[This question paper contains 6 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 738 **B**

Unique Paper Code : 32351201

Name of the Paper : BMATH203 – Real Analysis

Name of the Course : B.Sc. (H) Mathematics

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All Questions are Compulsory.
3. Attempt any two parts from each question.
4. All Questions are of equal marks.

1. (a) State the completeness property of $\mathbb{R}$, hence show that every non-empty set of real numbers which is bounded below, has an infimum in $\mathbb{R}$.

P.T.O.

(b) Show that if A and B are bounded subse then $A \cup B$ is a bounded set and sup (A max {sup A, sup B).

(c) State and prove nested interval property. 3.

(d) Define an open set and closed set in $\mathbb{R}$.

Show that if a, b $\in \mathbb{R}$, then the open i (a, b) is an open set.

Is a closed interval a closed set ?

2. (a) Let S be a bounded set in $\mathbb{R}$ and let S_0 be a empty subset of S. Show that

$$\inf S \leq \inf S_0 \leq \sup S_0 \leq \sup S$$

(b) State Archimedean property. Hence, prove th:

$S = \left\{\frac{1}{n}, n \in \mathbb{N}\right\}$ then $\inf S = 0$.

(c) If $S \subseteq \mathbb{R}$ is non empty. Show that S is bounc if and only if there exists a Closed bounded inte I such that $S \subseteq I$.

(d) If $x, y, z \in \mathbb{R}$ and $x \leq z$. Show that $x \leq y \leq z$ if and only if $|x-y| + |y-z| = |x-z|$. Interpret this geometrically.

3\. (a) Prove that a convergent sequence of real numbers is bounded.

Is the converse true? Justify.

(b) Let (x_n) be a sequence of positive real numbers

such that $\lim_{n\to\infty}\left(\frac{x_{n+1}}{x_n}\right) = L$ exists. If $L < 1$, then

(x_n) converges and $\lim_{n\to\infty}(x_n) = 0$.

(c) Prove that if $C > 0$, then $\lim_{n\to\infty}\left(C^{1/n}\right) = 1$.

(d) Let $x_1 > 1$ and $x_{n+1} = 2 - \frac{1}{x_n}$ for $n \in \mathbb{N}$. Show

that (x_n) is bounded and monotone. Also find the limit.

4. (a) Let $X = (x_n)$ and $Y = (y_n)$ be sequences of real numbers that converge to x and y respectively. Then the product sequence X.Y converges to x.y.

(b) Let $X = (x_n)$ be a bounded sequence of real numbers and let $x \in R$ have the property that every convergent subsequence of X converges to x. Then the sequence X is convergent to x.

(c) Discuss the convergence of the sequence (x_n).

where $x_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots\cdots + \frac{1}{2n}$ for $n \in N$.

(d) Use the definition of the limit of the sequence to find the following limits

(i) $\lim_{n \to \infty} \left(\frac{1}{n} - \frac{1}{n+1} \right)$

(ii) $\lim_{n \to \infty} \left(\frac{3n+1}{2n+5} \right)$

5. (a) Prove that a necessary condition for the convergence of an infinite series Σa_n is $\lim_{n \to \infty} a_n = 0$. Is the condition sufficient? Justify with the help of an example.

(b) Prove that the geometric series $1 + r + r^2 + \cdots$ converges for $0 \le r \le 1$ and diverges for $r \ge 1$.

(c) Test for convergence, the following series :

(i) $\frac{1}{5}+\frac{2!}{5^2}+\frac{3!}{5^3}+\cdots$

(ii) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n-1}}{n}$

(d) Prove that the series $x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots$ converges if and only if $-1 \le x \le 1$.

6. (a) State and prove Cauchy's n^{th} root test for positive term series.

(b) Prove that the series $1+\frac{1}{2^p}+\frac{1}{3^p}+\frac{1}{4^p}\cdots$ converges for $p > 1$ and diverges for $p \le 1$.

(c) Test for convergence, the following series :

(i) $\sum_{n=1}^{\infty}\left[\sqrt[3]{n^3+1}-n\right]$

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(ii) $\sum_{n=1}^{\infty} 2^{-n-(-1)^n}$

(d) Prove that every absolutely convergent series is convergent. Show that the series $\sum(-1)^n \frac{n+2}{2^n+5} x^n$ converges for all the real values of x.

[This question paper contains 6 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1395 A

Unique Paper Code : 32351403

Name of the Paper : BMATH-410 – Ring Theory and Linear Algebra – I

Name of the Course : CBCS (LOCF) B.Sc. (H) Mathematics

Semester : IV

Duration : 3.30 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory.

1. (a) Define zero divisors in a ring. Let R be the set of all real valued functions defined for all real numbers under function addition and multiplication. Determine all zero divisors of R. (6½)

P.T.O.

(b) What is nilpotent element? If a and b are nilpotent elements of a commutative ring, show that a + b is also nilpotent. Give an example to show that this may fail if the ring R is not commutative. (6½)

(c) Let R be a commutative ring with unity. Prove that U(R), the set of all units of R, form a group under multiplication of R. (6½)

(d) Determine all subrings of $\mathbb{Z}$, the set of integers. (6½)

2. (a) Define centre of a ring. Prove that centre of a ring R is a subring of R. (6)

(b) Suppose R is a ring with $a^2 = a$, for all $a \in R$. Show that R is a commutative ring. (6)

(c) Show that any finite field has order p^n, where p is prime. (6)

(d) Let R be a ring with unity 1. Prove that if 1 has infinite order under addition, then CharR = 0, and if 1 has order n under addition, then CharR = n. (6)

3. (a) Let R be a commutative ring with unity and let A be an ideal of R. Then show that R/A is a field if and only if A is maximal ideal. (6½)

(b) Prove that I = < 2 + 2i > is not prime ideal of $\mathbb{Z}[i]$.

How many elements are in $\frac{\mathbb{Z}[i]}{I}$? What is the characteristic of $\frac{\mathbb{Z}[i]}{I}$? (6½)

(c) In $\mathbb{Z}[x]$, the ring of polynomials with integer coefficients, let $I = \{f(x) \in \mathbb{Z}[x] \mid f(0) = 0\}$. Prove that I is not a maximal ideal. (6½)

(d) Let $\mathbb{R}[x]$ denote the ring of polynomials with real coefficients and let $< x^2 + 1 >$ denote the principal ideal generated by $x^2 + 1$. Then show that

$$\frac{\mathbb{R}[x]}{\langle x^2+1\rangle} = \{g(x) + \langle x^2+1\rangle \mid g(x) \in \mathbb{R}[x]\} =$$

$$\{ax + b + \langle x^2+1\rangle \mid a, b \in \mathbb{R}\}$$

(6½)

4. (a) If R is a ring with unity and the characteristic of R is $n > 0$, then show that R contains a subring isomorphic to $\mathbb{Z}_n$ and if the characteristic of R is 0 then R contains a subring isomorphic to $\mathbb{Z}$. (6)

(b) Determine all ring homomorphism from $\mathbb{Z}_{20}$ to $\mathbb{Z}_{30}$. (6)

(c) Let n be an integer with decimal representation $a_k a_{k-1} \cdots a_1 a_0$. Prove that n is divisible by 11 if and only if $a_0 - a_1 + a_2 - \cdots + (-1)^k a_k$ is divisible by 11. (6)

(d) Show that a homomorphism from a field onto a ring with more than one element must be an isomorphism. (6)

5. (a) Let W_1 and W_2 be subspaces of a vector space V. Prove that $W_1 + W_2$ is a smallest subspace of V that contains both W_1 and W_2. (6)

(b) For the following polynomials in $P_3(\mathbb{R})$, determine whether the first polynomial can be expressed as linear combination of other two. (6)

$$\{x^3 - 8x^2 + 4x,\ x^3 - 2x^2 + 3x - 1,\ x^3 - 2x + 3\}.$$

(c) Let $S = \{u_1, u_2, \ldots, u_n\}$ be a finite set of vectors. Prove that S is linearly dependent if and only if $u_1 = 0$ or $u_{k+1} \in \text{span}(\{u_1, u_2, \ldots, u_k\})$ for some $k\ (1 \leq k < n)$. (6)

(d) Let $W_1 = \{(a, b, 0) \in \mathbb{R}^3 : a, b \in \mathbb{R}\}$ and $W_2 = \{(0, b, c)\ E\ \mathbb{R}^3 : b, c \in \mathbb{R}\}$ be subspaces of $\mathbb{R}^3$. Determine $\dim(W_1)$, $\dim(W_2)$, $\dim(W_1 \cap W_2)$ and $\dim(W_1 + W_2)$. Hence deduce that $W_1 + W_2 = \mathbb{R}^3$. Is $\mathbb{R}^3 = W_1 \oplus W_2$? (6)

6. (a) Let V and W be finite-dimensional vector spaces having ordered bases β and γ respectively, and let $T : V \to W$ be linear. Then for each $u \in V$, show

$$[T(u)]_\gamma = [T]_\beta^\gamma [u]_\beta.$$ (6½)

(b) Let $T: P_2(\mathbb{R}) \to P_3(\mathbb{R})$ be linear transformation defined by

$$T(a + bx + cx^2) = (a - c) + (a - c)x + (b - a)x^2 + (c - b)x^3.$$

Find null space N(T) and range space R(T). Also verify Rank-Nullity Theorem. (6½)

P.T.O.

