Name of the Course: CBCS B.Sc.(H)MathematicsUnique Paper Code: 32351101_OCName of the Paper: C1-CalculusSemester: IDuration: 3 HoursMaximum Marks: 75

Attempt any four questions. All questions carry equal marks.

Sketch the graph of the function f(x) = x-6/(x-18) by determining intervals of increase and decrease, relative extrema, concavity, infection points and horizontal/vertical asymptotes (if any). Determine whether the graph of the given function has a vertical tangent or a cusp. If yes, find the same:

$$f(x) = (4x - 1)^{3/4} + 3$$

It is estimated that when x units of a certain commodity are produced, the total cost (in rupees) is given by $C(x) = \frac{x^2}{2} + 16x + 800$ and they can all be sold at a price of (150 - x) rupees per unit. Find the level of production that would minimize the average cost.

2. Identify and sketch the curve: $3x^2 + y^2 + 2\sqrt{3}x - 8x + 8\sqrt{3}y = 0$.

Sketch the polar curve: $r = 3cos3\theta$.

Evaluate $\lim_{x\to\infty} \left[\frac{e^{5x}}{\cosh 5x}\right]$.

3. Find the length of the arc of the curve $y = \log \left[\frac{e^x - 1}{e^x + 1}\right]$ from x = 1 to x = 2.

Find the area of surface generated by revolving the curve x = 3 + 2t, y = 9 - 3t; $1 \le t \le 4$ about y - axis.

Let R be the region between the curves $y^2 = x^3$ and $x^2 = y^3$. Find the volume of the solid generated when R is revolved about x - axis.

4. A ball is thrown from the ground level so as to just clear a wall 4 meters high at a distance of 4 meters from the point of projection and falling at a distance of 14 meters from the wall. Find the magnitude and direction of the velocity of the projection of the ball with the horizontal.

A particle moves in space with acceleration $\vec{a}(t) = e^t \hat{i} + 2t \hat{j} - 2\sin 2t \hat{k}$. Find the particle's position if the position at time t=0 is at the point (2, 1, -1) and $\vec{v}(0) = \hat{i} + \hat{k}$.

Find the arc length parametrization of the line x = -5 + 3t, y = 2t, z = 5 + t that has the same direction as the given line and has reference point (-5, 0, 5).

5. Find the curvature and radius of curvature for the graph of vector equation

 $\vec{r}(t) = e^t \cos t \,\hat{\imath} + e^t \sin t \,\hat{\jmath} + e^t \,\hat{k}$ at t=0.

Determine for what values of t the vector valued function

 $\vec{f}(t) = \langle ln(t+1), |t+2|, [t] \rangle$ is continuous?

If the foci of an ellipse are at points (3,1) and (3, -5) with length of major axis as 12, find its equation.

6. If $\vec{R}(t)$ is the position vector of a particle in a plane at time t, find the time in the given interval when the velocity and acceleration are orthogonal, where

 $\vec{R}(t) = (2t - 5 \operatorname{sint})\hat{\iota} + (2 - 5 \operatorname{cost})\hat{j} \quad ; \quad 0 \le t \le 2\pi.$

Evaluate the integral $\int_{-\pi/4}^{\pi/4} 6tan^4 x \, dx$ using the Reduction formula.

If $q = tan^{-1}p$, prove that $(1 + p^2)q_{n+2} + 2(n+1)pq_{n+1} + n(n+1)q_n = 0$.

Unique Paper Code	: 32351102_OC
Name of Paper	: C2 – Algebra
Name of Course	: CBCS BSc. (H) Mathematics
Semester	: I
Duration	: 3 hours
Maximum Marks	: 75

- Find the fifth root of the complex number (√3 + i) and represent them in the complex plane.
 Solve the equation z⁸ z⁵ + z³ 1 = 0.
 Express sin7θ in terms of powers of sinθ and cosθ.
- 2. For $a, b \in N$ define the relation ~ on N by $a \sim b$ if and only if $a \div b = 2^k$ for some integer k.
 - (i) Prove that \sim defines an equivalence relation on N
 - (ii) Find the equivalence class $\overline{1}$.

Let $A = \{x \in R \mid x \neq 1\}$. Show that the function $f: A \to R$ defined by $f(x) = 4 + \frac{1}{x-1}$, is one to one. Also find the range and suitable inverse of f(x).

Show that the composition of two one to one functions is also one to one but the converse is not true.

- 3. Prove that the intervals $(2, \infty)$ and $(4, \infty)$ have the same cardinality. Express gcd (260,154) in the form 260x + 154y, where x, y are integers. Find the value of $2^{100} (mod 11)$.
- 4. Describe all solutions of the following system in parametric vector form. Also give geometric description of the solution set.

$$2x + 2y + 2z = 0$$
$$-2x + 5y + 2z = 1$$
$$8x + y + 4z = -1$$

Show that the following set of vectors in \mathbb{R}^3 is linearly dependent.

$$\left\{ \begin{bmatrix} 4\\2\\1 \end{bmatrix}, \begin{bmatrix} 2\\6\\-5 \end{bmatrix}, \begin{bmatrix} 1\\-2\\3 \end{bmatrix} \right\}$$

5. $\beta = \{b_1, b_2, b_3\}$ is an ordered basis for R^3 where $b_1 = \begin{bmatrix} 2\\2\\2 \end{bmatrix}$, $b_2 = \begin{bmatrix} 0\\1\\-1 \end{bmatrix}$, $b_3 = \begin{bmatrix} 2\\1\\-3 \end{bmatrix}$.

For $v = \begin{bmatrix} 0 \\ -1 \\ 7 \end{bmatrix}$ find $[v]_{\beta}$, the coordinate vector of v relative to the ordered basis β .

Find the basis and dimensions of Col(A) and Nul(A) for the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 5 & -2 \\ 3 & 8 & 13 & -3 \end{bmatrix}$$

Also verify Rank - Nullity theorem for the Matrix A.

If the standard matrix of a linear transformation T: $R^4 \rightarrow R^3$

is the matrix A (given above) find T $\begin{bmatrix} 1 \\ -1 \\ 2 \\ 2 \end{bmatrix}$

6. Find the characteristic polynomial of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -3 & 0 \\ 4 & -13 & 1 \end{bmatrix}$

Show that $\lambda = -2$ is an eigen value of A and find one eigen vector

of A corresponding to $\lambda = -2$

- T: $R^3 \to R^3$ is a linear transformation such that T $(x_1, x_2, x_3) = (x_2 + 4x_3, x_1 + 2x_2 - x_3, 5x_1 + 8x_2)$
- (i) Find the standard matrix of T
- (ii) Is T one to one? Is T onto? Is T invertible? Justify your answer.

(iii) Find x such that $T(x) = \begin{bmatrix} 1 \\ 2 \\ 8 \end{bmatrix}$

Name of the course	:	B.Sc.(H) Mathematics
Unique Paper Code	:	32351101
Name of the Paper	:	C-1 Calculus (BMATH 101)
Semester	:	Ι
Duration	:	3hours
Maximum Marks	:	75

1. Sketch the graph of $f(x) = x^3 - 12x + 4$ by finding intervals of increase and decrease, critical points, relative extrema and concavity of the given function.

Find the *n*th derivative of

$$y = e^{2x} (\sin^3 x + \cos^2 x).$$

If $y = \log(x + \sqrt{x^2 + 1})$, prove that

$$(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2y_n = 0.$$

7+5+6.75

2. Find

i. $\lim_{x \to \frac{\pi}{2}} (tanx - secx)$

ii.
$$\lim_{x \to 1} \left(\frac{x}{x-1} - \frac{1}{\log x} \right)$$

iii.
$$\lim_{x\to 0} (\cot x - \frac{1}{x})$$

Sketch the parabola

$$\left(x-\frac{1}{2}\right)^2 = 2(y-1)$$

and label the focus, vertex, axis and directrix.

Find the centre, vertices, foci and ends of minor axis of the ellipse

$$3x^2 + 4y^2 - 30x - 8y + 67 = 0.$$
6+6.75+6

3 (i) Find the volume of the solid that is generated by revolving the region bounded by the lines y = x, y = 1 and x = 0 about the x - axis.

(ii) Use cylindrical shells to find the volume of the solid generated when the region bounded by the curves y = x and $y = x^2$ is revolved about the x - axis. (iii)Find the arc length of the curve $f(x) = 3x^{\frac{3}{2}} - 1$ over the interval [0,1]. (7+6.75+5)

4 (i) Find the tangent vector and parametric equations for the tangent line to the graph of the vector function

$$\overrightarrow{F(t)} = e^{2t}\hat{\imath} + (t^2 - t)\hat{\jmath} + (logt)\hat{k}$$

at the point P corresponding to t = 0.2.

(ii) If $\overline{r(t)}$ is the position of a particle in plane at time , find the time in the given interval when the velocity and acceleration are orthogonal, where

$$\overrightarrow{r(t)} = (t - sint)\hat{\imath} + (1 - cost)\hat{j}, 0 \le t \le 2\pi$$

(iii) A shell is fired from ground level with a muzzle speed of 280 ft/s and at an elevation of 45° from the ground level.

- a) Find the maximum height attained by the shell.
- b) Find the velocity and speed of the shell at impact. (6+6+6.75)

5 Find horizontal asymptotes (if any) for the graph of

$$y = \left(\frac{x+1}{x+2}\right)^x$$

Trace the curve $r = 2cos3\theta$

A firm determines that x units of its products can be sold daily at p dollars per unit

where
$$x = 1000 - p$$

The cost of producing x units per day is C(x) = 3000 + 20 x

Find average revenue, profit function, maximum profit. What price per unit should be

charged to obtain maximum profit. (6+6+6.75)

6 Find the curvature and radius of curvature for

$$r = 1 + \cos \theta$$
 at $\theta = \pi/2$

Given \vec{v} and \vec{a} are velocity and acceleration (respectively) of a moving particle at a certain instant of time.

$$\vec{v} = 3 \hat{\iota} - 4 \hat{j}$$
, $\vec{a} = \hat{\iota} - \hat{j} + 2\hat{k}$

Find tangential and normal components of velocity and acceleration, unit tangent vector and unit normal vector at this instant.

Evaluate

$$\int_0^{\pi} \sin^5\theta \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \, d\theta$$

(6+7.75+5)

Name of Course	: CBCS-2 (LOCF) B.Sc. (H) Mathematics
Unique Paper Code	: 32351102
Name of Paper	: C2-Algebra BMATH102
Semester	: I
Duration	: 3 hours
Maximum Marks	: 75 Marks

- **1.** Solve the equations:
 - (a) $6x^4 13x^3 35x^2 x + 3 = 0$ given that one root is $2 \sqrt{3}$.
 - (b) $2x^3 3x^2 11x + 6 = 0$ given that all roots are rational.
 - (c) $x^4 8x^3 + 14x^2 + 8x 5 = 0$ given that roots are in arithmetic progression.
- 2. Find |z|, arg z, Arg z, arg (-z) and arg \overline{z} for

$$z = (1+i)^6 (2\sqrt{3}+2i)^8.$$

Solve the equation $z^4 = -7 + 24i$.

- 3. Prove that the following relations '~' defined on \mathbb{R}^2 are equivalence relations. Also give the interpretation of the equivalence classes in each case.
 - (a) $(a,b) \sim (c,d)$ iff $3a^2 + 4b^2 = 3c^2 + 4d^2$.
 - (b) $(a,b) \sim (c,d)$ iff 2a + 5b = 2c + 5d.

Let $A = \{a, b, c\}$. List all the equivalence relations on A.

- 4. For what values of *x* the graph of the following functions break (jump):
 - (a) $y = \lfloor 4x 3 \rfloor$ where $\lfloor \cdot \rfloor$ denotes floor function.
 - (b) $y = \left\lceil \frac{x}{3} + 2 \right\rceil$ where $\left\lceil \cdot \right\rceil$ denotes ceiling function.

Find the greatest common divisor of 94 and 120. Express it in the form 94m + 120n, for some integers *m* and *n*.

Using principle of mathematical induction, prove that $7^n - 1$ is divisible by 6 for $n \ge 1$.

5. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be defined by

$$T(x, y, z) = (-x + 4y + 5z, x + z, 2y + z).$$

Check whether T is a linear transformation or not. Find $[T]_{\beta}$, where β is the standard ordered bases for \mathbb{R}^3 . Find the eigenvalues and the eigenvectors associated to $[T]_{\beta}$.

6. Solve the following system of linear equations:

$$x + 3y + 2z = 3$$
$$2x + y + 4z = 2$$
$$3x + 2y + 7z = 5$$

by reducing it to matrix form AX = B. Find the basis and dimension for the null space of A. Find row space of A and also find the rank of the matrix A.

Unique Paper Code	: 32353301_OC
Name of Paper	: Latex and HTML
Name of Course	: B.Sc.(Hons.)Mathematics-CBCS: Skill Enhancement
Semester	III
Duration: 3 Hours	Maximum N

Maximum Marks: 38

 $9\frac{1}{2}$

Attempt any four questions

1.	(a). Fill	in the blanks	(1x8=8)
	(i)	The command sets up a parametric plot	
	(ii)	To plot a function using PSTricks, we need thepackage.	
	(iii)	Thecommand draws an ellipse with center (2,2), horizontal radius 1.5 radius 1.	and vertical
	(iv)	Thecommand is used to produce $limit_{n ightarrow\infty}$	
	(v)	Multiline expressions are created in theenvironment.	
	(vi)	We usepackage to include image in LaTeX document.	
	(vii)	Postfix expression for t cos cos 2t is	
	(viii)	PSTricks is written by	
	(b). Wi	rite the LaTeX command to draw $f(x) = Sinx^2$.	$1\frac{1}{2}$

2. Write LaTeX code in beamer to prepare the following presentation





- 3. Write code in LaTeX to plot the lemniscate given by the parametric equations $9\frac{1}{2}$ $x = \frac{\cos \cos 2t}{1+2t}, y = \frac{\sin 2t \ \cos 2t}{1+2t}, 0 \le t \le 2\pi$
- 4. (i). Write code of picture environment to make a picture of circumscribing a triangle. $4\frac{1}{2}$



(ii) Write code to draw a picture of a circles as shown below

Q5 Write an HTML code to create following web page using

(a) Font face of the text should be "Calibri".

 $9\frac{1}{2}$

5

(b) Text colour of the main heading should be blue and of the sub-headings should be blue.

(c) The image (any image) to be included in the web page should be named as "image.jpg".



