0.27	0.05

Compute the following E(X), V(X) and σ(X).

(b) Find four moments about mean, where

$$f(x)=\begin{cases}\frac{4x}{81}\left(9-x^2\right) & 0\le x\le 3\\ 0 & \text{elsewhere}\end{cases}$$ (6)

(c) Suppose that X is a random variable with E(X)=10 and V(X)=25. Find the positive number a and b such that Y=aX-b, has mean 0 and variance 1. (6)

3. (a) A variate X has the probability distribution :

X	-3	6	9
p(x)	1/6	1/2	1/3

Find E(X) and $E(X^2)$ and using the law of expectation, evaluate $E[(2X + 1)^2]$. (6.5)

(b) Obtain the moment generating function of the Geometric distribution. Hence, find its mean. (6.5)

(c) Prove E(X+Y) = E(X) + E(Y), where X and Y are discrete random variables. (6)

4. (a) Let X and Y have joint pdf: (6)

$f(x, y) = cxy \quad 0 < x < 1, \quad 0 < y < 2.$

Find (i) the value of c, (ii) the marginal distributions.

(b) If X is Poisson variates such that

$$P(X = 2) = 9\ P(X = 4) + 90\ P(X = 6).$$

Find the mean of X. (6)

(c) For exponential distribution, find (i) its moment generating function (ii) P(X>1). (6)

5. (a) A random sample of 10 boys had the following IQs 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Does this data support the assumption of a population mean IQ of 100? (Given tabulated t(0.05) for 9 df is 2.262). (6.5)

(b) A dice is thrown 9000 times and a throw of 3 or 4 observed 3220 times. Can the dice be regarded as unbiased? (6.5)

(c) A random sample of 400 items has a mean of 82 and standard deviation of 18. Find 95% confidence limits for the population mean. (6.5)

6. (a) The theory predicts that the proportion of beans in 4 groups A, B, C, D should be 9:3:3:1. In an experiment among 1600 beans, the numbers in the 4 groups were 882, 313, 287 and 118. Does the experiment support the theory? (Given tabulated Chi-square(0.05) for 3 df is 7.81). (6)

(b) The following table gives the classification of 100 workers according to sex and nature of work. Test Whether nature of work is independent of the sex of the worker

	Skilled	Unskilled
Male	40	20
Female	10	30

(Given tabulated Chi-square (0.05) for 1 df is 3.84.)

(6)

(c) Examine the effect of a vaccine in controlling TB from the following data :

	Affected by TB	Unaffected
Inoculated	12	28
Not Inoculated	13	07

(Given tabulated chi-square (0.05) for 1 df is 3.84).

(6)