[This question paper contains 6 printed pages.]

Your Roll No.............

Sr. No. of Question Paper : 1114 A

Unique Paper Code : 32351601

Name of the Paper : BMATH 613 – Complex Analysis

Name of the Course : B.Sc. (H) Mathematics

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Attempt two parts from each question.

1. (a) Sketch the region onto which the sector $r \leq 1$, $0 \leq \theta \leq \pi$ is mapped by the transformation $w = z^2$ and $w = z^3$. (6)

(b) (i) Find the limit of the function $f(z) = \frac{(\bar{z})^2}{z}$ as z tends to 0.

P.T.O.

(ii) Show that $\lim_{z \to 1+\sqrt{3}i} \frac{z^2-2z+4}{z-1-\sqrt{3}i} = 2\sqrt{3}i.$ (3+3=6)

(c) Let u and v denote the real and imaginary components of the function f defined by means of the equations

$$f(z) = \begin{cases} \bar{z}^2/z & \text{when } z \neq 0 \\ 0 & \text{when } z = 0 \end{cases}$$

Verify that the Cauchy-Riemann equations are satisfied at the origin z = (0,0). (6)

(d) If $\lim_{z \to z_0} f(z) = F$ and $\lim_{z \to z_0} g(z) = G$, prove that

$\lim_{z \to z_0} \frac{f(z)}{g(z)} = \frac{F}{G}$ if $G \neq 0$. (6)

2. (a) Find the values of z such that

(i) $e^z = 1 + \sqrt{3}i$, (ii) $e^{(2z-1)} = 1$. (3.5+3=6.5)

(b) Show that the roots of the equation $\cos z = 2$ are $z = 2n\pi + i\cosh^{-1}2$ $(n = 0, \pm1, \pm2, \ldots)$. Then express them in the form $z = 2n\pi \pm i\ln(2+\sqrt{3})$ $(n = 0, \pm1, \pm2, \ldots)$. (3.5+3=6.5)

(c) Show that (3.5+3=6.5)

(i) $\log(1+i)^2 = 2Log(1+i)$,

(ii) $\log(-1+\sqrt{3}i) = \ln 2 + 2\left(n+\frac{1}{3}\right)\pi i \; (n = 0, \pm 1, \pm 2, \ldots)$

(d) Show that $\overline{\exp(iz)} = \exp(i\bar{z})$ if and only if $z = n\pi \; (n = 0, \pm 1, \pm 2, \ldots)$. (6.5)

3. (a) (i) State mean value theorem of integrals. Does it hold true for complex valued functions? Justify.

(ii) Evaluate $\int_0^{2\pi} e^{im\theta} e^{-in\theta} d\theta$. (3+3=6)

(b) Parametrize the curves C_1 and C_2, where

C_1: Semicircular path from −1 to 1

C_2: Polygonal path from the vertices $-1, -1+i, 1+i$ and 1

Evaluate $\int_{C_1} z\,dz$ and $\int_{C_2} z\,dz$. (3+3=6)

(c) For an arbitrary smooth curve $C: z = z(t),\ a \le t \le b$, from a fixed point z_1 to another fixed point z_2, show that the value of the integrals

(i) $\int_{z_1}^{z_2} z\,dz$ and

(ii) $\int_{z_1}^{z_2} dz$

depend only on the end points of C. (3+3=6)

(d) State ML inequality theorem. Use it to prove that $\left|\int_C \frac{dz}{z^4}\right| \leq 4\sqrt{2}$, where C denotes the line segment from z = i to z = 1. (2+4=6)

4\. (a) A function f(z) is continuous on a domain D such that all the integrals of f(z) around closed contours lying entirely in D have the value zero. Prove that f(z) has an antiderivative throughout D. (6.5)

(b) State Cauchy Goursat theorem. Use it to evaluate the integrals

(i) $\int_C \frac{1}{z^2+2z+2} dz$, where C is the unit circle $|z| = 1$

(ii) $\int_C \frac{2z}{z^2+2} dz$, where C is the circle $|z| = 2$

(2.5+2+2=6.5)

(c) State and prove Cauchy Integral Formula.

(2+4.5=6.5)

(d) (i) State Liouville's theorem. Is the function $f(z) = \cos z$ bounded? Justify.

(ii) Is it true that 'If $p(z)$ is a polynomial in z then the function $f(z) = 1/p(z)$ can never be an entire function'? Justify (4.5+2=6.5)

5. (a) If a series $\sum_{n=1}^{\infty} z_n$ of complex numbers converges then prove $\lim_{n\to\infty} z_n = 0$. Is the converse true? Justify. (6.5)

(b) Find the integral of $\int_C \frac{\cosh \pi z}{z^3+z}$ where C is the positively oriented circle $|z| = 2$. (6.5)

(c) Find the Taylor series representation for the function $f(z) = \frac{1}{z}$ about the point $z_0 = 2$. Hence prove that $\frac{1}{z^2} = \frac{1}{4}\sum_{n=0}^{\infty}(-1)^n(n+1)\left(\frac{z-2}{2}\right)^n$ for $|z - 2| < 2$. (6.5)

(d) If a series $\sum_{n=0}^{\infty} a_n(z - z_0)^n$ converges to $f(z)$ at all points interior to some circle $|z - z_0| = R$, then prove that it is the Taylor series for the function $f(z)$ in power of $z - z_0$.

(6.5)

6. (a) For the given function $f(z) = \frac{z+1}{z^2+9}$ find the poles, order of poles and their corresponding residue.

(6)

(b) Write the two Laurent Series in powers of z that represent the function $f(z) = \frac{1}{z+z^3}$ in certain domains and specify those domains. (6)

(c) Suppose that $z_n = x_n + iy_n$, $(n = 1,2,3,\dots)$ and $S = X + iY$. Then prove that

$\sum_{n=1}^{\infty} z_n = S$ iff $\sum_{n=1}^{\infty} x_n = X$ and $\sum_{n=1}^{\infty} y_n = Y$. (6)

(d) Define residue at infinity for a function f(z). If a function f(z) is analytic everywhere in the finite plane except for a finite number of singular points interior to a positively oriented simple closed contour C, then prove that

$$\operatorname*{Res}_{z=\infty} f(z) = -\operatorname*{Res}_{z=0}\left[\frac{1}{z^2} f\left(\frac{1}{z}\right)\right].$$ (6)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1299 A

Unique Paper Code : 32357616

Name of the Paper : DSE-4 Linear Programming and Applications

Name of the Course : CBCS (LOCF) – B.Sc. (II) Mathematics

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any two parts from each question.

3. All questions carry equal marks.

1. (a) Solve the following Linear Programming Problem by Graphical Method :

$$\begin{aligned} \text{Minimize} \quad & 3x + 2y \\ \text{subject to} \quad & 5x + y \geq 10 \\ & x + y \geq 6 \\ & x + 4y \geq 12 \\ & x \geq 0, y \geq 0. \end{aligned}$$

(b) Define a Convex Set. Show that the set S defined as :

$$S = \{(x,y) \mid x^2 + y^2 \leq 4\}$$ is a Convex Set.

(c) Find all basic feasible solutions of the equations:

$$x_1 + x_2 + 2x_3 + 3x_4 = 12$$

$$x_2 + 2x_3 + x_4 = 8$$

(d) Prove that to every basic feasible solution of the Linear Programming Problem:

$$\text{Minimize } z = cx$$

$$\text{subject to } Ax = b, \; x \geq 0$$

there corresponds an extreme point of the feasible region.

2. (a) Let us consider the following Linear Programming Problem:

$$\text{Minimize } z = cx$$

$$\text{subject to } Ax = b,\ x \geq 0$$

Let $(x_B, 0)$ be a basic feasible solution corresponding to a basis B having an a_j with $z_j - c_j > 0$ and all corresponding enteries $y_{ij} \leq 0$, then show that Linear Programming Problem has an unbounded solution.

(b) Let $x_1 = 2$, $x_2 = 1$, $x_3 = 1$ be a feasible solution to the system of equations:

$$x_1 + 4x_2 - x_3 = 5$$

$$2x_1 + 3x_2 + x_3 = 18$$

Is this a basic feasible solution? If not, reduce it to two different basic feasible solutions.