•	HTML	
•	CSS	
Q6. Write an HTML code to display the w	veb page as follows:	$9\frac{1}{2}$

- A. heading of the page as University Roll No. and Course of study.
- B. a table given below:

S.No.	Fruits	Vegetables
1	Apple	Carrot
2	Banana	Tomato
3	Guava	Potato

Name of Course	:	CBCS(LOCF) B.Sc.(H)Mathematics
Unique Paper Code	:	32351301
Name of Paper	:	BMATH305-Theory of Real Functions
Semester	:	III
Duration	:	3 hours
Maximum Marks	:	75 Marks

1. Prove that $\lim_{x \to -1} \frac{x^2 - 5}{x^2 + 7} \neq \frac{-5}{7}$.

Use $\varepsilon - \delta$ definition of limit to prove that $\lim_{x \to 1} \frac{x^{3}-3}{x^{2}+1} = -1$.

Also prove that $\lim_{x\to 0} x^2 sgn(x)$ exists. Here sgn denotes the signum function.

2. Let $A \subseteq \mathbb{R}$, functions $f, g: A \to \mathbb{R}$, *c* be a cluster point of A and $L \in \mathbb{R}$, $L \neq 0$.

If $\lim_{x\to c} f = L$ and $\lim_{x\to c} g = \infty$, then find $\lim_{x\to c} fg$.

If $\lim_{x\to c} f = 0$ and $\lim_{x\to c} g = \infty$ then justify by an example that $\lim_{x\to c} fg$ need not be infinity.

3. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x^2, & \text{if } x \text{ is rational} \\ 5x - 6, & \text{if } x \text{ is irrational} \end{cases}$$

Find all the points at which f is continuous.

Let $f: \mathbb{R} \to \mathbb{R}$ be an additive function that is, f(a + b) = f(a) + f(b) for all $a, b \in \mathbb{R}$. Prove that if f is continuous at some point x_0 , then f is continuous at every point of \mathbb{R} .

4. Show that $f(x) = \frac{x-1}{x+1}$ is uniformly continuous on $[0, \infty)$ and $g(x) = Cos(\frac{1}{x})$ is not uniformly continuous on $(0, \infty)$.

If *f* is continuous on [0, 2] and f(0) = f(2), then prove that there exists $x, y \in [0,2]$ such that |y - x| = 1 and f(x) = f(y).

5. Find the points of relative extrema of the following function on the specified domain $f(x) = |x^2 - 25|, \quad -7 \le x \le 7$.

Prove that $ex \leq e^x$, for all $x \in \mathbb{R}$.

Use Mean Value Theorem to find an approximate value of $\sqrt{51}$.

6. If $x \in [0,1]$ and $n \in \mathbb{N}$, show that

$$\left| ln(1+x) - \left(x - \frac{x^2}{2} + \frac{x^3}{3} + \dots + (-1)^{n-1} \frac{x^n}{n} \right) \right| < \frac{x^{n+1}}{n+1}.$$

Use this to approximate ln(1.5) with an error less than 0.01.

Use Taylor's theorem to prove that for all x > 0

$$1 + x + \frac{x^2}{2} + \frac{x^3}{3!} < e^x < 1 + x + \frac{x^2}{2} + \frac{x^3}{3!}e^x \,.$$

Name of Course	: CBCS B.Sc. (H) Mathematics
Unique Paper Code	: 32351302_OC
Name of Paper	: Core-Group Theory I
Semester	: III
Duration	: 3 hours
Maximum Marks	: 75 Marks

1. Describe all symmetries of the following pictures.



Write Cayley's table for each of the corresponding two symmetric groups. Further, find order of each element in the two symmetric groups. If F and R represent a fixed reflection and a fixed rotation respectively in any of the two groups, then find FR^7FR^2F . Find the center of each of the two groups. Explain whether the groups are abelian.

2. Show that the set of all elements of finite order in an abelian group is a subgroup of the group. Consider the following group under matrix multiplication.

 $GL(2, \mathbb{Z}_2) = \{A | A \text{ is a } 2 \times 2 \text{ matrix with entries from } \mathbb{Z}_2 \text{ and determinant } |A| \neq 0 \}$ Write $|GL(2, \mathbb{Z}_2)|$. Let

 $H = \{A \in GL(2, \mathbb{Z}_2) | A^t = A, \text{ where } A^t \text{ denotes the transpose matrix of } A \}$ Is H a subgroup of $GL(2, \mathbb{Z}_2)$? Justify! Compute order of the element $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ in $GL(2, \mathbb{Z}_2)$. Explain without actual calculation why there is no element in $GL(2, \mathbb{Z}_2)$ of order 6.

- 3. Let S_4 and S_5 denote symmetric groups of degree 4 and 5 respectively. What is the order of the permutation $\alpha = (1245)(314)$ in S_5 ? Find α^{-1} . Is the permutation α even? Give an element β of order 4 in S_4 . Write all the elements of the cyclic group generated by β . Let $H = \{\alpha \in S_4: \alpha(2) = 2\}$ and $K = \{\alpha \in S_4: \alpha(4) = 4\}$. Prove that H is isomorphic to K.
- 4. Let \mathbb{Z} be the additive group of integers. Let $H = \{0, \pm 5, \pm 10, \pm 15, ...\}$. Find all the left cosets of H in \mathbb{Z} . What is the index of H in \mathbb{Z} ? Find all the subgroups of \mathbb{Z}_{15} the

group of integers under addition modulo 15. Also write down all their generators. Show that U(25) is a cyclic group under multiplication modulo 25. Find all its generators.

- 5. If G is a group, N a normal subgroup of G and H any subgroup of G, show that
 - (i) NH is a subgroup of G,
 - (ii) HN is a subgroup of G,
 - (iii) N is a normal subgroup of HN.

If G = GL (2, \mathbb{R}) and H = $\left\{ \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} : x, y \in \mathbb{R} \text{ and } xy \neq 0 \right\}$, check whether H is a normal subgroup of G.

6. Find all homomorphism mappings from \mathbb{Z}_{10} to \mathbb{Z}_4 . If $\varphi : \mathbb{Z}_{12} \to \mathbb{Z}_{12}$ is defined as $\varphi(x) = 9x \quad \forall x \in \mathbb{Z}_{12}$, prove that φ is a homomorphism. Find its kernel.

Name of the course	: CBCS B.Sc. (H) Mathematics
Unique Paper Code	: 32351302
Name of Paper	: BMATH306-Group Theory-1
Semester	: 111
Duration	: 3 hours
Maximum Marks	: 75 Marks

1. Show that the set *S* of all ordered pairs (a, b) of non-zero real numbers is an abelian group under the multiplication defined by $(a, b)(c, d) = (ac, bd) \forall a, b, c, d \in S$

Consider the group $G = GL(2, \mathbf{R})$ under multiplication. Then find the centralizer of $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Also, find the center of *G*.

Let $A = \begin{pmatrix} 3 & 4 \\ 4 & 4 \end{pmatrix}$. Find A^{-1} in $SL(2, Z_5)$. Verify the answer by direct calculation.

2. Find all the subgroups of Z:

a) containing 20Z.

b) contained in 20Z.

Prove that an abelian group which contains two distinct elements which are their own inverses must have a subgroup of order 4.

Suppose a group contains elements *a* and *b* such that |a| = 4 and |b| = 5 and that $a^{3}b = ba$. Find |ab|.

3. State Cayley's theorem and verify theorem for U(10).

Let *a* and *b* be elements of a group *G*. If O(a) = 12, O(b) = 22 and $< a > \cap < b > \neq \{e\}$. Prove that $a^6 = b^{11}$.

Find a non-cyclic group of order 4 in U(40).

4. Let *p* be a prime. If a group has more than (p - 1) elements of order *p*. Then prove that the group cannot be cyclic.

Let $\beta = (1 \ 2 \ 3)(1 \ 4 \ 5)$. Write β^{99} as a cycle.

Given a permutation $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{pmatrix}$

- a) Write α as product of disjoint cycle.
- b) Find $|\alpha|$.
- c) Find α^{-1} and verify by calculation.
- 5. Let *G* be the additive group $\mathbf{R} \times \mathbf{R}$ and $H = \{(x, x) : x \in \mathbf{R}\}$ be a subgroup of *G*. Give a geometric description of cosets of *H*.

If *N* is a normal subgroup of order 2 of a group *G* then show that $N \subseteq Z(G)$.

If *H* is a subgroup of a group *G* such that (aH)(Hb) for any $a, b \in G$ is either a left or a right coset of *H* in *G*, prove that *H* is normal.

6. If \emptyset be a homomorphism from Z_{30} onto a group of order 5, determine Ker \emptyset .

Let N be a normal subgroup of a group G. If N is cyclic subgroup of G then prove that every subgroup of N is normal in G.

Prove that the mapping from $x \to x^6$ from C^* to C^* where C^* denotes the set of non -zero complex numbers is a homomorphism. What is the kernel?

Name of Course	: CBCS (LOCF) B.Sc. (H) Mathematics
Unique Paper Code	: 32351502
Name of Paper	: BMATH-512 GROUP THEORY-II
Semester	: V
Duration	: 3 hours
Maximum Marks	: 75 Marks

- Show that the automorphism groups of two isomorphic groups are isomorphic. Let G be a group and g ∈ G be an element of finite order. Show that |φ_g| divides |g|, where φ_g is the inner automorphism of G generated by g. Give an example of a group G and an element g ∈ G for which 1 < |φ_g| < |g|.
- 2. Determine the number of elements of order 15 in the group $\mathbb{Z}_{75} \bigoplus \mathbb{Z}_{20}$. Also determine the number of cyclic subgroups of order 15 in this group. Is the group $\mathbb{Z}_{75} \bigoplus \mathbb{Z}_{20}$ isomorphic to the group $\mathbb{Z}_{25} \bigoplus \mathbb{Z}_{60}$? Justify your answer.
- 3. Find all Abelian groups (up to isomorphism) of order 100. From these isomorphism classes determine those classes that have elements of order 25. Does every Abelian group of order 100 has a cyclic subgroup of order 10? Justify your answer.
- 4. State the following statements as True or False. Justify your answer with proper reasoning.
 - a. The action of D_{20} (the dihedral group of order 20) on itself by conjugation is faithful.
 - b. Let *S* be a finite set and *G* be a subgroup of *Sym* (*S*). Let $\sigma \in G$ and $s \in S$. If *G* acts transitively on *S*, then $\bigcap_{\sigma \in G} \sigma G_s \sigma^{-1} = \{I\}$, where G_s denotes the stabilizer of *s* in *G*.
 - c. A group of order 160 is not simple.
 - d. If two groups have same class equation, then the groups are isomorphic.
 - e. Any two 3-cycles in A_4 are conjugate.
- 5. Let $G = D_{10}$ (the dihedral group of order 10) and $A = \{1, r, r^2, r^3, r^4\}$, where *r* denotes the rotation of regular pentagon by 72° about the centre in clockwise direction. Find $C_G(A)$ and $N_G(A)$. Let *H* be a subgroup of order 2 in *G*. Show that $N_G(H) = C_G(H)$. Deduce that if $N_G(H) = G$, then *H* is a subgroup of Z(G). Let $G = D_8$ and *G* acts on itself by the left regular action. By labelling the elements $1, r, s, sr, r^2, sr^2, r^3, sr^3$ of *G* with the natural numbers 1, 3, 5, 7, 2, 4, 6, 8 respectively, where *r* denotes the rotation of a square by 90° in clockwise direction and *s* denotes the reflection of square about the line passing through the vertices 1 and 3, exhibit the image of each element of *G* under left regular representation of *G* into S_8 .
- 6. Find Sylow subgroups of a group G of order 108. Show that either G has a normal Sylow 3-subgroup or G contains a normal subgroup of order 9.

[This question paper contains 4 pages]

A

Sr. No. of the Question Paper :

Duration	:	3 Hours	Maximum Marks :	75
Semester	:	V		
Name of the Paper	:	C-11 (Metric Spaces)		
Unique Paper Code	:	32351501_OC		
Name of the Course	:	CBCS B.Sc. (Honors) Mathematics		

Instruction for candidates

- 1. Question No. 1 is compulsory. Attempt any 3 out of the remaining 5 questions.
- 2. All questions carry equal marks (18.75).
- 3. In the question paper, given notations have their usual meaning unless until stated otherwise. Also, if not mentioned specifically, $\mathbb{R}, \mathbb{R}^2, l_2$ or, any subset of \mathbb{R} etc. will be assumed to be endowed with the usual/standard metric.

.....

1 (a) State whether True or False. Justify your answer in brief.

(i)
$$d(x, y) = \sqrt{|x - y|}$$
 and $d(x, y) = (x - y)^2$ both define metrics on \mathbb{R} .

- (ii) A function with discrete metric space as its domain is continuous.
- (iii) $\mathbb{Q} \cup \{2^{1/2}\}$ is dense in \mathbb{R} .
- (iv) Not every subset of a metric space is a metric subspace.
- (v) A compact metric space is a complete metric space, but a complete metric space may not be compact metric space.

(vi) The set
$$[-1,0] \cup \left\{1,\frac{1}{2},\frac{1}{3},...,\frac{1}{n},...\right\}$$
 is compact in \mathbb{R} . $2 \times 6 = 12$

- (b) (i) Consider a metric space (X, d) and a 1-1 function $f: X \to \mathbb{R}$. Then verify whether the following defines a metric on $X: d^*(x, y) = d(x, y) + |f(x) f(y)|, x, y \in X$. 3
 - (ii) Show that the set $\{x \in \mathbb{Q} : x \in [0,1]\}$ is closed in (\mathbb{Q}, d_u) . 1.75

(iii) Let
$$A = \left\{ (x, y) \in \mathbb{R}^2 : y = \sin\left(\frac{1}{x}\right), 0 < x \le 1 \right\}$$
. Write \overline{A} in (\mathbb{R}^2, d_u) . 2

2 (a) (i) Prove that in a metric space (X, d) a sequence $\langle x_n \rangle$ is convergent and converges to a point x in X if and only if the sequence $\langle d(x_n, x) \rangle$ converges to 0 in \mathbb{R} . 2

- (ii) Show that the sequence $\langle x_n \rangle$, where $x_n = \langle 1, 1/2, 1/3, ..., 1/n, 0, 0, ... \rangle$ converges to $x = \langle 1, 1/2, 1/3, ..., 1/n, ... \rangle$ in l_2 -space. 2
- (iii) Write the boundary of the set $A = (0,1) \cup \{2\}$ in \mathbb{R} .
- (b) (i) Consider the subspace $X = I \cup J$ of \mathbb{R} , where I =]0,1[and J = [4,7[. Show that $d(\sup_X I, I) > 0$. 2.75
 - (ii) Show that every non-empty set is a bounded metric space.
 - (iii) Consider \mathbb{R} endowed with discrete metric. Show that every singleton in it is both open and closed ball of radius less than 1. 2

2

- (c) (i) Show that $A = \{ \langle x_n \rangle \mid x_n = 0 \text{ or } 1 \}$ is a metric subspace of l_{∞} -space. What is the induced metric on A? 3
 - (ii) Give an example of a sequence in the l_{∞} -space which is co-ordinate wise convergent but not convergent in l_{∞} .
- 3. (a) Let (X_1, d_1) and (X_2, d_2) be metric spaces. Let $f: X_1 \to X_2$ be a continuous function. Define a metric d on X_1 as $d(a, b) = d_1(a, b) + d_2(f(a), f(b))$. Show that d is equivalent to d_1 . Moreover, prove that $f: (X_1, d_1) \to (X_2, d_2)$ is uniformly continuous. **4+2**

(b) Let
$$X = C[0, 1]$$
. Consider the metrics d and ρ defined on $C[0, 1]$ by
 $d(f, g) = \sup \{|f(x) - g(x)| : x \in [0, 1]\};$
 $\rho(f, g) = \int_0^1 |f(x) - g(x)| dx$
Is the identity map $i: (X, d) \to (X, \rho)$ a homeomorphism? Justify your answer. 5

- (c) Suppose f (x, y) is a continuous real valued function of the two real variables x and y. Prove that the curve in the Euclidean plane whose equation is f (x, y) = 0 is a closed set. Is A = {(x, y) ∈ ℝ²: y = x²} an open set? Justify your answer. 2+1
- (d) Is (\mathbb{R}^2, d) isometric to (\mathbb{C}, e) , where d and e are standard metrics. Justify your answer. 3
- (e) Let $A = \left\{\frac{1}{n} : n \in \mathbb{N}\right\} \subset \mathbb{R}$. Find $d(\overline{A})$ when (i) d is usual metric on \mathbb{R} (ii) d is discrete metric on \mathbb{R} . 1.75
- 4. (a) Let (X, d) be any metric space. Does there exist a bounded metric on X which is equivalent to d? Justify your answer. 1+3
 - (b) Suppose f and g be real-valued functions on a metric space (X, d). Let $A = \{x \in X : f(x) < g(x)\}$ and $B = \{x \in X : f(x) \neq 0\}$

Is A open? Is B open? Justify your answer.

- (c) Let $f: \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = 1/(1+x^2)$. Is f continuous? Suppose $A \subseteq \mathbb{R}$ is a compact subset of \mathbb{R} . Is $f^{-1}(A)$ also compact? Give justification for your answer. 1+2
- (d) Let $X = [1, 5] \cup [9, 12]$ endowed with usual metric inherited from \mathbb{R} . Is $\overline{S(10, 5)} = \overline{S}(10, 5)$? Justify your answer.
- (e) Define $f:[0,1) \cup [2,3] \rightarrow [0,2]$ by $f(x) = \begin{cases} x & , x \in [0,1) \\ 4-x & , x \in [2,3] \end{cases}$ Is f continuous? Is f bijective? Is f^{-1} continuous? Justify your answer. 1+1+1.75

5. Give suitable justification to your answer.

(a)	Give an example of each of the following: A function defined as $f : \mathbb{R} \to \mathbb{R}$ having (i) no fixed point; (ii) having only 1 fixed point.	2
(b)	Is $f:([0,1], d_u) \rightarrow ([0,1], d_u)$ defined by $f(x) = x^2$ a contraction map?	2
(c)	Give an example of an open set in \mathbb{R} which is complete but not connected.	2
(d)	Give an example of a metric space where singletons are the only connected sets.	2
(e)	Show that in \mathbb{R} , the continuous image of an interval is an interval.	3
(f)	Can we have a countably infinite connected set in \mathbb{R} ?	2
(g)	Can we have an uncountable set in \mathbb{R} which is not connected?	2
(h)	Let $f:(X,d_X) \to (Y,d_Y)$ be a continuous, 1-1 and onto map, where (X,d_X) is a compact metric space. Let $A \subseteq Y$ be connected. Then what can be said about the	
	connectedness of $f^{-1}(A)$?	2
(i)	Comment upon the connectedness of the set $S = \{(x, y) \in \mathbb{R}^2 : xy = 1\}$.	1.75

6. Give suitable justification to your answer.

(a) Let $X = \mathbb{R} \setminus \{0\}$. Show that there exists a continuous map f defined on X such that

$$f(x) = \begin{cases} 1, & x \in (-\infty, 0) \\ 0, & x \in (0, \infty). \end{cases}$$

(b) Can the closure of a connected set be disconnected?

3+2

3

2

(h)	An arbitrary intersection of compact sets is compact?	2.75
(g)	Let $f:(X, d_X) \to (Y, d_Y)$ be a continuous functions and <i>A</i> be a closed and bounded subset of <i>X</i> . Then what can be said about the compactness of $f(A)$?	2
(f)	Give an example of a collection of closed sets whose union is closed but not compact.	2
(e)	Give an example to justify that the continuous image of a bounded set need not be compact.	2
(d)	Give an example of a countable infinite set which is not compact, but its closure is compact.	2
(c)	Give an example of a metric space whose diameter is finite, but it is not compact.	2

Name of the Course	: B.Sc. (H) Mathematics (CBCS)		
Unique Paper Code	: 32357505_OC		
Name of the Paper	: DSE- Discrete Mathematics		
Semester	: V Semester		
Duration	: 3 hours		
Maximum Marks	: 75 Marks		

Instructions for Candidates:

Attempt any four questions. All questions carry equal marks.

1. Let (P_1, \leq) be ordered set of all divisors of 5 and (P_2, \leq) be ordered set of all divisors of 9, both ordered with divisibility. Draw the diagram of direct product $P_1 \times P_2$. Is it same as the diagram of the lattice of factors of 20, justify ,also Prove or Disprove $P_1 \times P_2$ is a lattice. Further, prove the result that a lattice (L, \leq) is a chain if and only if every nonempty subset of *L* is a sublattice of *L*.

2. Use the Karnaugh map method to simplify the polynomial p given by

$$p = (x' + y)(x' + z) + xyz$$

Does the expression x'y' imply the expression xy'z' + x'y'z + x'y'z'. Give reason for your answer Give the symbolic representation of the circuit q given by

q = xy'z((x' + z) + (y + z'))

Also, draw the contact diagram of above circuit q.

3. Let $A = \{a, b, c, d, e\}$ be a partial ordered set with the given Hasse diagram



Find the meet (Λ) and join (V) of subsets of {a,c} and {b,e}. Let $B = \{1, 2, 3, 4, 5\}$ be order isomorphic to *A* under the following order isomorphism: f(a) = 1, f(b) = 3, f(c) = 5, f(d) = 2, f(e) = 4. Draw the Hasse diagram of poset *B*. Also prove that in a lattice every non empty subset has a supremum and infimium.

4. Draw the graph $K_{4,4}$. How many edges does this graph has? Also, find the adjacency matrix of this graph. Does there exist a graph with 32 edges and 12 vertices, each of degree 4 or 6? Justify your answer. For the graph given below, Either give the bipartition sets or explain why the graph is not bipartite.



5. Use the original form of Dijkstra's Algorithm to find shortest path from A to F in the graph given below. Label all vertices. Explain algorithm.



Consider the following Graph.Check whether the graph is Eulerian. Explain your Answer.



6. Find elements a, b, c in N_5 which do not satisfy the following law:

 $(a \lor b = c \lor b \text{ and } a \land b = c \land b) => a = c$

Deduce that a lattice L is distributive if and only if the above law holds for all $a,b,c \in L$. Verify whether the lattice L in the figure given below is modular and /or distributive, by using M_3-N_5 theorem.



Name of the Course	: B.Sc. (H) Mathematics (CBCS)		
Unique Paper Code	: 32357505_OC		
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Maximum Marks	: 75 Marks		

Instructions for Candidates:

Attempt any four questions. All questions carry equal marks.

- Let P = {1, 2, 3, 5, 6, 10, 15, 30}. Show that P is a partial ordered set with divisibility as partial order relation. Draw Hasse Diagram of P. Find lower bounds and upper bounds of 10 and 15. Draw the diagrams for the ordered sets 2 ∪ 3, 2 × 3, and 2 ⊕ 3 where n denotes the chain 0 < 1 < 2 < 3 < n 1.
- 2. Given lattice L = {1,3,5,7,15,21,35,105} of positive divisors of 105 with respect to the given order x ≤ y ⇔ x divides y, Find sublattices S and T of L such that the union, S ∪ T is not a sublattice of L,but S ∩ T is a lattice. Prove or disprove that every chain is a lattice. Is every lattice also a chain? Justify your answer by verifying the same for the lattice L defined above. Prove or disprove that lattice (P(X),∩,∪), where X = {1,2} is isomorphic to the sublattice R = {1,3,15,105} of L. Hence or otherwise, find a sublattice of L that is isomorphic to (P(X),∩,∪). Justify your answer.
- **3.** Verify whether the lattice L₁ in the figure given below is modular and /or distributive, by using M₃-N₅ theorem.



Find elements a, b, c in M₃ which do not satisfy the following law:

 $(a \lor b = c \lor b \text{ and } a \land b = c \land b) \Longrightarrow a = c$

Also prove that in a distributive lattice L the above law holds for all $a,b,c \in L$.

4. Find the disjunctive normal form of p = x'y + xy'z + xy'z' + x'y'z in three variables and simplify it using Karnaugh Map or Quine-McCluskey method. Give the symbolic representation of the circuit q given by

$$q = x'yz((x + z') + (y' + z'))$$

Also,draw the contact diagram of above circuit q.

5. Use the improved version of Dijkstra's Algorithm to find the length of shortest path from A to F in the graph given below. Show the final labels on all vertices, Explain Algorithm.



Find the adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 shown below. Find an isomorphism between the graph G_1 and G_2 .



6. Does there exist a graph with 28 edges and 10 vertices, each of degree 4 or 6? Justify your answer. Does there exist the graphs whose degree sequence are (a) 4,3,2,2,1 (b) 5,4,3,2,1 ? In each case either draw the graph or explain why no graph exists. For pair of graphs shown below, either label the graphs so as to exhibit an isomorphism or explain why graphs are not isomorphic.



Name of the Course	: B.Sc. (H) Mathematics (CBCS)		
Unique Paper Code	: 32357505_OC		
Name of the Paper	: DSE- Discrete Mathematics		
Semester	: V Semester		
Duration	: 3 hours		
Maximum Marks	: 75 Marks		

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Find the meet (Λ) and join (V) of subsets of {a,c} and {b,e}. Let $B = \{1, 2, 3, 4, 5\}$ be order isomorphic to *A* under the following order isomorphism: f(a) = 1, f(b) = 3, f(c) = 5, f(d) = 2, f(e) = 4. Draw the Hasse diagram of poset *B*. Also prove that in a lattice every non empty subset has a supremum and infimium.

4. Draw the graph $K_{4,4}$. How many edges does this graph has? Also, find the adjacency matrix of this graph. Does there exist a graph with 32 edges and 12 vertices, each of degree 4 or 6? Justify your answer. For the graph given below, Either give the bipartition sets or explain why the graph is not bipartite.



5. Use the original form of Dijkstra's Algorithm to find shortest path from A to F in the graph given below. Label all vertices. Explain algorithm.



Consider the following Graph.Check whether the graph is Eulerian. Explain your Answer.



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Deduce that a lattice L is distributive if and only if the above law holds for all $a,b,c \in L$. Verify whether the lattice L in the figure given below is modular and /or distributive, by using M_3-N_5 theorem.



Name of Course	: CBCS B.Sc. (H) Mathematics
Unique Paper Code	: 32357502_LOCF
Name of Paper	: DSE1-Mathematical Modeling and Graph Theory
Semester	: V
Duration	: 3 hours
Maximum Marks	: 75 Marks

1. Consider a spring-mass dashpot system with mass m = 1, the viscous damping c = 0 and the spring modulus k = 9 with external force $f(t) = 6 \cos 3t$. Let y(t) denotes the displacement of the mass from its equilibrium position. Find y(t).

Determine the Laplace Transform of the square-wave function shown in the figure below.



Use the factorization:

$$s^{4} + 4a^{4} = (s^{2} - 2as + 2a^{2})(s^{2} + 2as + 2a^{2})$$

and apply inverse Laplace transform to show that:

$$L^{-1}\left\{\frac{1}{s^{4}+4a^{4}}\right\} = \frac{1}{4a^{3}}\left(\cos h \, at \sin at - \sin h \, at \cos at\right)$$

2. Find the power series solution of the IVP y'' + 9y = 0, y(0) = 1, y'(0) = 0. What is an Analytic function and a regular singular point? Find the singular points for the ordinary differential equation $x(x-2)^3 y'' + 2(x-2)^3 y' + 3y = 0$. Also check whether the singular points are regular or irregular.

Find two linearly independent Frobenius series solutions of the differential equation

$$x^{2} y'' - x y' + \left(\frac{3}{4} - x^{2}\right) y = 0$$

3. Consider the following Linear Programming Problem: Maximize $Z = 6X_1 + 5X_2$ Subject to

$$X_1 + X_2 \le 5$$

 $3X_1 + 2X_2 \le 12$
 $X_1, X_2 \ge 0$

Solve this Problem using tabular form of Simplex Method.

Explain middle square method. Is there any drawback of this method? Take initial seed

 $x_0 = 14$ and generate two digit eight random numbers. Is there any problem to generate

eleven, two-digit random numbers when seed is $x_0 = 14$.

Write steps of Monte Carlo simulation algorithm to calculate the volume

of the following ellipsoid in the first octant:

$$\frac{x^2}{4} + \frac{y^2}{8} + \frac{z^2}{16} \le 32$$

That lies in the first octant x > 0, y > 0, z > 0

4. Consider the following Linear Programming Problem: Minimize $Z = X_1 + 3X_2$ Subject to

$$5X_1 + 4X_2 \ge 20 3X_1 + 4X_2 \le 24 X_1, X_2 \ge 0$$

Plot the feasible reason of the problem. Determine the optimal solution from the graph. What would be the optimal solution if the objective function were to be maximized?