(c) Using Simplex method, find the solution of the following Linear Programming Problem:

Minimize $x_1 - 3x_2 + 2x_3$

subject to $3x_1 - x_2 + 2x_3 \leq 7$

$$2x_1 - 4x_2 \geq -12$$

$$-4x_1 + 3x_2 + 8x_3 \leq 10$$

$$x_1, x_2, x_3 \geq 0.$$

(d) Solve the following Linear Programming Problem by Big-M method:

Maximize $x_1 - 4x_2 + 3x_3$

subject to

$$2x_1 - x_2 + 5x_3 = 40$$
$$x_1 + 2x_2 - 3x_3 \geq 22$$
$$3x_1 + x_2 + 2x_3 = 30$$
$$x_1, x_2, x_3 \geq 0.$$

3. (a) Solve the following Linear Programming Problem by Two Phase Method :

Maximize $x_1 + 4x_2 + 3x_3$

subject to

$$x_1 + x_2 + x_3 \geq 4$$
$$-2x_1 + 3x_2 - x_3 \leq 2$$
$$x_2 - 2x_3 \leq 1$$
$$x_1, x_2, x_3 \geq 0.$$

(b) Find the solution of given system of equations using Simplex Method:

$$3x_1 - 2x_2 = 8$$
$$x_1 + 2x_2 = 4$$

Also find the inverse of A where $A = \begin{bmatrix} 3 & -2 \\ 1 & 2 \end{bmatrix}$.

(c) Using Simplex method, find the solution of the following Linear Programming Problem :

Maximize $2x_1 + x_2$

subject to

$$x_1 - x_2 \leq 10$$
$$2x_1 - x_2 \leq 40$$
$$x_1 \geq 0,\ x_2 \geq 0.$$

(d) Find the optimal solution of the Assignment Problem with the following cost matrix :

Job \ Machines	I	II	III	IV	V	VI
A	4	8	5	4	6	9
B	8	3	8	4	11	7
C	9	5	7	9	8	7
D	10	9	5	6	9	9
E	5	11	9	10	10	9
F	9	5	7	10	8	[illegible]

4\. (a) Find the Dual of following Linear Programming Problem :

Minimize $x_1 + x_2 + 3x_3$

subject to

$$4x_1 + 8x_2 \geq 3$$
$$7x_2 + 4x_3 \leq 6$$
$$3x_1 - 2x_2 + 5x_3 = 7$$
$x_1 \leq 0,\ x_2 \geq 0,\ x_3$ is unrestricted.

(b) State and prove the Weak Duality Theorem. Also show that if the objective function values corresponding to feasible solutions of the Primal and Dual Problem are equal then the respective solutions are optimal for the respective Problems.

(c) Using Complementary Slackness Theorem, find optimal solutions of the following Linear Programming Problem and its Dual:

Maximize $4x_1 + 3x_2$
subject to

$$x_1 + 2x_2 \leq 2$$
$$x_1 - 2x_2 \leq 3$$
$$2x_1 + 3x_2 \leq 5$$
$$x_1 + x_2 \leq 2$$
$$3x_1 + x_2 \leq 3$$
$$x_1, x_2 \geq 0.$$

(d) For the following cost minimization Transportation Problem find initial basic feasible solutions by using North West Comer rule, Least Cost Method and Vogel's Approximation Method. Compare the three solutions (in terms of the cost):

Source \ Destination	A	B	C	D	E	Supply
I	15	15	16	17	15	24
II	18	19	16	20	15	38
III	16	15	22	17	20	43
Demand	27	12	32	17	17.	

5. (a) Solve the following cost minimization Transportation Problem :

Destinations / Origin	I	II	III	IV	Availability
A	10	11	10	13	30
B	12	12	11	10	50
C	13	11	14	18	20
Requirements	20	40	30	10	

(b) Four new machines are to be installed in a machine shop and there are five vacant places available. Each machine can be installed at to one and only one place. The cost of installation of each job on each place is given in table below. Find the Optimal Assignment. Also find which place remains vacant.

Place / Machine	A	B	C	D	E
I	13	15	19	14	15
II	16	13	13	14	13
III	14	15	18	15	11
IV	18	12	16	12	10

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 762 **B**

Unique Paper Code : 42341202

Name of the Paper : Database Management Systems

Name of the Course : **B.Sc. (Prog.) / Math. Science**

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Question 1 is compulsory.
3. Answer any five questions out of remaining questions (Q2–Q8).
4. Answer all parts of a question together.

1. Answer the following :

(a) What is data independence? Differentiate between physical and logical data independence. (4)

P.T.O.

(b) What are the responsibilities of DBA? (2)

(c) Define the following terms : (2)

(i) Metadata

(ii) Derived attribute

(d) What is meant by a recursive relationship type? Give its example. (2)

(e) Differentiate between Specialization hierarchy and Specialization lattice. (2)

(f) What are the various reasons that lead to the occurrence of NULL values in relations? Explain with the help of example. (3)

(g) Explain the *DROP* command with *cascade* option in SQL with the help of an example. (3)

(h) What are the various update-anomalies that can occur in a relation? (4)

(i) Given the following table and its associated functional dependencies : (3)

Emp_proj

<u>*Emp_ssn*</u>	<u>*Project_no*</u>	*Hours*	*Emp_name*	*Proj_name*

Emp_ssn → Emp_name

Project_no → Proj_name

Emp_ssn, Project_no → Hours

What is the highest normal form that the relation *Emp_proj* satisfies? Justify your answer.

2. (a) Write any four functionalities of DBMS. (4)

(b) Differentiate between the following : (6)

(i) Entity type and Entity set.

(ii) Centralized and Distributed DBMS.

(iii) Casual End user and Sophisticated end user.

3. Consider a MUSICAL COMPANY database in which data is recorded about the music industry. The data requirements are as follows : (10)

(i) Each musician has an SSN, a name, an address and a phone number.

(ii) Each instrument that is used in the songs has a name and a musical key.

(iii) Each album that is recorded on the company label has a title, a copyright date, a format and an album identifier.

(iv) Each song recorded at the company has a title and an author.

(v) Each musician may play several instruments and several musician may play a given instrument.

(vi) One or more musician perform each song and a musician may perform in a number of songs.

(vii) Each album has exactly one musician who acts as its producer. A musician may produce several albums.

Design an ER diagram for the above specifications and indicate all keys and cardinality constraints. Also state any assumptions that are made.

4. Consider the following schema about *Supplier - Part* database, primary key is underlined. (10)

Part (*<u>Partno</u>, Partname, Color, Weight*)

Project (*<u>Pjno</u>, Pjnante, City*)

Shipment (*<u>Sno, Partno, Pjno,</u> Qty*)

Supplier (*<u>Sno</u>, Sname, Status, City*)

Write SQL commands to express each of the following queries :

(i) Find the Project number of all the projects using the parts that are supplied by *supplier* 'S1'.

(ii) Retrieve supplier names for suppliers who supply *part* 'P4'.

(iii) For each part supplied, retrieve the part number and total quantity supplied for that part.

(iv) Change the color of *part* 'P6' to red and increase its weight by 7.

(v) Insert a new tuple into the relation ***Project***.

5. (a) Describe the three schema architecture. Why do we need mappings between schema levels? (4)

(b) Consider the following schema : (6)

Sailors (<u>Sailor id</u>, Sname, Rating, Age)

Boats (<u>Boat id</u>, Bname, Color)

Reserves(<u>Sailor id, Boat id, Date</u>)

Write the following queries in relational algebra:

(i) Find the names of sailors who have reserved boat 102.

(ii) Find the names of sailors who have reserved a red or a yellow boat.

(iii) Find the Sailor_id of sailors with age over 30 who have not reserved a red boat.

6. (a) Consider the following relations for a database that keeps track of student enrollment in courses and the books adopted for each course : (5)

Student (Ssn, Name, Major, Bdate)

Course (Course#, Cname, Dept)

Enroll(Ssn, Course#, Semester, Grade)

Book_Adoption (Course#, Semester, Book_isbn)

Text (Book_isbn, Book_title, Publisher, Author)

Specify the primary keys and foreign keys for this schema, stating any assumptions you make.

(b) Explain the entity integrity and referential integrity constraints. Why is each considered important? (5)

7. (a) Consider the relation: **R (<u>Dentist no, Appt dt, Appt time</u>, Dentist_Name, Patient_no, Patient_Name, Surgery_No)** with the following FDs : (8)

Dentist_no, Appt_dt, Appt_time → Patient_no, Patient_Name

Dentist_no → Dentist_Name

Patient_no → Patient_Name, Surgery_No

Dentist_no, Appt_dt → Surgery_No

Appt_dt,Appt_time → Dentist_no, Dentist_Name

Patient_no → Dentist_Name

Decompose the above relation to 3 NF. State the reason behind each decomposition.

(b) Consider the following Table 1. (2)

Table 1

X	Y	Z
1	3	8
3	4	2
4	5	3
5	6	4
6	7	8
1	3	8

Which of the following functional dependency constraints do not hold in the Table 1.