Consider the following Linear Programming Problem

Maximize $Z = 3X_1 + 5X_2$ Subject to $X_1 + 2X_2 \le 2000$

$$X_{1} + Z_{2} \leq 2000$$
$$X_{1} + X_{2} \leq 1500$$
$$X_{2} \leq 600$$
$$X_{1}, X_{2} \geq 0$$

Determine the sensitivity of the optimal solution to change in C_1 using the objective function $C_1x_1 + 5x_2$

A small harbour has unloading facilities for ships. Only one ship can be unloaded at any one time. The unloading time require for a ship depends on the type and the amount of cargo and varies from 45 to 90 minutes. Below is given a situation with 5 ships:

	Ship 1	Ship 2	Ship 3	Ship 4	Ship 5
Time between successive ships	10	20	5	110	15
Unload time	45	35	50	65	70

Draw the time-line diagram depicting clearly the situation for each ship, the idle time for harbour and the waiting time. Also list the waiting times for all the ships and find the average waiting time.

5. Verify and explain whether the following graphs are isomorphic:



True or false: A graph is bipartite iff every cycle of a graph has even number of edges. Explain?

Can, an Eulerian graph can be split into cycles, no two of which have an edge in common? Justify your answer. Also illustrate the result using the following graph



6. Is a bipartite graph with odd number of vertices not Hamiltonian. Explain? Verify whether the Petersen graph is semi-Hamiltonian or not?Show how a Gray code represents a Hamiltonian path in 2-cube and 3-cube.For the graph shown below, write down a walk of length 7 between A and I and a cycle of length 3, 4 and 5.


Name of the Course:	LOCF B.Sc. (H)Mathematics
Unique Paper Code:	32357501
Name of the Paper:	DSE-1 (i) Numerical Analysis
Semester:	V
Duration:	3 hours
Maximum Marks:	75 Marks

1. (a) Find the smallest positive root of the given equation by performing three iterations of the Bisection method

 $f(x)=e^{x}-3x=0.$

(b) Apply four iterations of the Fixed Point Iteration Method to find an approximate root of the following equation by taking the initial approximation as $p_0=0$

$$f(x) = 3x - (1 + \cos x) = 0.$$

Also represent the root graphically.

(c) Find LU decomposition for the matrix

$$A = \begin{pmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & -2 & -1 \end{pmatrix}$$

and then solve the system

$$3x + y + 2z = 4$$
$$2x - 3y - z = -6$$
$$x - 2y - z = -4$$

2. (a)Set up the Gauss-Seidel iteration scheme for the following system of equations

$$2x - y + 2z = 3$$
$$x + 3y + 3z = -1$$
$$x + 2y + 5z = 1$$

and iterate three times starting with the initial vector $X^{(0)} = (0,0,0.5)^T$.

(b) Apply Secant method to find a root of the equation $\ln(1+x) - \frac{1}{2}\cos x = 0$

(0,1). Perform three iterations. What order of convergence do you expect?(c)Solve the following system of equations using SOR iteration method

$$5x_1 + x_2 - 2x_3 = 2$$

$$3x_1 + 4x_2 - x_3 = -2$$

$$2x_1 - 3x_2 + 5x_3 = 10$$

Take w = 0.9 with $X^{(0)} = (0,0,0)^T$ and iterate three times.

3. (a) Find a polynomial of degree 3 or less passing through the points (-1,9), (0,5),(1,3) and (2,1) using Lagrange interpolation. Use this polynomial to estimate the ordinate for x = 1.5.

(b) Use central difference formula to approximate f'(0.5) for the function $f(x) = 4e^{-2x}$ by taking two step sizes h = 0.1 and 0.05. What is the order of approximation?

4. (a) Derive the following approximation of function $f'(x_0)$ for an arbitrary function f(x):

$$f'(x_0) = \frac{-3f_0 + 4f_1 - f_2}{2h}$$

Hence show that the above approximation provides exact value of the first order derivatives for f(x) = 1, x and x²but not for $f(x) = x^3$.

(b) Determine the step size h in an equidistant table for f(x) = sinx in $[0, \pi/4]$, if the error in magnitude in quadratic interpolation is less than or equal to 5×10^{-8} in magnitude.

(c) Obtain the piecewise linear interpolating polynomials for the function f(x) defined by the given data .

Х	0.5	1.5	2.5
f(x)	0.125	3.375	15.625

Interpolate at x = 1.0 and 2.0.

5. (a) Approximate the value of the given integral by Simpson's 1/3 rule and the Trapezoidal rule

$$\int_0^3 \frac{1}{4+x^2} dx.$$

Calculate the difference between the actual value and the approximate value. Also find the error term for both.

(b)Apply the Newton Raphson Method to find a root of the equation

 $f(x)=x^3 - x^2 - 10x + 6 = 0$. Perform three iterations with $x_0=1$.

6 (a) Construct the Richardson extrapolation table to find the derivative of the function $f(x) = \tan^{-1} x$ at $x_0 = 3$ using the first-order forward difference approximation

$$D_{h}^{(1)} = \frac{f(x_{o} + h) - f(x_{o})}{h}$$

taking h=1,0.5,0.25,0.125.

(b) Apply the Modified Euler method to find approximate solution of the following initial value problem with four numbers of steps

$$\frac{dy}{dx} = xy, \quad (1 \le x \le 2), \quad y(1) = 1.$$

Also compare with the exact solution.

Name of Course	: CBCS B.Sc. (H) Mathematics
Unique Paper Code	: 32357501_OC
Name of Paper	: DSE-Numerical Methods
Semester	: V
Duration	: 3 hours
Maximum Marks	: 75 Marks

1. Let *a* be a nonzero real number. For any x_0 , the recursive sequence defined by $x_{n+1} = (a^3 + \frac{2x_n^3}{3x_n^2})$ converges to *a*. Determine the order of convergence and the asymptotic error constant.

Find the root of the equation tan x + x = 0 correct to two decimal places which lies between 2 and 2.1 using bisection method.

Find the smallest root of the equation $x^3 - 6x^2 + 11x - 6 = 0$ by Regula-Falsi method. Perform three iterations.

2. Using scaled partial pivoting during the factor step, find matrices *L*, *U* and *P* such that LU = PA, where

$$\mathbf{A} = \begin{bmatrix} 2 & -6 & 10 \\ 1 & 5 & 1 \\ -1 & 15 & -5 \end{bmatrix}$$

Perform two iterations for finding the root of $f(x) = x^3 - 13$ by secant method starting with $p_0 = 3$ and $p_1 = 2$. Further, compute the ratio $\frac{|p_3 - p|}{|p_2 - p|^{1.618}}$ and show that this value approaches $\frac{|f''(p)|}{|2f'(p)|}$ with $p = \sqrt[3]{13}$.

Find the positive root of the equation $x^2 + e^{-x} - 5 = 0$ using the Newton's method with $p_0 = 2$. Perform three iterations.

3. Solve the following system of equations using Gauss Jacobi iteration method:

$$4x_1 + 2x_2 + x_3 = -2$$

$$2x_1 + 5x_2 + 2x_3 = -6$$

$$x_1 + 2x_2 + 6x_3 = 10$$

Take $X^{(0)} = [0.5, -0.5, -0.5]^T$ and iterate three times.

Solve the following system of equations using SOR iteration method:

$$2x_1 - x_2 + x_3 = 7$$
$$-x_1 + 2x_2 - x_3 = 1$$
$$x_1 - x_2 + 2x_3 = 1$$

Take $\omega = 0.7$ with $X^{(0)} = [0,0,0]^T$ and iterate three times.

Write the following data in the divided difference tabular form and determine the missing values:

$$x_0 = 0, \quad x_1 = 1, \quad x_2 = 2, \quad x_3 = 3$$
$$f[x_0] = 2, \quad f[x_1] = 6, \quad f[x_2] = 6$$
$$f[x_0, x_1] = 4, \quad f[x_2, x_3] = 0, \quad f[x_1, x_2, x_3] = 0$$

Also, write the Newton form of the interpolating polynomial for the data and estimate the value at x = 1.5.

4. Solve the following system of equations using Gauss Seidel iteration method:

$$2x_{1} - x_{2} = 1$$

$$-2x_{1} + 4x_{2} - x_{3} = 0$$

$$-x_{2} + 2x_{3} + x_{4} = 0$$

$$-x_{3} + x_{4} = 1$$

Take $X^{(0)} = [0,0,0]^T$ and iterate three times.

Obtain the interpolating polynomial using Lagrange interpolating formula for the following data:

(-2, -8), (-1, -6), (0, 5), (1, 10), (2, 20), (3, 50)

Hence, estimate the value of the polynomial at x = 0.5 and x = 2.5.

5. State the Simpson's Rule for finding the integral of a continuous function f(x) over a closed interval [a, b] and use it to find the approximate value of the definite integral $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$. Also obtain the exact integral and find the absolute error. Further, state the theoretical error bound for the Simpson's Rule and verify that the theoretical error bound holds in this case.

Approximate the derivative of $f(x) = \cos x$ at $x_0 = 0$ using the formula

$$f'(x_0) = \frac{f(x_0 + h) - f(x_0 - h)}{2h}$$

taking the values of h as 1, 0.1 and 0.01. Also find the error corresponding to each value of h.

6. Approximate the second derivative of $f(x) = e^{2x}$ at $x_0 = 0$ using the formula

$$f''(x_0) = \frac{f(x_0) - 2 f(x_0 + h) + f(x_0 + 2h)}{h^2}$$

taking the values of h as 0.1, 0.01 and 0.001. Also find the error corresponding to each value of h.

Obtain an approximate solution of the Initial Value Problem (IVP)

$$\frac{dx}{dt} = \frac{\sqrt{1-x^2}}{t}, \ 1 \le t \le 5$$
$$x(1) = 0$$

in 4 steps using Euler's method. Also find the absolute error at each step given that the exact solution of the IVP is $x(t) = Sin(\ln t)$.

Name of Course	: CBCS (LOCF) B.Sc. (Hons) Mathematics
Unique Paper Code	: 32357507
Name of Paper	DSE-2: Probability Theory and Statistics
Semester	: V
Duration	: 3 hours
Maximum Marks	: 75 Marks

1. Suppose that the cumulative distribution function of the random variable X is given by

 $F(x) = 1 - e^{-x^2}, x > 0.$

Evaluate P(X>2), E(X) and Var(X). Find the 25th percentile(pth percentile is a value ξp such that $P(X < \xi p) \le p$ and $P(X \le \xi p) \ge p$), the mode and the median of this distribution.

- 2. Let C be the set of points interior to or on the boundary of a square with side of length 1. Moreover, say that the square is in the first quadrant with one vertex at the point (0, 0) and an opposite vertex at the point (1, 1). Let P(A) be the probability of region A contained in C. If A={(x, y): 0<x<y<1}, compute P(A), and what will be P(A) if A={(x, y): 0<x=y<1}. Suppose, two points are independently chosen at random in the interval (-1, 1). Obtain the probability that the three parts into which the interval is divided can form the sides of a triangle.</p>
- **3.** State the memory-less property of the exponential distribution. Let the time (in hours) required to repair a smart mobile is exponentially distributed with mean 3. What is the probability that the repair time exceeds 3 hours? Also, find the probability that a repair takes at least 5 hours given that its duration exceeds 4 hours?
- **4.** Let

f(x, y) = 24xy, 0 < x < 1, 0 < y < 1, 0 < x + y < 1, and =0, otherwise.

Find the moment generating function of X and Y, and hence, find whether X and Y are independent? Further obtain the coefficient of correlation between X and Y.

5. Let

 $f(x, y) = 10xy^{2}$, 0 < x < y < 1, and =0 elsewhere, be the joint pdf of X and Y.

Find the conditional mean and variance of X, given Y=y, 0 < y < 1. Hence find the distribution of Z=E(X|Y) and determine E(Z) and Var(Z) and compare these to E(X) and Var(X), respectively.

- **6.** (i) State the Chebyshev's Theorem (or Inequality). Let the number of customer's visiting a bike showroom is a random variable with mean 12 and standard deviation 2. With what probability can we assert that there will be more than 6 but fewer than 18 customers visiting the showroom?
 - (ii) Let {X_i}, i=1, 2, ... be a sequence of i.i.d. Poisson variables with E[X_i]=1.5. Find P(160 < Y < 200), where $Y = X_1 + X_2 + ... + X_{100}$

Name of Course	: CBCS B.A. (Prog)
Unique Paper Code	: 62351101_OC
Name of Paper	: Calculus
Part	: I
Duration	: 3 hours
Maximum Marks	: 75 Marks

1.
$$f(x) = \begin{cases} \frac{x(e^{1/x} - e^{-1/x})}{e^{1/x} + e^{-1/x}}, & x \neq 0\\ 0, & x = 0 \end{cases}$$

Show that f(x) is continuous but not derivable at x = 0and also for the function

$$y = \frac{\sin^{-1}x}{\sqrt{1-x^2}},$$

show that

$$(1 - x2)yn+2 - (2n + 3)x yn+1 - (n + 1)2yn = 0.$$

2. Let
$$f(x) = \begin{cases} \frac{1}{2} - x, & \text{if } 0 < x < \frac{1}{2} \\ \frac{3}{2} - x, & \text{if } \frac{1}{2} < x < 1 \end{cases}$$
$$f(0) = 0, f\left(\frac{1}{2}\right) = \frac{1}{2}, f(1) = 1.$$

Show that f(x) is discontinuous at $x = \frac{1}{2}$, 1 and also if $z = sec^{-1}\left(\frac{x^2+y^2}{x+y}\right)$

show that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2cotz.$$

and also if $u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$

show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0.$$

3. Show that the tangents to the curve

$$x^3 + y^3 = 3axy$$

at the points where it meets the parabola $y^2 = ax$ are parallel to the axis of y

and find the asymptotes of the curve

$$4x^3 - 3xy^2 - y^3 + 2x^2 - xy - y^2 - 1 = 0.$$

4. Find the position and nature of the double points on the curve

$$y(y-6) = x^2(x-2)^3 - 9$$

and trace the curve $ay^2 = (x - a)(x - 5a)^2$.