(i) YZ → X

(ii) X → Z

(iii) X → Y

(iv) Z → X

8. (a) Map the ER diagram given in Figure 1 to a relational database. Cardinality constraints are given as follows : (8)

(i) BANK and BANK-BRANCH (1 : N)

(ii) BANK-BRANCH and ACCOUNT (1 : N)

(iii) BANK-BRANCH and LOAN (1 : N)

(iv) LOAN and CUSTOMER (M : N)

(v) ACCOUNT and CUSTOMER (M : N)

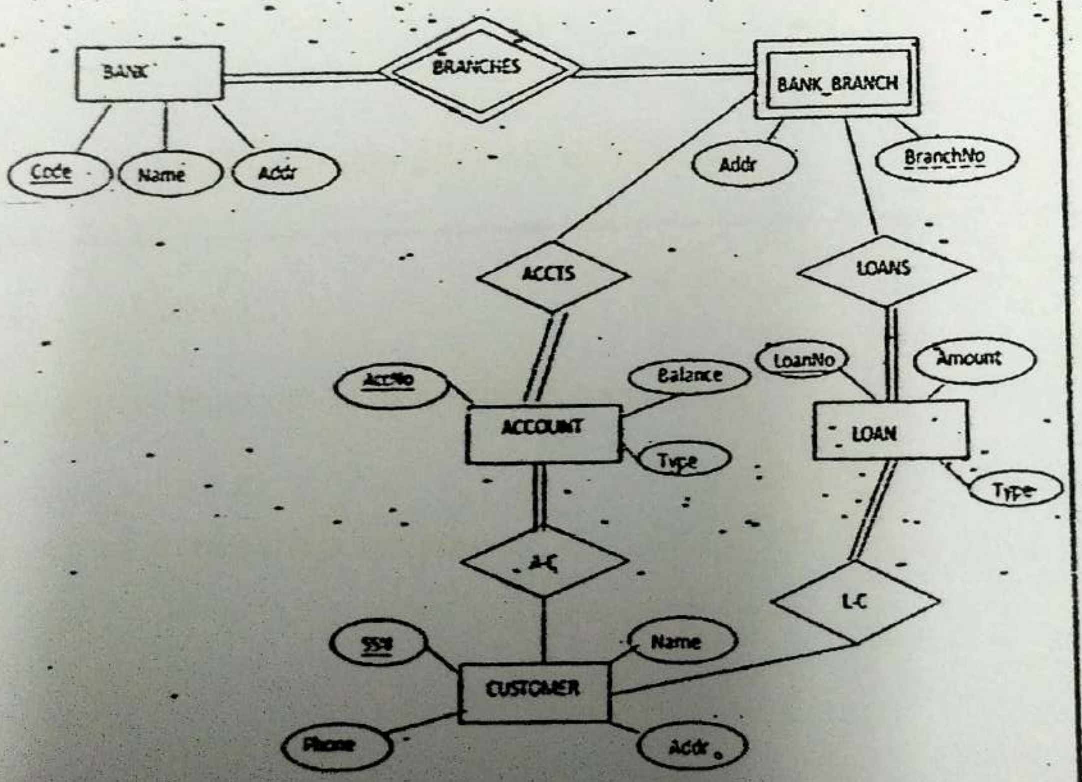


Figure 1

(b) How is EER model different from the ER model? (2)

6

[This question paper contains 4 printed pages.]

Your Roll No..............

B

Sr. No. of Question Paper : 915

Unique Paper Code : 62351201

Name of the Paper : Algebra

Name of the Course : B.A. (Prog.)

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions carry equal marks.

1. (a) Show that the vectors {(1,2,1), (2,1,0), (1,–1,2)} form a basis of $R^3(R)$.

(b) Prove that the set $S=\left\{\begin{bmatrix} x & 0 \\ 0 & 0 \end{bmatrix} : x \in R\right\}$ is a vector space over the field R w.r.t. usual matrix addition and multiplication of a matrix by a scalar.

P.T.O.

(c) Define subspace of a vector space. Show that the set $W = \{(a_1, a_2, a_3): a_3 = 3a_1; a_1, a_2, a_3 \in R\}$ is a subspace of the vector space $R^3(R)$.

2. (a) Find the inverse of the following matrix :

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

(b) Find the rank of the following matrix by reducing it to its normal form:

$$\begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}$$

(c) For what value of λ, the following system of equations has a unique solution and then find the solution :

$$\lambda x + 2y - 2z = 1$$

$$4x + 2\lambda y - z = 2$$

$$6x + 6y + \lambda z = 3$$

3. (a) If $\cos\theta + \cos\varphi + \cos\psi = \sin\theta + \sin\varphi + \sin\psi = 0$, Prove that $\cos 3\theta + \cos 3\varphi + \cos 3\psi = 3\cos(\theta + \varphi + \psi)$, and $\sin 3\theta + \sin 3\varphi + \sin 3\psi = 3\sin(\theta + \varphi + \psi)$.

(b) Prove that

$$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta.$$

(c) Solve the equation

$$z^7 - z = 0.$$

4. (a) Find the sum of the cubes of the roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$.

(b) If α, β, γ be the roots of the equation $x^3 - px^2 + qx - r = 0$, find the value of

(i) $\Sigma(\beta + \gamma)(\gamma + \alpha)(\alpha + \beta)$

(ii) $\Sigma\alpha/\beta$.

(c) Solve the equation $x^3 - 5x^2 - 16x + 80 = 0$, the sum of two of its roots being zero.

5. (a) Find the order of the permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 6 & 1 & 2 & 3 & 4 & 5 \end{pmatrix}.$$

(b) Let $G = \{x \in \mathbb{R} : x > 1\}$ be the set of all real numbers greater than 1. For $x, y \in G$. define $x * y = xy - x - y + 2$. Show that G forms a group under the defined operation $*$.

(c) Give an example of a non-commutative ring with 16 elements.

6. (a) Find the inverse of $\begin{bmatrix} 2 & 6 \\ 3 & 5 \end{bmatrix}$ in the group $GL_2(\mathbb{Z}_{11})$.

(b) Let R be a ring of all continuous functions on the interval [0,1] and $S = \left\{ f \in R : f(x) = 0 \ \forall \frac{1}{2} < x \leq \frac{3}{4} \right\}$.

Prove or disprove that S is a subring of R.

(c) If D_n denotes a Dihedral group of order 2n, then list all the elements of order 2 in the Dihedral group D_4.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1403 A

Unique Paper Code : 42341202

Name of the Paper : Database Management Systems

Name of the Course : B.Sc. (Prog.) / Math. Science

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Question **1** is compulsory.
3. Answer any **five** questions out of remaining questions **(Q2–Q8).**
4. Answer all parts of a question together.

SECTION A

1. Answer the following :

   (a) Draw the notation for the following ER conceptual diagram (3)

(i) Weak entity

(ii) Multivalued attribute

(iii) Derived attribute

(b) For the following binary relationships, find the cardinality ratio. Clearly, state any assumption you make. (3)

(i) Student and teacher

(ii) Student and class

(iii) Course and Teacher

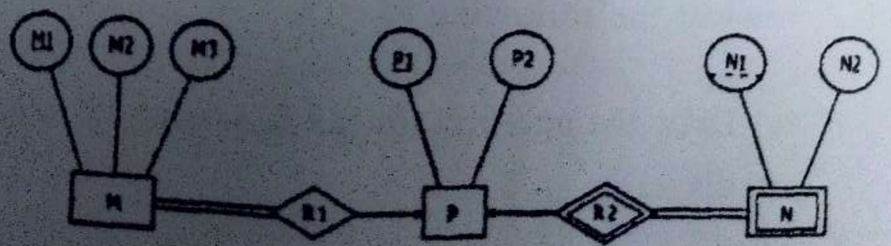
(c) Explain the problem of spurious tuple and how can it be prevented? (3)

(d) Differentiate between the following : (4)

(i) Delete and Drop SQL command

(ii) Database and DBMS

(e) Consider the following ER diagram : (4)



Identify the entities and its attributes in above ER diagram.

(f) What is meant by recursive relationship type? Give an example. (2)

(g) Consider the table named **Customer** having one of the attribute as **CITY**.

Write an SQL query to find all the cities having 'GAR' somewhere in its name. (2)

(h) List any two responsibilities of DBA. (2)

(i) Which SQL command is used to modify the structure of the table? Write its syntax. (2)

SECTION B

2. (a) What is the difference between logical data independence and physical data independence? Which one is harder to achieve? Why? (5)

(b) Discuss the main characteristics of DBMS and how does it differ from traditional file system? (5)

3. (a) Consider the following table Order : (6)

Order_ID	Order_Price	Customers
1	500	Rajesh
2	300	Ramesh
3	100	Suresh
4	600	Rajesh
5	800	Rajesh
6	900	Suresh
7	400	Rakesh

(i) Write SQL command to create the above table with appropriate constraints.