5. Verify the Rolle's Theorem for the function $f(x) = x(x-2)^3$ in [0,2]

and show that for x > 0

$$\frac{tanx}{x} > \frac{x}{sinx}, 0 < x < \pi/2.$$

6. Assuming the possibility of expansion prove that

$$\log \sec x = \frac{x^2}{2} + \frac{x^4}{12} + \cdots$$

and also verify Lagrange's Mean Value Theorem for $f(x) = x^2 - 3x - 1$ in [1,3].

Unique Paper Code	:	42341102-OC
Name of the Course	:	B.Sc. Prog. / Mathematical Science (CBCS)
Name of the Paper	:	Problem Solving using Computers
Semester	:	Ι
Year of Admission	:	2015, 2016, 2017, 2018

Duration: 3 Hours

Maximum Marks: 75

Instructions for the candidates:

- 1. All questions carry equal marks.
- 2. Attempt any four questions.
 - (a) Define a class Rectangle that has the two data members Length and Breadth and three methods given below:
 - Constructor to initialize the data members Length and Breadth.
 - Area This computes the area of the rectangle.
 - Perimeter This computes the perimeter of the rectangle.
 - __str__ function to generate a string that may be used to display the data members of the rectangle.

Also, write Python statements to perform the following:

- Create an object of the class Rectangle with suitable values.
- Invoke the method Area
- Print all the data members of the class

(b) Find and correct the errors in the code snippet given below:

```
DEF execmain()
    x = input("Enter a number:")
    if (abs(x) = x):
        print("You entered a positive number")
    else
```

x=*-1

print "Number made positive:"x

2. (a) Check the validity of variable names with respect to the naming conventions of

Python.		Justify	your	answer.
а.	new_var			
b.	_newvar			
с.	lvar			
d.	new%var			
е.	new var			

(b) Give an algorithm for each of the following problems:

I. To compute whether a number is a prime number or not.

II. To determine whether a string is a palindrome.

(c) Give the output corresponding to the statements mentioned in the following code segment.

```
a = 35
b = 17
c = a & b
print(c) #Statement 1
c = a | b
print(c) #Statement 2
c = a ^ b
print(c) #Statement 3
c = ~b
print(c) #Statement 4
print(b<<3) #Statement 5
print(a>>2) #Statement 6
```

 a) Apply binary search to search key value 89 in the following list of numbers. Show all the intermediate steps required to do the search.

[8, 12, 39, 45, 57, 67, 78, 89, 91]

b) Apply the selection sort to arrange the following list of elements in the ascending order. Show all the intermediate steps of each pass. Determine the

number of passes to sort the entire list.

[21, 71, 43, 10, 60, 37, 91, 2, 55, 42]

c) Evaluate the following expressions and justify your answers:

I. 9 * 2 - 3 ** 2 // 6 + 5
II. 10 < 5 or 7 < 12 and not 1 > 3
III. 'hello'<'hi' or 'Iamfine' > 'I am not'

4. a) Write a program to print the sum of the first n terms of the following series:

 $1/3 + 1/6 - 1/9 + 1/12 + \dots$

- b) Define a function that accepts a number n and a string mystr as input arguments and returns a list lst containing n elements. Each element of the list must be a string repeated i times where i ranges from 1 to n. For example, if the arguments are 3 and 'Hi' then the function should return ['Hi', 'HiHi','HiHiHi'].
- c) Define a function count that accepts a string s as an input and counts the vowels, consonants, and spaces that occur in s and print their counts in the result.
- d) Define a recursive function fibonacci that accepts i as an input and returns the ith term of the Fibonacci series. Define a function main that accepts a umber a number n and invokes the function fibonacci to display the fibonacci series upto n terms.
- (a) What will be the output produced on execution of the following code segments? Justify your answers.

5

```
L1.append(i)
         print(L3)
                        #Statement 2
         for i in range(len(L2)):
              L3.append(L2[i] + L1[i])
              L3.append(len(L1)-len(L2))
                       #Statement 3
         print(L3)
      changelist()
 ii)
      list1 = ['english', 'geography', 'history', 2014,
      2021]
      print(list1[1][-3:-9:-1]) #Statement 4
      list1[3] = 2001
      del list1[3]
      print(list1[2]) #Statement 5
      print(list1)
                        #Statement 6
      print(list1[2:4]) #Statement 7
iii) txt='Hi! Good morning! Have a great day! Enjoy.'
     txt = txt.split('!')
                         #Statement 8
     print(txt)
     print('**'.join(txt)) #Statement 9
     txt = txt.partition('!')
     print(txt) #Statement 9
     print(txt.find('good')) #Statement 10
     print(s1.count('o'))
                               #Statement 11
```

(b) Define a Python function triangle that accepts a positive integer n as an argument and produces the pattern given below as output. In the pattern, for example, when 4 is entered as the value of n, the output will be as follows.

```
1
2 2
3 3 3
4 4 4 4
3 3 3
2 2
1
```

- a) Show the contents of the stack on execution of each of the following sequence of operation: push('A'), isempty(), push('B'), pop(), push('C'), pop(), pop(). Assume that the stack is initially empty. What will be the effect of another pop() operation invoked after the execution of the above sequence of operations?
 - b) Enumerate any four types of Exception. Explain under what conditions these exceptions are raised by the Python interpreter. How are they handled?
 - c) Perform the following operations on a queue in the sequence given below (assume queue is initially empty)

Enqueue 3 Enqueue 23 Enqueue 5 Dequeue 11 Dequeue 11 Dequeue Dequeue

d) Define a function prime_value_remove that accepts a string s as an input and returns a string res with the characters removed at indexes (starting with 0) corresponding to prime numbers. Example: for a string s="Helloworld", it should return res="Heoold".

Unique Paper Code	:	42341102
Name of the Course	:	B.Sc. Prog./Mathematical Sc
Name of the Paper	:	BSCS01 Problem Solving Using Computers
Semester	:	Ι
Year of Admission	:	2019 onwards

Duration: 3 hours

Maximum Marks: 75

Instructions for Candidates:

- All questions carry equal marks. Attempt any Four out of Six questions.
- All the coding/error/output statements are based on Python programming language.
- 1. Identify the errors in the following statements and rewrite them after removing the errors:

```
a) j = 10
while j != 0:
    print("j =" j)
    j =- 1
b) For i in range(4,6):
c) 1 + "Hello"
d) if i =< 4:
e) True = 1
f) a = [16, 82, 39]
print(A)
```

Which of the following are valid identifiers? Also mention the cause of violation if an identifier is invalid.

a) Rangeb) 2Subjectsc) abc-1d) NaNe) No of elements

Write statements/code snippet for the following expression:

$$c = \sqrt{a^2 + b^2 - 2ab\cos\gamma}$$

- 2. Write a single for loop to display the following pattern for n number of rows (in the example below, the number of rows is 3).
 - \$ \$\$ \$\$\$

Write a program to read the names and marks of 10 students and store them in a dictionary. Also write the functions to perform the following:

- Print the maximum marks
- Calculate the average marks
- Find the number of students who have scored less marks than the average marks
- 3. Define a class Employee with the following specifications:
 - Data members:

EmpID, Name, Age, Department, Salary

- Methods:
 - o Constructor to initialize the data members.
 - o setValues method to assign values to the Department and Salary data fields
 - o _____str___ method to display employee details

Also write statements to:

- Create an object for an employee 'Sunil' of 34 years who is working in the HR department, drawing a salary of Rs.50,000/- with employee id as 123.
- Display the details of the employee created in the statement above.
- Invoke setValues method to update the department of 'Sunil' to Sales and set the salary to Rs.55,000/-.
- Display employee details with new values.
- Write a function that accepts the name of a file and returns the number of lines and the characters in the file. The function should return -1 if the specified file does not exist.
- 4. What will be the output on the execution of the following code segments? Justify your answers.

a) lst = [x * y for x in range(3, 9, 2) for y in range(3, 1, -1) if x * y % 3 == 0] print(lst)

```
b) import copy
  list 1 = [10, 20, [30, 40], 50]
  list 2 = copy.deepcopy(list 1)
  list 2[3] = 70
  list 2[2].append(60)
  print(list 2)
  print(list 1)
c) str = "Computer Fundamentals"
  print(str[10:2:-2])
  print(str[5:-2:2])
d) t = (1, 2, 3, 4, 5, 6)
  for i in range(len(t) - 1):
      print("t[%d] = %d"%(i, t[i]))
e) def fn():
     try:
         s = "Hello"
         print(s[10])
     except ValueError:
         print("Value error")
     except IndexError:
         print("Index error")
         raise ValueError("Some error")
     except:
         print("Default error")
     print("Out")
  try:
     fn()
  except ValueError as msg:
     print(msg)
```

5. Define a function checkDivisors that takes an integer N as an argument and returns True when the number of divisors of N is even, False otherwise. Define another function that takes an integer num as an input from the user and invokes checkDivisors. If num is a positive integer and displays the result, otherwise display the message "Invalid Input".

Write statements to perform the following:

- create a set S1 having "Violet", "Indigo", "Blue", "Green", "Yellow", "Orange" and "Red" colors
- create a set S2 having "Cyan" and "Magenta" colors
- ♦ add "Yellow" and "Black" colors to S2
- find colors that are common to both the sets
- find colors that are present in S2 but not in S1
- find all the colors that are present in both the sets

6. Apply selection sort to arrange the elements of the following list in descending order:

lst = [12, 5, 2, 4, 17, 44, 7, 6, 9]

Show all the intermediate steps of each pass. Determine the number of passes to sort the entire list.

Define a function for linear search that accepts two arguments: a list lst and the element ele to be searched. Assume that lst can have duplicate elements. If ele is present in the list, the function should return a list of all the indices corresponding to ele, [-1] otherwise. For example, if the list is [24, 68, 68, 24, 14, 68] and 68 is to be searched then the function should return [1, 2, 5].

Name of Course	: B.Sc. (Math. Sci.)-I, B.Sc. (Phy. Sci.)-I,
	B.Sc. (Life Sci.)-I
Unique Paper Code	: 42351101_OC
Name of Paper	: Calculus and Matrices
Semester	: I
Duration	: 3 hours
Maximum Marks	: 75 Marks

Qs 1: Is set $S = \{(1, 2, 0), (0, 1, 2), (1, 0, 2)\}$ a basis of \mathbb{R}^3 ?

Let
$$P = \{(a, b): a, b \in \mathbb{R}, a + b \ge 0\}$$
 and $Q = \{(a, b): a, b \in \mathbb{R}, b = 4a + 10\}$

Are *P* and *Q* subspaces of \mathbb{R}^2 ?

Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be given by T(x, y) = (x + y, x - 7y)

Show that T is a linear transformation and find the matrix representation of T.

Qs 2: Find the inverse of the matrix P, if it exists, using elementary row operations where

$$P = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Find the rank of the matrix *B* where

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & -1 \\ 2 & 3 & 0 \\ 4 & 6 & 0 \end{bmatrix}$$

Using elementary row operations, find the solution of the following system of equations, if it exists

$$x + y + z = 2$$
$$2x - z = 0$$
$$3x + y + z = 4$$

Qs 3: Find the eigen values of matrix *A* and corresponding eigen vectors for each of the eigen values of *A* where

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Find the limit of the sequence, if it converges:

a.
$$< 10 + \frac{n+1}{n^2+1} >$$

b. $< (-1)^{n+1} \frac{\sin n}{n^2} >$

Let $f(x) = x^3 - 3x + 1$. Answer the following for f(x):

(i) Is f(x) symmetric about y - axis?

(ii) Find formula to new function which is obtained after translating f(x) by 4 units to the left.

Qs 4. Assuming the validity of expansion, find the Taylor series generated by $f(x) = e^{2x} at x = 0$. If $y = \tan^{-1} x$, show that $(1 + x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$. Find the second-order partial derivatives of $f(x, y) = x^3 + y^3 - 2x^2y^2$.

Qs 5. Verify that $z(x, y) = e^{-x} \sin y$ is a solution of the Laplace equation.

Find |z|, arg z, arg \overline{z} , arg(-z) for the complex number $z = (1 - \sqrt{3}i)(-1 - i)$.

Find the equation of the circle described on the line joining the points given by 1 + 2i and

5 - 6i as diameter.

Qs 6. Form an equation of lowest degree with real coefficients whose roots are 1, -1, i.

If *n* is a positive integer, then show that $(1+i)^n - (1-i)^n = i 2^{\frac{n}{2}+1} \sin(\frac{n\pi}{4})$.

A body with initial temperature of 120° C is allowed to cool in air which remains at a constant temperature of 30° C. It is given that after 10 minutes, the body has cooled to 60° C. Find the temperature of the body after 20 minutes.

Unique Paper Code : 62354343

Name of the Course : B.A. (Prog.) Mathematics

Name of the Paper : Analytic Geometry and Applied Algebra

Semester : III (CBCS)

Time: 3 Hours

Maximum Marks:75

- Attempt any four questions in all.
- All questions carry equal marks.
- 1. (a) Identify and sketch the curve:

$$y = 4x^2 + 8x + 5$$

Also label the focus, vertex and directrix

(b) Describe the graph of the curve:

$$x^2 + 9y^2 + 2x - 18y + 1 = 0$$

Find its foci, vertices and the ends of the minor axis.

(c) Find an equation for the parabolic arch with base b and height h, shown in the accompanying figure



- 2. (a) Find the equation of parabola with vertex (2,4) and focus (3,4).
 - (b) Find the equation for the ellipse that has ends of major axis (± 6 , 0) and passes through (2, 3)
 - (c) Find the equation for a hyperbola that satisfies the given conditions: Asymptotes y = 2x + 1, y = -2x + 3 and passes through the origin.
- 3. (a) Find an equation of the sphere with centre (2,-1,-3) and satisfyingi) Tangent to the x-y plane.

- ii) Tangent to the x-z plane.
- iii) Tangent to the y-z plane.
- b) Show that the graph of the equation:

$$\sqrt{x} + \sqrt{y} = 1$$
, $\forall x \in [0,1], y \in [0,1]$

is a portion of a parabola.

(c) Describe the surface whose equation is given as

$$x^2 + y^2 + z^2 + 2y - 6z + 5 = 0$$

- 4. (a) Find \vec{u} and \vec{v} if $5\vec{u}+2\vec{v}=6\hat{i}-5\hat{j}+4\hat{k}$ and $3\vec{u}-4\vec{v}=\hat{i}+2\hat{j}+9\hat{k}$ Also find a vector of length 3 and oppositely directed to \vec{v} .
 - (b) (i) Find the projection of $\vec{a} = 2\hat{i} \hat{j} + \hat{k}$ on $\vec{b} = \hat{i} 2\hat{j} + \hat{k}$
 - (ii) Determine whether $\vec{u} = < 6, 1, 3 >$ and $\vec{v} = < 4, -6, -7 >$ make an acute angle, an obtuse angle or are orthogonal? Justify your answer.
 - (c) Find the volume of the parallelopiped with adjacent edges $\vec{u} = 3\hat{i} + 2\hat{j} + \hat{k}$, $\vec{v} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{w} = \hat{i} + 3\hat{j} + 3\hat{k}$. Also find the area of the face determined by \vec{u} and \vec{v} .
- 5. (a) Find the distance of the point P(2, 5, -3) from the plane

$$\vec{r}.(6\hat{i}-3\hat{j}+2k) = 4$$

- (b) Find the equation of the plane through the points $P_1(2, 1, 4)$, $P_2(1, 0, -3)$ that is perpendicular to the plane 4x + y + 3z = 2.
- (c) Show that the lines L_1 and L_2 are parallel and find the distance between them

$$L_1: x = 2-t,$$
 $y = 2t,$ $z = 3+t$
 $L_2: x = -1+2t,$ $y = 3-4t,$ $z = 5-2t$

6. (a) Suppose a job placement agency wants to schedule interviews for candidates Ann, Judy and Carol with interviewers Al, Brian and Carl on Monday, Tuesday and Wednesday in such a way that each candidate gets interviewed by each interviewer. Solve this problem using a Latin Square.

(b) Find a vertex basis for the following graph:



(c) For the following graph, find a minimal edge cover and a maximal independent set of vertices.



Name of Course	: BA. (Prog.) Mathematics
Unique Paper Code	: 62354343_OC
Name of Paper	: Analytical Geometry and Applied Algebra
Semester	: III
Duration	: 3 hours
Maximum Marks	: 75 Marks

1. Describe sketch and label the focus, vertex and directrix of the parabola

$$4x^2 - 2y + 8x + 5 = 0.$$

Find the centre, vertices, foci and ends of minor axis of the ellipse

$$9x^2 + 4y^2 - 18x + 24y + 9 = 0$$

Describe sketch and label the centre, vertices, foci and asymptotes of hyperbola

$$16x^2 - y^2 - 32x - 6y - 57 = 0.$$

2. Find equation of the parabola that has the focus (-1, 4) and directrix x = 5.

Find equation of the hyperbola having foci at $(0, \pm 5)$ and asymptotes $y = \pm 2x$.

Rotate the coordinate axes to remove *xy* term and identify the curve

$$31x^2 + 10\sqrt{3}xy + 21y^2 - 144 = 0.$$

Find the vector projection of v = 6i + 3j + 2k on b = i − 2j − 2k and find the scalar component of v in the direction of b.

Find the direction cosine of the vector $\mathbf{u} = 2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$.

Determine whether the vectors $\mathbf{u} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\mathbf{v} = 3\mathbf{i} - 2\mathbf{k}$, and $\mathbf{w} = 5\mathbf{i} - 4\mathbf{j}$ lie in the same plane?

4. Find the distance between the skew lines:

$$L_1: x = 1 + 7t, y = 3 + t, z = 5 - 3t;$$

$$L_2: x = 4 - t, \quad y = 6, \quad z = 7 + 2t;$$

Determine whether the planes 3x - 4y + 5z = 0 and 6x - 8y - 10z - 4 = 0 are parallel or perpendicular to each other.

Find the angle between the planes

$$3x - 6y - 2z = 15$$
 and $2x + y - 2z = 5$.

5. Find the parametric equations of the line through the point (5, 0, -2) and is parallel to the planes x - 4y + 2z = 0 and 2x + 3y - z + 1 = 0.

Define a Latin Square. Give an example of a Latin Square of order 4.

Find a matching or explain why none exists for the following graph.



6 Three pitchers of sizes 8 litre, 5 litre and 3 litre are given. If, initially, 8 litre pitcher is full and another two empty, find a minimal sequence of pouring so as to have exactly 1 litre of water in one pitcher.

Give a model of overlapping intervals for the following graph.



Find a maximum independent set in the following graph. Justify your answer.



Unique Paper Code	: 42354302_OC	
Name of the Course	: B.Sc. (Math Sci)-II/B.Sc. (Phy Sci)-II/ B.Sc. (Life Sci)-II/ B.Sc. (Industrial Chemistry)-II/Analytical Chemistry –II	
Name of the Paper	: Algebra-DSC	
Semester	: III	
Duration	: 3 Hours	Maximum Marks: 75

Instructions for Candidates:

- 1. Attempt any **four** questions out of six questions.
- 2. All questions carry equal marks.

Is the group U (15) cyclic. List all the elements of the group. What is the order of U (15). Check that H= { [a b] | ad ≠ 0; a, b, d ∈ ℝ } is a normal subgroup of GL (2, ℝ) or not? Also, determine whether the following permutations are even or odd:
(a) (1 3 5)

- (b) (1 3 5 6 7) (c) (1 2 4 3)(3 5 2 1)
- 2. Consider the set {4, 8, 12, 16}. Show that this set is a group under multiplication modulo 20 by constructing its Cayley table. What is the identity element? Is the group cyclic? If so, find all its generators. Show that $\mathbb{Z}_{10} = \langle 3 \rangle = \langle 7 \rangle = \langle 9 \rangle$. Is $\mathbb{Z}_{10} = \langle 2 \rangle$?
- 3. Let G be a group such that $a^3 = a \forall a \in G$. Show that G is abelian. Does the converse hold? Justify.

Let $M_2(\mathbb{Q})$ be the ring of 2×2 matrices with entries from \mathbb{Q} (the set of rationals). Let

$$S = \left\{ \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \mid a, b \in \mathbb{Q} \right\}.$$

Prove or disprove *S* is a subring of $M_2(\mathbb{Q})$.

Prove that S is commutative with respect to matrix multiplication.

Find the inverse of $\begin{bmatrix} 0 & -b \\ b & 0 \end{bmatrix}$ for $b \neq 0$ in *S*.

4. Let $R = \mathbb{Z} \oplus \mathbb{Z}$, where $\mathbb{Z} \oplus \mathbb{Z} = \{(a, b) \mid a, b \in \mathbb{Z}\}$ with the operations

$$(a,b) + (c,d) = (a + c, b + d),$$

 $(a,b)\cdot(c,d)=(ac,bd).$

Find all zero divisors of *R*. Find $x \neq 0$, 1_R such that $x^2 = x$.

Let $S = \mathbb{Z}_5 \oplus \mathbb{Z}_6 = \{(a, b) \mid a \in \mathbb{Z}_5, b \in \mathbb{Z}_6\}$. Find the characteristic of *S*.

Let $H = \{a_0 + a_1i + a_2j + a_3k \mid a_0, a_1, a_2, a_3 \in \mathbb{R}\}$. Is H a field?

Is $\alpha = 1 + i + j + k$ invertible in *H*? If so, find its inverse β and show that $\alpha\beta = \beta\alpha = 1$.

5. Let $V = \mathbb{R}^3$ and $W = \{(a, b, c) \mid a^2 + b^2 + c^2 \le 1\}$. Is W a subspace of V?

Determine whether or not the polynomials $x^2 + 3x - 2$, $2x^2 + 5x - 3$ and $-x^2 - 4x + 4$ generate $P_2(\mathbb{R})$.

Let U be the subspace of \mathbb{R}^3 generated by {(1,1,0), (1,2,3), (2,3,3)} and W be the subspace of \mathbb{R}^3 generated by {(1,2,2), (2,3,2), (1,3,4)}. Find the dimension of U+W.

6. Let T: $\mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation defined by

T(x, y, z) = (2x+3y-z, x+z)

Find a basis and dimension of (i) R(T), the range space of T (ii) N(T), the null space of T.

Is there a linear transformation T: $\mathbb{R}^2 \to \mathbb{R}^2$ such that

T (
$$x_i$$
)= y_i for i=1, 2, 3

where $x_1 = (1, -1), y_1 = (1, 0)$

 $x_2=(2,\,-1),\ y_2=(0,\,1)$

 $x_3 = (-3, 2), y_3 = (1, 1)$

Unique Paper Code	:	42347902-OC
Name of the Course	:	B.Sc. Programme / B.Sc. Mathematical Science
Name of the Paper	:	Analysis of Algorithms and Data Structures
Semester	:	V
Year of Admission	:	2015/2016/2017/2018

Duration: 3 Hours

Maximum Marks: 75

Instructions for Candidates

- 1. All questions carry equal marks.
- 2. Attempt any **FOUR** Questions.
- 1 Consider the class Node having info and next fields. Further, the class SinglyLinkedList maintains a single pointer, namely, head and implements the operations defined for maintaining the linked connection of nodes. Determine the functionality of following piece of code:

1 function modify()	
2 if head != NULL and head.next !=	NULL
3 prevNode = None	
4 nextNode = head.next	
5 while head != None	
6 head.next = prevNode	
7 prevNode = head	
8 head = nextNode	
9 if head != None	
10 nextNode =head.next	
11 head = prevNode	

Illustrate the step by step execution of the above piece of code for the linked list consisting of elements 13, 19, 5, 20, 18, 25 in sequence. State whether maintaining an extra pointer, namely tail will assist in achieving the above functionality. Justify your answer. Also, write a pseudocode for the doubly and circular linked list achieving the same operation.

2 Consider the following modification in (binary) search algorithm on an array A[1...n], sorted in increasing order. Instead of finding one middle index, we find two middle indices, mid1 and mid2, that divide the array into three equal parts. If an element at mid1 or mid2 equals the element 'x' that we are trying to search, we stop. Else, we do either of the following: search x from A[1...mid1 -1] if x is less than element at mid1 or search x from A[mid1 +1...mid2 -1] if x is greater than element at mid1 but less than element at mid2 x or search x from A[mid2 +1 ... n] if element at mid2 is less than x.

How will you compute mid1 and mid2? Write the pseudocode for the above-mentioned search strategy. What is the running time of the algorithm?

```
Consider the array: -5,-3,1,4,2,-6,1,5
```

Can you use Count sort to sort the above array? Justify your answer.

3 Given a binary search tree and the following code to traverse the tree, modify it to count the number of nodes in the tree.

```
fun traverse(Node root)
    if root==NULL
        return
    else
        traverse(left child of root)
        print(value of root)
        traverse(right child of root)
```

Also, determine which of the following statements is/are false about tree Traversals? Justify your answer in each case.

- a. If values 1,2,3,4,5,6 are inserted in the given order in a Binary search tree, then inorder, pre-order and level-by-level traversal are all the same.
- b. If values 6,5,4,3,2,1 are inserted in the given order in a Binary search tree, then inorder and preorder traversal are the same.
- c. If values 6,5,4,3,2,1 are inserted in the given order in a Binary search tree, then preorder and level-by-level traversal are the same.
- d. If values 6,5,4,3,2,1 are inserted in the given order in a Binary search tree, then inorder and postorder traversal are the same.
- 4 Consider the infix expression: A+B/C-(D-E)\$F*G (where \$ stands for exponentiation). Represent the given infix expression using a binary tree. Identify the ways in which the constructed binary tree can be traversed and write the traversal sequence for each one of them.

Suppose, we wish to represent the infix expression in postfix notation. Identify the data structure that you will employ for this conversion. Show the step-by-step sequence of operations involved. Further, determine the data structure that you will use for evaluating the postfix notation. Using the identified data structure and the values A=5, B=6, C=3, D=5, E=3, F=2, G=3; evaluate the postfix expression.

- 5 Which data structure would you choose for the following tasks and why? Justify your answer for each case.
 - a. To implement undo/redo operation in a text editor.
 - b. Bookmarks tab of a web-browser.
 - c. You are looking for a way out of a maze and you are not allowed to use recursion. You have to store the path that you are currently exploring and be able to go back one step whenever you find yourself in a dead-end and explore a new possibility from there.
 - d. You want to store a sorted list of strings and support the merging of two sorted lists into one, in-place (i.e., without creating a copy of the lists).
 - e. You are writing software for a call center. When a client calls, his call should be stored until there is a free operator to pick it up. Calls should be processed in the same order in which they were received.
 - f. You want to store the stations of a public transportation line. New stations can be added at both ends of the line but not between existing stations. You should be able to traverse the line in both directions.
 - g. You need to create a table of contents, i.e., the index page of a book. It must show every chapter, its sections and corresponding sub-sections, to all levels.
 - h. You need to create a glossary for a book. It must show important terms in sorted order. Synonyms or inter-related terms must have each other's reference.
- 6 Consider the following piece of code:

```
1 function enigma(integer n)
2
      if (n>0)
3
          enigma(n-2)
4
          print(n)
5
           enigma(n-2)
6
      else
7
          print(`*')
8
9 function main()
10 enigma(7)
11
12 main()
```

Determine the output of above code. How many times will enigma(n) be called to determine the value of enigma(7)? Draw the tree showing all the calls generated by enigma(7). Which data structure is used for tracking each recursive function call?

Determine the output for the function call enigma(7) after eliminating the statement in line 5.

Further, write the iterative version for this modified code (after eliminating statement in line 5), and derive its time complexity, give reasoning for the answer.

Unique paper code : 62357502_OC

Name of the paper : Differential Equations

Name of the course: CBCS B.A.(Prog.)

Semester : V

Duration : 3 Hours

Maximum Marks : 75

Instructions for the candidate:

This question paper has six questions in all. Attempt any four. All questions carry equal marks.

1. Solve the initial value problem

$$(4x + y - 3)dy = (2x + 3y - 4)dx; y(1) = 1.$$

Solve $p^{3}(x+2y) + 3p^{2}(x+y) + (y+2x)p = 0$.

Find an integrating factor and solve the differential equation:

$$(e^{x+y} - y)dx + (xe^{x+y} + 1)dy = 0.$$

2. Find complete solution of the differential equation: $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = \sin(2x+3).$

Consider the differential equation: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$. Show that $e^x \sin x$ and $e^x \cos x$ are linearly independent solutions of the differential equation. Find the general solution. Also find the solution y(x) with the property y(0) = 2, y'(0) = -3.