(ii) Write the output when following query is executed:

1. *Select Customer, sum(Order_Price) from orders group by Customers having sum (Order_Price)<=1000*

2. *Select * from Order where Customer Like '%esh'*

(b) Refer to the relational schema given below :

Supplier(Sno, Sname, city)

Parts(Pno, Pname, colour, city)

Projects(Projno, Projectname, city)

Sup_par_proj(Sno, Pno, Projno, Quantity)

Identify the primary key and foreign key in the above relational schema. (4)

4. (a) Consider the two tables T1 and T2 as given below. Show the results of following operations given in Relational Algebra. (10)

T1

P	Q	R
10	a	5
15	b	8
25	a	6

T2

A	B	C
10	a	6
25	c	3
10	b	5

(i) $T1 \cup T2$

(ii) $T1 - T2$

(iii) $T2 - T1$

(iv) $T1 \cap T2$

(v) $T1 \times T2$

5. A college Library maintains a database about students and books having the following information

- Book including ISBN, title, price and author
- Student includes name, Student ID, address, phone, age and Bdate
- Publisher including Publisher_ID, name, phone and address
- Section includes S_id, name and phone

Construct an ER diagram for the above database.

Specify all entities, their attributes and cardinality relational. State assumptions if any. (10)

6. (a) For the relation given below (in tabular format) identify which of the functional dependency holds true :

J	K	L	M
x	1	2	5
x	1	2	6
y	1	3	7
y	1	3	8
z	2	4	9
p	4	7	5

(i) J, K → L

(ii) J → K

(iii) J, K → L, M

(iv) L → K (4)

(b) What does the term unnormalized relation refers to?

Consider the following relation :

Car_Sale(Car_no, Date_Sold, Salesman_no. Commission, Discount_amt)

Normalize it in third normal form given that the additional dependencies are :

Date_Sold → Discount_amt

Salesman_no → Commission (6)

7. Consider the following relations(key of each relation is underlined) : (10)

Sales Person(S_No, S_Name, Commission)

Product(P_Id, Description)

Sale(Date, C_No, S_No, P_Id, Qty)

Customer(C_No, C_Name, C_Address)

Write the following queries in SQL and Relational Algebra

(i) Get the name of the Sales Person who sold Product with P_Id=25

(ii) Get the name of Customers who bought "Table Fans"

(iii) Get the total number of Products sold on "15-09-2020"

(iv) Get the total number of products purchased by each customer (10)

8. Write short note on the following : (10)

(i) Specialization

(ii) Generalization

(iii) Weak Entity Type

(iv) Referential Integrity

(v) Database State

(100)

[This question paper contains 6 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : **1564** A

Unique Paper Code : 42351201

Name of the Paper : Calculus and Geometry, CBCS (LOCF)

Name of the Course : **B.Sc. (Programme) Mathematical Sciences / Physical Sciences**

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has **six** questions in all.
3. Attempt any **two** parts from each question.
4. **All** questions are compulsory.
5. Marks are indicated.

1. (a) Sketch the graph of the function $f(x) = x^4 + 2x^3$. (6.5)

P.T.O.

(b) Sketch a graph of a function f with all the following properties :

The graph has $y = 1$ and $x = 3$ as asymptotes; f is increasing for $x < 3$ and $3 < x < 5$ and is decreasing elsewhere; the graph is concave up for $x < 3$ and for $x > 7$ and concave down for $3 < x < 7$; $f(0) = 4 = f(5)$ and $f(7) = 2$. (6.5)

(c) Identify the symmetries of the curve $r = 1 + 2\cos\theta$ and then sketch the curve. (6.5)

(d) Let $x = 4\tan 2t$, $y = 3\sec 2t$ where $0 \leq t \leq \pi$. Find an explicit relation between x and y. Also, sketch the path described by the given parametric equations over the prescribed interval. (6.5)

2. (a) Evaluate the following limits using L'Hospital's Rule

$\lim_{x\to\pi/2}(\tan x)^{(\pi/2)-x}$ and $\lim_{x\to 0^+}[\tan x \log x]$. (6)

(b) Sketch the graph of $y = (x - 4)^{2/3}$. (6)

(c) Use cylindrical shells to find the volume of the solid that is generated when the region R in the first quadrant enclosed between $y = x$ and $y = x^2$ is revolved about the line $x = 0$. (6)

(d) Find the volume of the solid formed when the region between the graphs of $y = 1 + 2x^2$ and $y = 3 - 2x^2$ is revolved about the x-axis. (6)

3. (a) Find the volume of the solid generated when the region enclosed by $x = 0$, $y = 0$, $x = 1$ and $y = x^2 + 1$ is revolved about the y-axis. (6.5)

(b) Find the length of the curve $y = 2x^2 + 1$ over the interval [1,3]. (6.5)

(c) Find the area of the surface swept out by revolving $y = \sqrt{9 - x^2}$ about the x-axis. (6.5)

(d) Find the arc length of the curve $x = t^3$, $y = 3\frac{t^2}{2}$, $0 \le t \le \sqrt{3}$. (6.5)

4. (a) Derive the reduction formula

$$\int \sin^n x \, dx = -\frac{1}{n}\sin^{n-1} x \cos x + \frac{n-1}{n}\int \sin^{n-2} x \, dx$$

and evaluate the integral $\int_0^{\pi/2} \sin^5 x \, dx$. (6)

(b) Prove that for nonnegative integers m and n,

$$\int_0^{2\pi} \cos mx \cos nx \, dx = 0.$$

(6)

(c) Obtain reduction formula

$$\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx, \quad n > 2$$

and hence using that evaluate the integral $\int \tan^4 x \, dx$.

(6)

(d) Evaluate $\int_0^{\pi/2} \sin^2 \frac{x}{2} \cos^2 \frac{x}{2} dx$ after obtaining the reduction formula for the integral

$$\int \sin^m x \cos^n x \, dx.$$

(6)

5. (a) Describe the graph of the equation

$$y = 4x^2 + 8x + 5.$$

(6.5)

(b) Find equation for the hyperbola that has vertices $(0, \pm 2)$ and asymptotes

$$y = \pm \frac{2}{3} x.$$

(6.5)

(c) Sketch the graph of ellipse and label the foci, vertices and ends of the minor axis

$(x + 3)^2 + 4(y - 5)^2 = 16.$ (6.5)

(d) Rotate the axes of coordinates to get rid of the xy-term from the equation, name the conic $x^2 + 4xy - 2y^2 - 6 = 0$ and sketch its graph. (6.5)

6. (a) Show that the graphs of given $r_1(t)$ and $r_2(t)$ intersect at the point P(1, 1, 3). Find the acute angle between the tangent lines to the graphs of $r_1(t)$ and $r_2(t)$ at this point, where

$$r_1(t) = t^2\hat{i} + t\hat{j} + 3t^3\hat{k}$$

$$r_2(t) = (t-1)\hat{i} + \frac{1}{4}t^2\hat{j} + (5-t)\hat{k}.$$ (6)

(b) Sketch the graph and show direction of increasing t for

$$r(t) = 9\cos t\,\hat{i} + 4\sin t\,\hat{j} + t\hat{k}.$$ (6)

(c) Evaluate $\nabla \times (\nabla U \times \nabla V)$ where $U = x^2yz$, $V = xy - 3z^2$. (6)

(d) Show that divergence of the field

$$F(x, y, z) = \frac{c}{\left(x^2+y^2+z^2\right)^{3/2}}\left(x\hat{i}+y\hat{j}+z\hat{k}\right)$$ is zero.

(6)

[This question paper contains 6 printed pages.]

Your Roll No..............

A

Sr. No. of Question Paper : 1401

Unique Paper Code : 42344403

Name of the Paper : Computer System Architecture

Name of the Course : **B.Sc. (Prog) / Mathematical Science**

Semester : IV

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Question No. **1** is compulsory.
3. Attempt any **5** of questions Nos. **2** to **9**.
4. Parts of a question must be answered together.

1. (a) How many 256 words x 8 bits per word RAM chips are needed to provide a memory capacity of 4096 words x 16 bits per word? (2)

(b) What is radix of the numbers if the solution to the quadratic equation (2)

$x^2 - 10x + 31 = 0$ is $x = 5$ and $x = 8$?

(c) Represent the following conditional control statement by two register transfer statements with control functions.

If $(P = 1)$ then $(R1 \leftarrow R2)$ else if $(Q = 1)$ then $(R1 \leftarrow R3)$ (2)

(d) State any two differences between combinational and sequential circuit. (2)

(e) Give the characteristic table of JK flip-flop. (2)

(f) What is a binary counter? How many flip-flops will be required for an n-bit binary counter? (2)

(g) Consider a memory of capacity 16M words x 32 bits per word. How many address lines and input-output data lines are needed? (2)

(h) Simplify the following expressions using Boolean algebra.