Solve $(2x+3)^2 \frac{d^2y}{dx^2} - (2x+3)\frac{dy}{dx} - 12y = 6x.$

3. Solve the differential equation using the method of variation of parameters:

$$y'' + 9y = 2\sec 3x.$$

Solve $(x^2D^2 - xD - 3)y = x^2 log x$. (where differential operator $D = \frac{d}{dx}$)

Solve the following differential equation given that $y = e^x$ is an integral:

$$xy'' - (2x - 1)y' + (x - 1)y = 0.$$

4. Solve the following system of differential equation

$$\frac{dx}{dt} - 7x + y = 0, \frac{dy}{dt} - 2x - 5y = 0.$$

Solve $\frac{dx}{x^2yz(y^2-z^2)} = \frac{dy}{y} = \frac{dz}{z}$.

Solve:

 $zydx - zxdy - y^2dz = 0.$

5. Eliminate the arbitrary function f from the equation

$$f(x + y + z, x^2 + y^2 - z^2) = 0$$

to find the corresponding partial differential equation.

Find the general solution of the differential equation

$$px(x + y) - qy(x + y) = -(x - y)(2x + 2y + 2z).$$

Find the complete integral of the partial differential equation

$$2xz - px^2 - 2qxy + pq = 0.$$

6. Classify the following partial differential equation into elliptic, parabolic or hyperbolic:

$$x^{2}(y-1)r - x(y^{2}-1)s + y(y-1)t + xyp - q = 0$$

where $r = \frac{\partial^{2}z}{\partial x^{2}}$, $s = \frac{\partial^{2}z}{\partial x\partial y}$, $t = \frac{\partial^{2}z}{\partial y^{2}}$, $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$

Form a partial differential equation by eliminating constants a, b from the relation:

$$ax^2 + by^2 + cz^2 = 1.$$

Find the general solution of the differential equation

$$x^2p + y^2q = x + y.$$

Find the complete integral of

$$px + qy + pq = z$$
.

Name of Course	: B.A.(Prog.) DSE : Mathematics				
Unique Paper Code	: 62357503				
Name of Paper	: DSC- Statistics				
Semester	: V				
Duration	: 3 hours				
Maximum Marks	: 75 Marks				

- 1. An oil exploration company currently has two active projects, one in Asia and the other in Europe. Let A be the event that the Asian project is successful and B be the event that the European project is successful. Suppose that A and B are independent events with P(A) = 0.4 and P(B) = 0.7.
 - i. If the Asian project is not successful, what is the probability that the European project is also not successful?
 - ii. What is the probability that at least one of the two projects will be successful?

If *A* and *B* are two events and the probability $P(B) \neq 1$, prove that

$$P(A|\bar{B}) = \frac{P(A) - P(A \cap B)}{1 - P(B)}$$

Hence show that $P(A \cap B) \ge P(A) + P(B) - 1$

2. The probability density of the random variable *X* is given by

$$f(x) = \begin{cases} \frac{x}{4}, & 0 < x \le 2\\ \frac{4}{x^3}, & 2 < x\\ 0 & elsewhere \end{cases}$$

Find the mean, variance and cumulative distribution function of *X*. Find $P(X \le 3)$ and P(X > 6)

Three urns contain respectively 3 green and 2 white balls, 5 green and 6 white balls and 2 green and 4 white balls. One ball is drawn from each urn. Find the expected number of white balls drawn out.

3. An insurance company finds that point 0.005% of the population die from a certain kind of accident each year. What is the probability that the company must pay off on more than 3 of 10,000 insured against such accidents in a given year.

The life of electronic tubes of a certain type may be assumed to be normally distributed with mean 155 hours and standard deviation 19 hours. What is the probability

- i. that the life of a random chosen tube is between 136 hours and 174 hours
- ii. that the life of a randomly chosen tube is less than 117 hours

[Given
$$\frac{1}{\sqrt{2\pi}} \int_0^1 e^{-\frac{t^2}{2}} = 0.3413$$
, $\frac{1}{\sqrt{2\pi}} \int_0^2 e^{-\frac{t^2}{2}} = 0.4772$]

- **4.** Consider a sample of size 2 drawn without replacement from an urn containing three balls numbered 1, 2 and 3. let *X* be the smaller of the two numbers drawn and *Y* the larger of the two.
 - i. Find the joint discrete density function of *X* and *Y*
 - ii. Find the conditional distribution of *Y* given X = 1
 - iii. Find the correlation coefficient of X and Y

Husband's age X:	23	27	28	28	29	30	31	33	35	36
Wife's age Y:	18	20	22	27	21	29	27	29	28	29

Calculate the coefficient of correlation for the following ages of husbands and wives

5. Ten individuals are chosen at random from a normal population and their heights are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71 inches. Test if the sample belongs to the population whose mean height is 66 inches. [Given t at 9 degrees of freedom at 5% level of significance is 2.62]

The students of same age group from two different schools were compared for variability in their mathematical skill. A random sample of 25 pupils from one school had a variance of 16 marks while a random sample of 22 pupils from the other school had a variance of 8 marks. Examine if the difference in variability is significant. [F at (24,21) at 5% level of significance is 2.05]

6. A die is tossed 120 times and each outcome is recorded as under

 Faces:
 1
 2
 3
 4
 5
 6

 Frequency:
 20
 22
 17
 18
 19
 24

Is the distribution of outcomes uniform? [Given chi-square at 5 degrees of freedom at 5% level of significance is 11.07]

A sample of 900 members is found to have a mean of 3.4 cm. Can it be reasonably regarded as a simple sample from a large population with mean 3.25 cm and standard deviation 2.61 cm?
Name of Course	: CBCS B.Sc. Phy. Sci
Unique Paper Code	: 42354302
Name of Paper	: DSC-Algebra
Semester	: III
Duration	: 3 hours
Maximum Marks	: 75 Marks

- 1. (a) Let $G = \{a + b\sqrt{2}\}$ where a and b are rational numbers not both 0. Prove that G is a group under ordinary multiplication. Is this group Abelian? Verify your answer.
 - (b) Let $A = \begin{bmatrix} 1 & 5 \\ 6 & 3 \end{bmatrix}$. Find A^{-1} in $GL(2,\mathbb{Z}_7)$.
- 2. (a) Prove that if 'a' is any integer relatively prime to n, then $a^{\phi(n)} = 1 \pmod{n}$ where $\phi(n)$ denotes the number of integers (positive) less than n and co-prime to n.
 - (b) Compute 5^{15} modulo7 and 7^{13} modulo11.
 - (c) Find all the left cosets of $H=\{1, 11\}$ in U(30).
- **3.** (a) Make a Cayley's Table for the group G of symmetries of a rectangle.
 - (b) Write all the proper non-trivial subgroups of G.
 - (c) What is the centre of this group?
- (a) Show that the set 2Z₁₀ = {0, 2, 4, 6, 8}⊕₁₀⊙₁₀ is a ring. By constructing the multiplication table, show that ring has unity.
 - (b) Show that $\mathbb{Z}_5[i]$ is not an Integral Domain.
 - (c) Let A and B be ideals of a ring R. If $A \cap B = \{0\}$, show that ab = 0 when $a \in A$ and $b \in B$.
- (a) Determine whether or not the set{(2, 21, 0), (1, 2, 5), (7, 21, 5)} is linearly independent over ℝ.
 - (b) Define T: $\mathbb{R}^3 \to \mathbb{R}^2$ by T(a₁,a₂,a₃) = (a₁-a₂,2a₃). Prove that T is a linear transformation and find a basis of both N(T) and R(T). Also verify the dimension theorem.
- 6. (a) For the vector space, Let $V = \{ \begin{bmatrix} a & a+b \\ a+b & b \end{bmatrix} : a, b \in \mathbb{R} \}$. Find a basis of V over \mathbb{R} .
 - (b) Let T: $\mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that T(1,2)=(2,3) and T(0,1)=(1,1). Find T(a,b) for any $(a,b) \in \mathbb{R}^2$.

Name of the Course	: CBCS B.Sc. (Mathematical Sciences) / B.Sc. (Physical Sciences)/ B.Sc. (Life Sciences)/Applied Sciences	
Unique Paper Code	: 42357501	
Name of the Paper	: DSC-Differential Equations	
Semester	: V	
Duration	: 3 Hours	
Maximum Marks	: 75	

1. Find an integrating factor and solve the differential equation $(y^2x + y^2 + y)dx + (2xy + 1)dy = 0$

Solve the differential equation

$$3 x (1 - x^2) y^2 \frac{dy}{dx} + (2x^2 - 1) y^3 = x^3$$

Also, solve the differential equation

$$(3y - 7x + 7)dx + (7y - 3x + 3)dy = 0$$

2. Solve the differential equation

$$\frac{d^2y}{dx^2} - y = x^2 \cos x$$

Also, solve the following initial value problem

$$x^{2}\frac{d^{2}y}{dx^{2}} - 2y = 4x - 8,$$
 $y(1) = 4, y'(1) = -1$

3. Given that $y = e^{2x}$ is a solution of

$$(2x+1)\frac{d^2y}{dx^2} - 4(x+1)\frac{dy}{dx} + 4y = 0,$$

Find the linearly independent solution by reducing the order and also write the general solution.

Use the method of variation of parameters to solve the equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = \frac{e^{-3x}}{x^3}$$

4. Find a family of oblique trajectories that intersect the family of parabolas $y^2 = cx$ at angle 60^o .

Also, solve the system of linear differential equations

$$2\frac{dx}{dt} + 4\frac{dy}{dt} + x - y = 3e^{t}$$
$$\frac{dx}{dt} + \frac{dy}{dt} + 2x + 2y = e^{t}$$

5. Find the general solution of the equation

$$(y - xu)\frac{\partial u}{\partial x} + (x + yu)\frac{\partial u}{\partial y} = x^2 - y^2$$

Apply the method of separation of variables to solve the equation:

$$\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} + u = 0, \quad \text{with } u(x,0) = 4e^{-3x}$$

6. Find the partial differential equation arising from the surfaces

$$z = f(x^2 + y^2)$$

Also, reduce the equation

$$\frac{\partial^2 u}{\partial x^2} + 2x \frac{\partial^2 u}{\partial x \partial y} + x^2 \frac{\partial^2 u}{\partial y^2} = 0$$

to canonical form.

Name of Course	:CBCS B.Sc. (Math Sci)- II /B.Sc. (Phy Sci)-II/
Unique Paper Code	: 42357501_OC
Name of Paper	:DSE-Differential Equations
Semester	:V
Duration	:3 Hours
Maximum Marks	:75 Marks

1. Find the Wronskian of

 $\{e^{t}, e^{-t}, e^{3t}\}$. Are these vectors linearly independent?

Solve

- $y (xy + 2x^2y^2)dx + x(xy x^2y^2) dy = 0$
- 2. Solve

$$\frac{d^2y}{dx^2} + 2y = x^2 e^{3x}$$

Solve

$$x log x \frac{dy}{dx} + y = 2 log x$$

3. Solve the Initial Value Problem

$$\frac{d^2y}{dx^2} + 4y = \frac{1}{2}Sin3x, \quad y(0) = 0, \quad y'(0) = 0$$

Solve

$$(2x+1)^2 \frac{d^2y}{dx^2} - 2(2x+1)\frac{dy}{dx} - 12y = 6x$$

4. Solve the equation using method of Variation of Parameters

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^x + x^2$$

Solve the Simultaneous Equations

$$\frac{dx}{dt} - \frac{dy}{dt} + 2y = Cos2t, \qquad \frac{dx}{dt} + \frac{dy}{dt} - 2x = sin2t$$

5. Find the general solution of the partial differential equation:

$$2xy\frac{\partial z}{\partial x} + (x^2 + y^2)\frac{\partial z}{\partial y} = 0$$

Find the complete integral of the partial differential equation (p+q)(px+qy) = 1, by using Charpit's method, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$.

6. Form a partial differential equation given by $z = xy + f(x^2 + y^2)$, where f is an arbitrary function.

Classify and transform the equation $\frac{\partial^2 z}{\partial x^2} + y \frac{\partial^2 z}{\partial y^2} - x \frac{\partial z}{\partial y} + y = 0$ into the canonical form and then find the general solution.

Name of Course	: CBCS (LOCF) Generic Elective- Mathematics
Unique Paper Code	: 32355101
Name of Paper	: GE-1 Calculus
Semester	: I
Duration	: 3 hours
Maximum Marks	: 75 Marks

1. Consider the function $f(x) = 2x - 3x^{2/3}$.

- a) Find the intervals on which f(x) is increasing and the interval on which f(x) is decreasing.
- b) Find where the graph of f(x) is concave up and where it is concave down.
- c) Find points of maxima, minima and inflection.
- d) Find asymptotes, vertical tangents and cusps if any.
- e) Sketch the graph of f(x)
- 2. Evaluate the following limits:
 - a) $\lim_{x\to 0^+} x^m (\ln x)^n$; *m*, *n* are positive integers
 - b) $\lim_{x \to \pi/4} (2 \tan x)^{1/\ln(\tan x)}$
- 3. Sketch the graph of $r + \theta = 0$, $\theta \in (-\infty, \infty)$ in polar coordinates.
- 4. (i) Find the volume of the solid obtained by revolving the region bounded by the curves y = 4x and y = x² + 3x/2 for x ≥ 0, y ≥ 0 about x-axis using washer's method.
 (ii) Find the volume of the solid obtained by revolving the region bounded by the curves xy = 2, xy = 1, x = 1, x = 4 about y-axis using cylindrical shell method.
- 5. Discuss the continuity of the function $f = \begin{cases} \frac{\cos(x^4 + y^4)}{x^4 + y^4}, & (x, y) \neq (0, 0) \\ 1 & (x, y) = (0, 0) \end{cases}$ Also find $f_x(x, y)$ and $f_y(x, y)$ for all $(x, y) \in R^2$.
- 6. (i) The Temperature (in degree Celsius) at a point (x, y) on a metal plate in xy-plane is $T = \frac{x^2 y^2}{1 + x^4}$

A men at (1, 2) wants to walk in the direction in which the temperature drops most rapidly. Find a unit vector in the direction.

(ii) Find the directions in which the function $f(x, y, z) = \ln xy + \ln yz + \ln xz$ increase and decrease most rapidly at the point $P_0(1, 1, 1)$. Then find the derivatives of the function in those directions.

Name of Course	: Generic Elective	
Unique Paper Code	: 32355101_OC	
Name of Paper	: GE-1 Calculus	
Semester	: I	
Duration	: 3 hours	
Maximum Marks	: 75 Marks	

1.

a. For the function $f(x) = x^3 - 3x^2$,

- (i) Find the interval in which the function is increasing or decreasing.
- (ii) Find the points of relative maxima, relative minima or none.
- (iii) Find the interval where the graph is concave up and where it is concave down.
- (iv) Determine points of inflexion, if any.
- (v) Sketch the graph.

b. Find all the vertical and horizontal asymptotes of the function $g(x) = \frac{3x^3 + 4x - 9}{x^3 - 27}$.