(BC'+A'D) (AB'+CD') (2)

(i) Can the following microoperation be executed during a single clock pulse in the system? Specify a sequence of microoperations that will perform the operation

IR ← M[PC] (2)

(j) How many flip-flops will be complemented in an 8-bit counter to reach the next count after :

(i) 01100111

(ii) 11111111 (2)

(k) Convert the following decimal numbers to the base indicated

(i) 7562 to octal

(ii) 1938 to hexadecimal (2)

(l) Write a short note on input-output interface. (3)

2. (a) Define the full adder. Illustrate same with the help of truth table and logic diagram. Also write Boolean expression for carry and sum operations. (6)

(b) Given two registers A and B with contents as follow –

Register A (before operation)	1010
Register B (logic operand)	1100

Show the contents of A using the contents of B after performing the following operations.

(i) Mask operation

(ii) Selective Complement (4)

3. (a) Design a 4-bit combinational circuit decrementer using four full-adder circuit. Explain its working. (6)

(b) Simplify the given Boolean function using four-variable maps. [Sum of the Products (SOP) form.]

$F(A, B, C, D) = \Sigma(0,2,4,5,6,7,8,10,13,15)$. (4)

4. (a) An instruction is stored at location 300 with its address field at location 301. The address field has the value 400. A processor register R1 contains the number 200. Evaluate the effective address if the addressing mode of the instruction is

(i) immediate

(ii) relative

(iii) index with R1 as the index register. (6)

(b) Explain the concept of Direct Memory Access using block diagram? How does DMA transfer take place? (4)

5. (a) What are the different types of instruction formats?

Given the following instructions (in hexadecimal), identify the category to which each of these belong.

(i) F800

(ii) 7800 (6)

(b) Design a 3x8 decoder using 2x4 decoders. Explain its working. (4)

6. (a) Write a program to evaluate the arithmetic statement :

$$X = (A+B) * (C+D)$$

using zero address and one address instructions. (6)

(b) What is hardwired control unit? Explain its working with a suitable diagram. (4)

[This question paper contains 4 printed pages.]

Your Roll No.............

'A

Sr. No. of Question Paper : 2819

Unique Paper Code : 62354443

Name of the Paper : Analysis (LOCF)

Name of the Course : B.A. (Prog.)

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Attempt any two parts from each question.
4. All questions carry equal marks.

1. (a) Let $S = \{x \in \mathbb{R} : x \geq 0\}$. Show in detail that the set S has lower bounds, but no upper bounds. Show that inf S=0. Verify your answer.

(b) Define continuity of a real valued function at a point.

Show that the function defined as $f(x) = \begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ 6, & x = 3 \end{cases}$

is continuous at $x = 3$.

(c) Let S be a non empty bounded set in $\mathbb{R}$. Let $a > 0$, and let $aS = \{as : s \in S\}$. Prove that $\inf aS = a \inf S$, $\sup aS = a \sup S$.

(d) Test for convergence the series those nth term is $\left(\frac{\sqrt{n+1}-\sqrt{n-1}}{n}\right)$.

2. (a) A function f is defined by

$$f(x)=\begin{cases}\frac{1}{2}-x, & \text{if } 0<x<\frac{1}{2}\\ \frac{3}{2}-x, & \text{if } \frac{1}{2}\le x<1\end{cases}$$

Evaluate $\lim_{x\to\frac{1}{2}} f(x)$

(b) Define order completeness property of real numbers. State and prove Archimedean Property of real numbers.

(c) Show that the function f defined by $f(x)=x^3$ is uniformly continuous in the interval $[0, 3]$.

(d) Prove that a necessary and sufficient condition for a monotonically increasing sequence to be convergent is that it is bounded above.

3. (a) State Cauchy's second Theorem on Limits. Prove that

$$\lim_{n\to\infty}\left[\frac{(2n)!}{(n!)^2}\right]^{1/n}=4$$

(b) Test for convergence the series whose nth term is $u_n=\frac{n^{n^2}}{(n+1)^{n^2}}$.

(c) State Cauchy's general principle of convergence. Apply it to prove that the sequence $\langle a_n\rangle$ defined by

$$a_n=1+\frac{1}{4}+\frac{1}{7}+\dots+\frac{1}{3n-2}$$ is not convergent.

(d) Prove that a sequence of real numbers converges if and only if it is a Cauchy sequence.

4 (a) State D'Alembert's ratio test for the convergence of a positive term series. Use it to test for convergence the series $\sum_{n=1}^{\infty} \frac{2^{n-1}}{3^n+1}$.

(b) A sequence $\langle a_n \rangle$ is defined as follows:

$$a_1 = 1, \quad a_{n+1} = \frac{4+3a_n}{3+2a_n}, \quad n \geq 1$$

Show that sequence $\langle a_n \rangle$ converges and find its limit.

(c) Show that the series

$$\sum_{n=1}^{\infty} \frac{2.4.6.....2n}{1.3.5.....(2n+1)} \text{ diverges.}$$

(d) Prove that if a function f is continuous on a closed and bounded interval $[a, b]$, then it is uniformly continuous on $[a, b]$.

5 (a) State Leibnitz test for convergence of an alternating series of real numbers. Apply it to test for convergence the series $\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{7}} + ...$

(b) Show that the function f defined by

$$f(x) = \begin{cases} 0, & \text{when } x \text{ is rational} \\ 1, & \text{when } x \text{ is irrational} \end{cases}$$

is not integrable on any interval.

(c) Test for convergence and absolute convergence of the following series.

$$\frac{1}{1.2} - \frac{1}{3.4} + \frac{1}{5.6} - \frac{1}{7.8} + ...$$

(d) Show that the sequence defined by $\langle a_n \rangle = \left\langle \frac{n}{n+1} \right\rangle$ is a Cauchy sequence.

6 (a) Show that every Monotonic function on $[a, b]$ is integrable on $[a, b]$

(b) Test the convergence of the series $\sum_{n=1}^{\infty} (-1)^n \frac{\sin n\alpha}{n^p}$, $p > 0$. Is this series absolutely convergent.

(c) Show that the function $f(x) = [x]$, where $[x]$ denotes the greatest integer not greater than x, is integrable over [0, 3] and $\int_0^3 [x]dx = 3$

(d) Show that the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \tan\frac{1}{n}$ is convergent.

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 2779 A

Unique Paper Code : 62357603

Name of the Paper : Numerical Methods

Name of the Course : B.A. (Prog.)

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All six questions are compulsory.
3. Attempt any two parts from each question.
4. Use of Non – Programmable Scientific Calculator is allowed.

Q-1. (a) Find the real root of the equation $2x^3 - 3x + 1 = 0$ by Regula Falsi method. Perform two iterations. [6]

(b) Use secant method to find root of $3x + \sin(x) - e^x = 0$ in $]0,1[$. Perform two iterations. [6]

(c) Define the floating point representation, Global error and Truncation error with examples. [6]

(d) Obtain the Rate of Convergence of Bisection method. [6]

Q-2. (a) Find the value of :-

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

with an absolute error smaller than 0.005 for x = 0.2145E0 using Normalized floating point arithmetic with 4 digit mantissa. [6.5]

(b) Evaluate the sum $S = \sqrt{3} + \sqrt{8} + \sqrt{10}$ to four significant digits and find its absolute and relative error. [6.5]

(c) Use Bisection method to find a real root of the equation

$$f(x) = 2x - \sqrt{(1+\sin x)} = 0.$$ [6.5]

(d) Find a real root of the equation $3x = \cos x + 2$ by Newton- Raphson method. [6.5]

Q-3. (a) The function $y = f(x)$ is given at the point $(7,3), (8,1), (9,1)$ and $(10,9)$. Find $f(8.5)$ using Lagrange's interpolation technique. [6]

(b) Find the inverse of the following matrix using the Gauss-Jordan method:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}.$$ [6]

(c) For the following system of equations:

$$4x + y + 2z = 4$$
$$3x + 5y + z = 7$$
$$x + y + 3z = 3.$$

Use Gauss-Jacobi iteration method by performing three iterations. Take the initial approximation as $(x, y, z) = (0,0,0)$. [6]

(d) If $f(x) = \frac{1}{x}$ then evaluate Newton Dividend difference $f[a,b,c,d]$. Also prove the following relation:

$$(1 - \nabla)^{-1} = 1 + \frac{\delta^2}{2} + \delta\sqrt{1 + \frac{\delta^2}{4}}$$ [6]

Q-4. (a) Solve the linear system $Ax = b$ using Gauss-Elimination method with row pivoting:

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 4 & -2 & 1 \\ 3 & -1 & 3 \end{bmatrix} \text{ and } b = \begin{bmatrix} 3 \\ 5 \\ 8 \end{bmatrix}$$ [6.5]

(b) Starting with initial vector $(x, y, z) = (1, 1, 1)$, perform three iterations of Gauss-Seidel method to solve the following system of equation:

$$10x + 2y + z = 9$$

$$2x + 20y - 2z = -44$$

$$-2x + 3y + 10z = 22.$$ [6.5]

(c) Find the cubic polynomial which takes the following values: [6.5]

x	0	1	2	3
$f(x)$	1	2	1	0

(d) Apply Gauss-Jordan method to solve:

$$x + 2y + z = 8$$

$$2x + 3y + 4z = 20$$

$$4x + 3y + 2z = 16.$$ [6.5]

Q-5. (a) The velocity v (km/min) of a moped which starts from rest, is given at fixed intervals of time t (min) as follows: [6]

t	0	2	4	6	8	10	12
v	0	10	18	25	29	32	20

Estimate approximately the distance covered in 12 minutes by Simpson $\frac{1}{3}$rd rule.

(b) Solve $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$, at $y = 0.1$ by taking $h = 0.02$ by using Euler's method. [6]

P.T.O.

(c) Apply modified Heun's method to calculate $y(1)$, given that

$$\frac{dy}{dx} = x + 2y; y(0) = 0; h = 0.5.$$ [6]

(d) Evaluate $I = \int_0^1 x\sqrt{1+x}\, dx$ using Trapezoidal rule with 4 subintervals. [6]

Q-6. (a) Evaluate $\int_0^{\pi} \sin x \, dx$ using Simpson rule by dividing interval into four equal parts.

[6.5]

(b) Calculate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = 5$, given the following table: [6.5]

x	2	4	9	10
$f(x)$	4	56	71	90

(c) Apply modified Euler's method to approximate the solution of the initial value problem and calculate $y(1.3)$ by using $h = 0.1$:

$$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}, \; y(1) = 1.$$ [6.5]

(d) The following table of values is given:

x	0.6	0.8	0.9	1.0	1.1	1.2	1.4
$f(x)$	0.7072	0.8599	0.9259	0.9840	1.0337	1.0746	1.1280

using the formula $f'(x_0) = \frac{f(x_0+h)-f(x_0-h)}{2h}$, with $h = 0.4, 0.2, 0.1$ and the

find $f'(1)$ [6.5]

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1607 A

Unique Paper Code : 42357618

Name of the Paper : DSE – NUMERICAL METHODS

Name of the Course : B.Sc. Mathematical Sciences / B.Sc. (Prog.)

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any two parts of each question.
3. All questions carry equal marks.

Q 1(a) If $X = 2.536$, find the absolute error and relative error when, X is truncated to two decimal digits. (6.25)

(b) Find the relative error of the number 6.4, if both of its digits are correct. (6.25)

(c) Write the order of convergence of Bisection Method, Secant Method and Newton-Raphson Method. Also name these methods in decreasing order w.r.t. rate of convergence. (6.25)

P.T.O.

(d) Determine the number of significant digits in the following numbers. (6.25)

(I) 0.6500025, (II) 0.000232317, (III) 50.00045, (IV) 8×10^{-9}.

Q. 2(a) Perform five iterations of the Bisection method to obtain the smallest positive root of the equation $f(x) = x^3 + 2x - 2 = 0$. (6.25)

(b) Using Regula-Falsi method compute the real root of the equation $x^2 = 7$. Correct to four decimal places. (6.25)

(c) Using Newton-Raphson Method compute $\sqrt{17}$ correct to four decimal places. (6.25)

(d) Using Secant method, find the smallest positive root of the equation $x^3 + 3x^2 = 1$ correct to three decimal digits. (6.25)

Q. 3(a) Solve the following system of equations AX= b where (6.25)

$$A = \begin{bmatrix} 2 & 0 & -2 \\ 3 & -4 & -4 \\ -2 & 2 & -1 \end{bmatrix}, \quad b = \begin{bmatrix} -10 \\ -8 \\ 3 \end{bmatrix},$$

using Gauss elimination method by using partial pivoting.

(b) Find the interpolating formula for (6.25)

x	0	1	2	3
y	1	2	4	8

In Newton form.

(c) Consider the following table: (6.25)

x	-1	2	4	5
y	-5	13	255	625

Use Lagrange interpolation to estimate (0.25).

(d) Find a cubic polynomial which take the following values using Newton forward difference formula y(1)= 24, y(3)= 120, y(5)= 336, y(7)= 720. Also, find y(8)? (6.25)

Q. 4(a) Perform three iterations to solve the linear system (6.25)

$$8x + y - z = 8,$$
$$-x + 7y - 2z = 4,$$
$$2x + y + 9z = 12,$$

Using Gauss-Jacobi iteration method by taking the initial approximation as $(x,,z)=(0,0,0)$.

(b) Find the unique polynomial P(x) of degree 2 or less such that P(1)= 1, P(3)= 27, P(4)= 64, using Newton divided difference formula. Estimate P(1.5). (6.25)

(c) Solve the following system of equations (6.25)

$$6x + 2y + 2z = 8,$$
$$6x + 2y + z = 4,$$
$$x + 2y - z = 12,$$

using Gauss Jordan Method.

(d) Obtain the piecewise linear interpolating polynomial for (6.25)

x	0.5	1.5	2.5
$f(x)$	0.125	3.375	15.625

Interpolate at x = 2.0.

Q. 5(a) Apply Richardson extrapolation when $f(x) = e^{-x} + Sinx + x^3, x = 1.2, h = 0.4$ using central divided difference formula $f'(x) = \frac{f(x+h)-f(x-h)}{2h}$. (6.25)

(b) Compute $\int_1^6 x^3\, dx$ by trapezoidal rule with n = 4. (6.25)

(c) Find Richardson extrapolation of $f(x) = e^x - Sin\, x$ when x = 1, h = 0.5, h = 0.25 using central divided difference formula $f'(x) = \frac{f(x+h)-f(x-h)}{2h}$. (6.25)

(d) Compute the value of $\int_1^2 \frac{dx}{x}$ with h = 0.25 using Simpson's rule. (6.25)

Q. 6(a) Using Euler modified method, obtain a solution of $\frac{dy}{dx} = x + |\sqrt{y}|$, $y(0) = 1$ for the range [0, 0.6] in steps of 0.2. (6.25)

(b) Use Euler's method and its modified form to obtain y(0.2), y(0.4), and y(0.6) correct to three decimal places, given that $y' = y - x^2$ with initial condition $y(0) = 1$. (6.25)

(c) Compute the value of $\int_0^1 \frac{dx}{1+x^2}$ using trapezoidal rule with h = 0.25. (6.25)

(d) Compute the value of $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's three eight rule. (6.25)

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B.Sc. (P)
Maths

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s question paper contains 8 printed pages.]

Your Roll No..............

No. of Question Paper : 1608 A

ue Paper Code : 42357602

e of the Paper : DSE – Probability and Statistics

e of the Course : B.Sc. Mathematical Sciences

ester : VI

tion : 3 Hours Maximum Marks : 75

ructions for Candidates

Write your Roll No. on the top immediately on receipt of this question paper.

Attempt all the six questions.

Each question has four parts. Attempt any two parts from each question.

Each part in Question 1, 3, 5 carries 6 marks.

Each part in Question 2, 4, 6 carries 6.5 marks.

Use of scientific calculator is allowed.

(a) A secretary types three letters and three corresponding envelopes. In a hurry, he places at random one letter in each envelope. What is the probability that at least one letter is in correct envelope?

(b) Let $\{C_n\}$ be a nondecreasing sequence of events. Show that

$$\lim_{n\to\infty} P(C_n) = P\left(\lim_{n\to\infty} C_n\right) = P\left(\bigcup_{n=1}^{\infty} C_n\right).$$

(c) Let $p_X(x)$ be the pmf of a random variable X. Find and sketch the cdf $F_X(x)$ of X, where

$$p_X(x) = \begin{cases} \frac{x}{15} & x = 1, 2, 3, 4, 5 \\ 0 & \text{elsewhere} \end{cases}$$

Also find

(i) $P(X = 1 \text{ or } 2)$

(ii) $P\left(\frac{1}{2} < X < \frac{5}{2}\right)$

(iii) $P(1 \le X \le 4)$

(d) Define cumulative distribution function. Let X be a random variable with cumulative distribution function F(x). Show that

$$\lim_{x \downarrow x_0} F(x) = F(x_0)$$

for all $x_0 \in \mathbb{R}$.

2. (a) A bowl contains 10 chips, of which 8 are marked \$2 each and 2 are marked \$5 each. A person chooses three chips at random without replacement from this bowl. If person is to receive the sum of the resulting amounts. Find his expectation.

(b) Find the moment generating function of Normal Distribution. Also, find its mean and variance using moment generating function.

(c) Let a random variable of continuous type have a pdf $f(x)$ whose graph is symmetric with respect to the line $x = c$. If the mean value of X exists. Show that $E(X) = c$.

(d) If the probability is 0.75 that a person will believe a rumor about a certain actor. Find the probability that

(i) the fifth person to hear the rumor will be the second to believe it.

(ii) the sixth person to hear the rumor will be the fourth to believe it.

3. (a) Find the joint probability density of the two random variables X and Y whose joint distribution function is given by :

$$F_{X,Y}(x,y)=\begin{cases}\left(1-e^{-x^2}\right)\left(1-e^{-y^2}\right); & x>0,\ y>0\\ 0 & ;\ \text{otherwise}\end{cases}$$

Then use the joint pdf to obtain $P[1 < X \leq 2,\ 1 < Y \leq 2]$.

(b) Let X and Y have the joint probability mass function :

$$p(m,n)=\begin{cases}\dfrac{1}{2^{(m+1)}}; & m \geq n\\ 0 & ;\ m < n\end{cases}$$

for m, n = 1,2,...... Verify that it satisfies the properties of a joint pmf. Compute the marginal probability mass functions.

(c) Consider the experiment of tossing two tetrahedra with sides numbered 1 to 4. Let X denotes the smaller of the two downturned numbers and Y be the larger.

(i) Find the joint mass function of X and Y.

(ii) Find $P[X \geq 2,\ Y \geq 2]$.

(iii) Find conditional pmf of X given Y.

(d) Suppose that X and Y are jointly continuous random variables with joint density :

$$f_{X,Y(x,y)} = \begin{cases} c\,x^2y : & 0 < x < y < 2 \\ 0 : & \text{otherwise} \end{cases}$$

(i) Find the value of c ?

(ii) What is the probability that X < 2Y ?

(iii) What are the marginal densities f_X and f_Y ?

4. (a) Let X and Y be two random variables with joint pdf :

$$f(x,y) = \begin{cases} 5\,xy : & 0 < x,\ y < 1 \\ 0 : & \text{otherwise} \end{cases}$$

(i) Find joint moment generating function of X and Y.

(ii) Using joint mgf, compute E(XY) and E(X).

(iii) Compute E(2X − 4XY).

(b) If the joint probability density of X and Y is given by :

$$f(x, y) = \begin{cases} 24y(1-x-y) & x > 0, y > 0, x + y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Compute conditional mean and variance of Y given $X = x,\ x > 0$.

(c) Let X and Y be discrete random variables. Then prove the following :

(i) $E[E(Y \mid X)] = E(Y)$

(ii) $\text{var}(Y) = E[\text{var}(Y \mid X)] + \text{var}(E(Y \mid X))$.

(d) If two cards are randomly drawn (without replacement) from an ordinary deck of 52 playing cards, X is the number of spades obtained in the first draw, and Y is the total number of spades obtained in both draws, find

(i) the joint cumulative distribution function of X and Y;

(ii) conditional cdf of X given Y = y.

5. (a) If X is a random variable with mean μ and variance σ^2, then prove that for any $k > 0$

$$P\{|X-\mu| < k\sigma\} \geq 1-\frac{1}{k^2}.$$

(b) Use the Central limit theorem to prove that if X is a random variable having binomial distribution with parameters n and θ, then

$$\frac{X-n\theta}{\sqrt{n\theta(1-\theta)}} \to N(0,1) \text{ as } n\to\infty.$$

(c) Given the joint density

$$f(x,y)=\begin{cases} 2 & \text{for } 0<y<x<1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the regression equation of Y on X.

(d) Using method of least squares fit a straight line for the following data

X	1	2	3	4	6	8
Y	2.4	3	3.6	4	5	6

6. (a) Let X and Y have joint probability mass function described as follows

(x, y)	(1, 1)	(1, 2)	(1, 3)	(2, 1)	(2, 2)	(2, 3)
p(x, y)	$\frac{2}{15}$	$\frac{4}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{4}{15}$

Find the coefficient of correlation of X and Y.

(b) Find $P\left(0<X<\frac{1}{3},\ 0<Y<\frac{1}{3}\right)$, if the random variable X and Y have joint probability density function

$$f(x,y)=\begin{cases}4x(1-y) & 0<x<1,\ 0<y<1\\ 0 & \text{elsewhere}\end{cases}$$

(c) If the regression of Y on X is linear, then show that

$$E[Y|X]=\mu_2+\frac{\rho\sigma_2}{\sigma_1}(X-\mu_1).$$

(d) Let X_i, i = 1 to 5 be independent random variables, each being uniformly distributed over (0,1). Use the Markov's inequality to get bound on

$$P[X_1+X_2+X_3+X_4+X_5\geq 8].$$

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1666 A

Unique Paper Code : 42357618

Name of the Paper : DSE – NUMERICAL METHODS

Name of the Course : **B.Sc. Mathematical Sciences / B.Sc. (Prog.)**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts of each question.
3. **All** questions carry equal marks.

Q 1.(a) If X = 2.536, find the absolute error and relative error when X is rounded-off to two decimal digits. (6.25)

(b) Find the relative error of the number 9.5, if both of its digits are correct. (6.25)

(c) Write the order of convergence of Bisection Method, Secant Method and Newton-Raphson Method. Also name these methods in decreasing order w.r.t. rate of convergence. (6.25)

(d) Determine the number of significant digits in the following numbers. (6.25)

(I) 0.60549, (II) 8.00889, (III) 458900, (IV) 0.87×10^{-9}.

Q. 2(a) Perform five iterations of the Bisection method to obtain the smallest positive root of the equation $f(x)=x^3+4x-1=0$. (6.25)

(b) Using Regula-Falsi method compute the real root of the equation $x^2=3$. Correct up to four decimal places. (6.25)

(c) Using Newton-Raphson Method compute $\sqrt{19}$ correct to four decimal places. (6.25)

(d) Using Secant method find the smallest positive root of the equation $x^3-2x^2=2$ correct up to three decimal digits. (6.25)

Q. 3 (a) Explain the term partial pivoting. Solve the following system of equations using Gauss-Jordan method (6.25)

$$2x+6y+10z=1,$$
$$x+3y+33z=2,$$
$$3x+14y+28z=33,$$

(b) Approximate the solution of AX= b where (6.25)

$$A=\begin{bmatrix}5 & -1 & 0\\ -1 & 5 & -1\\ 0 & -1 & 5\end{bmatrix},\ b=\begin{bmatrix}9\\ 4\\ -6\end{bmatrix},$$

with $x^{(0)}=[0,0,0]^{(t)}$ using Gauss- Seidel iteration method by performing three iterations.

(c) Consider the following table: (6.25)

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	1.40	1.56	1.76	2.00	2.28

Obtain the Newton forward and Newton backward difference polynomials. Are they same? Estimate f(0.35).

(d) Obtain the piecewise quadratic interpolating polynomial for (6.25)

x	-2	-1	1	2	4
$f(x)$	-29	-8	-2	-5	7

interpolate at x = 3.0.

Q. 4 (a) Perform three iterations to solve the linear system (6.25)

$$2x - y + z = -1,$$
$$x + 2y - z = 6,$$
$$x - y + 2z = -3,$$

using Gauss-Jacobi iteration method by taking the initial approximation as $(x,y,z) = (0,0,0)$.

(b) Construct the interpolating polynomial by using Gregory- Newton backward difference interpolation formula for the given data: (6.25)

x	1	1.5	2.0	2.5
$f(x)$	2.7183	4.4817	7.3891	12.1825

Estimate the value of f(2.25).

(c) Show that (6.25)

(i) $\mu = \left[1 + \frac{\delta^2}{4}\right]^{1/2}$, (ii) $\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$.

Also, if $f(x) = \frac{1}{x^2}$, find the divided difference $f[x_1, x_2, x_3, x_4]$.

(d) Given that f(0)= 1, f(1)= 3, f(3)= 55, find the unique polynomial of degree 2 or less, which fits the given data by Lagrange interpolation. (6.25)

Q. 5(a) Compute the value of $\int_0^6 \frac{dx}{1+x^2}$ using trapezoidal rule taking n = 6. (6.25)

(b) Find Richardson extrapolation of $f(x) = -2e^{-2x}$ when x=0.35, h=0.25 with the help of central divided difference formula $f'(x) = \frac{f(x+h) - f(x-h)}{2h}$. (6.25)

(c) Compute the value of $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's one third rule. (6.25)

(d) Evaluate $\int_1^7 \frac{dx}{x}$ by Richardson's extrapolation method using central divided difference formula $f'(x) = \frac{f(x+h) - f(x-h)}{2h}$. (6.25)

Q. 6(a) Calculate by Simpson's rule an approximate value of $\int_{-3}^{3} x^4 \, dx$ by taking seven equidistant ordinates. (6.25)

(b) Solve by Euler's method, the initial value problem (6.25)

$\frac{dy}{dx} = \frac{x-y}{2}, y(0) = 1$ over [0, 3], using step size 0.5.

(c) Apply Heun's method to compute $y(0.2)$ where $\frac{dy}{dx} = x + 2y,\ y(0) = 0,\ h = 0.1.$ (6.25)

(d) Compute $\int_1^6 x^3 \, dx$ by trapezoidal rule with n = 4. (6.25)