2.

a. By using L' Hospital's Rule, find the following limits

(i)
$$\lim_{x \to 1} (\cos x)^{\left(\frac{1}{x^2}\right)}$$
 (ii) $\lim_{x \to +\infty} \left(1 + \frac{2}{x^2}\right)^{3x^2}$

b. If the limit of $\left(\frac{a \sin 3x - \sin x}{x^3}\right)$ as $x \to 0$ is finite, find the value of *a* and the limit.

3.

a. Find the volume by washers of the solid that results when the region enclosed by the curve $y = \sqrt{100 - x^2}$ and the line 2y = x + 10 is revolved about the x-axis.

b. Find the arc length of the curve
$$x = 7t^3$$
 and $y = 4t^2$; $1 \le t \le 5$.

4.

- a. Find T(t), N(t) and B(t) at the given point $r(t) = 6\cos t i + 6\sin t j + (t/4) k$; $t = \pi/2$.
- b. Find the curvature and the radius of curvature at the stated point. $x = 5 \sin t$, $y = 5 \cos t$, z = 7t; $t = \pi/4$.

5.

a. Find the absolute maxima and minima of function $f(x, y) = x^3 + 4x - 4xy + y^2$ over the rectangle $R: -2 \le x \le 2, 0 \le y \le 2$.

b. Sketch the curve $r = 2 - sin\theta$.

6.

- a. Find the equation of hyperbola with foci (0, ± 6) and asymptotes $y = \pm \frac{2}{3}x$.
- b. Check whether $\lim_{(x,y\to(0,0)} \frac{x^2y^3}{(x^4+2y^2)^{\wedge}2}$ exists or not.
- c. Find the equation of tangent plane of the function $f(x, y) = x^2 + 2x + e^{-x} \sin y + y$ at the point $(0, \frac{\pi}{2}, 1 + \frac{\pi}{2})$.

Name of the Course	: Generic Elective
Unique Paper Code	: 32355301
Name of the Paper	: GE-3 Differential Equations
Semester	: 111
Duration	: 3 Hours
Maximum Marks	: 75 Marks

1. (i) Solve

$$xdy - ydx = \sqrt{x^2 + y^2} dx.$$

(ii) Solve the initial value problem

$$(x^{2} + y^{2} + x)dx + xy dy = 0, \quad y(1) = 1.$$

(iii) Solve the initial value problem

$$x\frac{dy}{dx} + y = y^2 \log x, \qquad y(1) = -1.$$

2. (i) Find the orthogonal trajectories of the family of curves $3xy = x^3 - a^3$, *a* being parameter of the family.

(ii) Find a family of oblique trajectories that intersect the family of circles $x^2 + y^2 = c^2$ at angle 45°.

(iii) Solve

$$\frac{dy}{dx} = e^{x+y} + x^2 e^{x^3+y} \, .$$

3. (i) Solve by method of variation of parameters

$$y'' + y = \csc ec x.$$

(ii) Solve by method of undetermined coefficients

 $y'' + 1.44 y = 24 \cos 1.2 x.$

(iii) Solve

$$y''' - 2y'' + 4y' - 8y = 0$$
, $y(0) = -1$, $y'(0) = 30$, $y''(0) = 28$.

4. (i) Show that $\{e^{-x}, e^{3x}, e^{4x}\}$ forms a basis of the solution set of the equation

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 12y = 0.$$

(ii) Solve the initial value problem

$$x^{2} y'' - 2x y' - 10y = 0, y(1) = 5, y'(1) = 4.$$

(iii) Solve the linear system

$$y'_{1} = 2y_{1} + 5y_{2}$$
$$y'_{2} = 5y_{1} + 12.5y_{2}$$

- 5. (i) Find the partial differential equation arising from the surface $z = xy + f(x^2 + y^2)$.
 - (ii) Find the general solution of the partial differential equation $u_x + 2xy^2 u_y = 0.$
 - (iii) Apply the method of separation of variables u(x, y) = f(x)g(y) to solve

$$yu_x + xu_y = 0$$
 on $u(0, y) = y^2$.

6. Reduce each of the following equations into canonical form and find the general solution:

(i)
$$u_x - u_y = u$$
, $u(x, 0) = 4e^{-3x}$.

- (ii) $u_{xx} + 6u_{xy} + 9u_{yy} + 3yu_{y} = 0.$
- (iii) $u_{xx} 3u_{xy} + 2u_{yy} = 0.$

Name of the Course	: CBCS-(LOCF)-Generic Elective B.A.(Prog.)/ B.Com(Prog)
Unique Paper Code	: 62355503
Name of the Paper	: GE- General Mathematics-I
Semester	: V
Duration	: 3 hours
Maximum Marks	: 75 Marks

1. Write a short note on the life and mathematical contribution of any of three of the following Mathematicians:

- a. Aryabhata
- b. Bhaskara- II
- c. Paramesvara
- d. Brahmagupta
- 2. Define Perfect numbers and Amicable numbers. State the properties of Perfect numbers.

Define unit fraction and express $\frac{3}{4}$ and $\frac{5}{8}$ as unit fraction. Define algebraic numbers and transcendental numbers. Why π is not an algebraic number?

3. Define the Inversion and explain The Fifteen Puzzle.

Find the remainder when

 $12345 \times 123456 \times 1234567$ is divided by 13 .

What is the Euclidean algorithm? Find the greatest common divisor of 60 and 25.

4. Find the number of distinct permutations of the letters in "Karnataka" and "Chennai"?

Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 6 \\ 0 & 1 & 3 \\ 2 & 1 & 4 \end{bmatrix}$. Compute the product AB and BA whichever exists.

5. Express the matrix A as the sum of a symmetric and skew symmetric matrix

$$A = \begin{bmatrix} 2 & 0 & -3 \\ 4 & 3 & 1 \\ -5 & 7 & 2 \end{bmatrix}$$

Let
$$C = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$
 and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, Find λ so that $C^2 = 5C + \lambda I$

6. Use Cramer's Rule to solve for x and y in the below two equations

$$\begin{aligned} x - 2y &= 4 \\ -3x + 5y &= -7 \end{aligned}$$

If $A = \begin{bmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ -2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 & 4 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$, Find det(AB), det(A) and det(B) and Verify

whether det(AB) = det(A) * det(B)

Name of Course	: LOCF B.Sc. (Math Sci)-II/ B.Sc. (Phy Sci)-II/		
B.Sc. (Life Sci)-II/ B.Sc. (Industrial Chemistry)-II/ Analytical Chemistry-II			
Unique Paper Code	: 42353328		
Name of Paper	: SEC-1 Computer Algebra System		
Semester	: III		
Duration	: 3 hours		
Maximum Marks	: 38 Marks		

1. Plot the piecewise function

$$f(x) = \begin{cases} 2-x & x < -1 \\ x & -1 \le x < 1 \\ (-1+x)^2 & x \ge 1 \end{cases}$$

Plot the following graph of a circle with orange and dashed grid lines



2. For c = -1 + 2i and z = 2 + 3i, iterate the function f(z) = 3z + c five times. Manipulate the function, $f(x) = a x^2$ for $-2 \le x \le 2$ and $-3 \le x \le 3$ using the slider.

3. Write some similarities and differences between reduce, solve and NSolve commands. Integrate $\ln(x + 1)^m$ for integers m = 1 to 6, identify the pattern, and propose a general formula for

$$\int \ln(x+1)^m \, dx$$

for any positive integer *m*.

4. Give the syntax to find the first derivative and indefinite integral of the function $x^3 + \cos x$ and plot the function using Mathematica/Maxima/Matlab/etc. Find the maximum value of the function

$$f(x) = \sin x + \frac{\sin 2x}{2} + \frac{\sin 3x}{3}$$
, for all $x \in [0, \pi]$.

5. Write the syntax to obtain a matrix of order 5×5 with all the diagonal entries as 4, all entries on the sub-diagonal as 6 and all entries on the super-diagonal as 7. Find the cofactors and eigenvalues of the matrix.

6. Write a syntax to obtain 4×4 lower triangular matrix with entries on and below the diagonal equal to i + 3ij, and above the diagonal equal to 0. Find adjoint of the matrix.

Unique Paper Code	:	62353326_OC
Name of the Paper	:	SEC-Mathematical Typesetting System
Name of the Course	:	B.A. (Prog.) CBCS Skill Enhancement Course
Semester	:	III
Duration	:	3 Hours
Maximum Marks	:	38Marks

Instructions for Candidates

Attempt any four questions. All questions carry equal marks.

1 (i) What is wrong with the following LaTeX input? What is the correct way to do it?

If m=1 and n=2, then m+n=3

- (ii) Which command is used to draw a circle with center(x, y) and radius r.
- (iii) What is the output of the command $frac{d}{dx}\left(\frac{0}^{x}f(t),\frac{1}{y}=f(x)\right).$
- (iv) In beamer, frames are made with a which command ?
- (v) Write a code in LaTeX to get the following: if x < y 1 then x < y
- (vi) What is the output of the command $y=\sqrt{x}$.
- (vii) What is the difference between "itemize" and "enumerate" environment.
- 2(a) Write the command in LaTeX to obtain the expression $e^{i\theta} = \cos \theta + i \sin \theta$.
 - (b) Explain the command $\product{psarc[linewidth=0.1pt](1,1){2}{0}{70}}$.
 - (c) Typeset the following in LaTeX:

The *derivative* of a function f, denoted f', is defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

3(a) Write the command in PSTricks to draw the following picture



(b) Write the command in PSTricks to plot the function $y = \sin(1/x)$.

4(a) Write the command in PSTricks to draw the following picture



(b) Plot $y = \sqrt{x} \sin(1/x)$, for $0 < x \le 2$. On the same co-ordinate system, plot the functions $y = \sqrt{x}$ and $y = -\sqrt{x}$, for $0 \le x \le 2$, with these functions shown as dotted curves.

5 (a) Write the code to make the following multi-line equations

$$1+2 = 3$$

$$4+5+6 = 7+8$$

$$9+10+11+12 = 13+14+15$$

$$16+17+18+19+20 = 21+22+23+24$$

$$25+26+27+28+29+30 = 31+32+33+34+35$$

(b) Write the code to typeset the following:

$$f(x) = \begin{cases} -x^2, & x < 0\\ x^2, & 0 \le x \le 2\\ 4, & x > 2 \end{cases}$$

6. Write a code to make a beamer presentation of 5 pages (including title and thank you page) on any topic with diagram/picture.

Name of Course	: CBCS B.A. (Prog.)
Unique Paper Code	: 62353327_LOCF
Name of Paper	: SEC-Computer Algebra System
Semester	: III
Duration	: 2 hours
Maximum Marks	: 38 Marks

Q1. Form the following grid using appropriate functions

1	1	1	1
2	4	8	16
3	9	27	81
4	16	64	256
5	25	125	625

Each column is right aligned.

Graph each of the functions

a. $g(x, y) = 19\cos(x) + \sin(\sqrt{7} x), -20 \le x \le 20$ b. $2x + 1, 1 = x^2, 1 = x = x^2/3, -2 \le x \le 2$

b.
$$2x + 1$$
, $1 - x^2$, $1 - x - x^2/3$, $-2 \le x \le 2$

Q2.



Create the above plot of $f(x) = cos(x^2)$, $0 \le x \le 2\pi$, with thick, red and dotted grid lines. Change the code so that the grid lines appear only at $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ and y=.1,.3,.5,.7,.9

Compute each expression. a. $3^8 \mod 7$ b. $5^{10} \mod 11$

Q3. Write the command to define a polynomial

$$f(x) = x(x^2 + y^2) - a(x^2 - y^2)$$

Plot it in $-a \le x \le a$. Write a syntax to solve

$$g(x) = 3x^3 - 16x^2 + 23x - 6$$

and find the numerical solution of g(x) = 0.

Q4. Give the syntax to find the derivative of $f(x) = \frac{1}{(1+4x^2)\sqrt{1+3x^2}}$ with respect to x, then evaluate

$$\int_0^\infty \frac{1}{(1+4x^2)\sqrt{1+3x^2}} dx$$

and write a command to compute:

$$\sum_{p=0}^{100} (p+2)^5$$

Q5. Find the minimum value of the function $f(x) = x^{2/3}$ and write syntax for it also. Find the extreme point of the function $g(t) = 9t^2 + 2t + 4$ and also plot the given function.

Q6. Generate a 9×9 matrix whose i, j th entry is i^2+2ij by using Table command. Also write syntax to get the fifth column of the matrix. Find the trace of the matrix.

Name of the Course: B. Sc. (H) Mathematics CBCS(LOCF)

Unique Paper Code: 32353301

Name of the Paper: SEC : LaTeX and HTML

Semester: III

Time: 2 Hrs

Maximum Marks: 38

Instructions for Candidates:

- (i) This question paper has **six** questions in all.
- (ii) Attempt any **four** questions.

(iii)All questions carry equal marks.

1. (a) Write codes in LaTeX to typeset the following :

If *B* is any **square matrix** of order *m*, *I* is the m^{th} order identity matrix and α is a scalar, then the **characteristic equation** of *B* is given as :

$$|B - \alpha I| = \begin{vmatrix} a_{11} - \alpha & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} - \alpha & \cdots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mm} - \alpha \end{vmatrix} = 0.$$

- (b) Write codes in LaTeX to typeset a filled ellipse of horizontal radius a=3 and vertical radius b=1 with center (1,1). Also label its radii and circumference.
- (c) Write codes in HTML to create a web page showing the ordered list of the following subjects:

Calculus, Algebra, Real Analysis, Differential Equations, Group Theory The font of the text in the list should be "italics".

- (a) Elaborate the LaTeX commands to draw straight lines, arrows, circles and ovals using picture environment with at least one example of each type.
 - (b) Write codes in HTML to create a web page consisting a link "**my interests**" and a photograph . Keep the background color of the page yellow.

(c) Write codes in LaTeX to create a **colorful** beamer presentation on an equilateral trianglehaving at least **four** slides. Also, include a picture of the equilateral triangle.

3. (a) Write codes in LaTeX to typeset the following:

$$\begin{aligned} x_{2^n} &= 1 + \frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4}\right) + \dots + \left(\frac{1}{2^{n-1}} + \dots + \frac{1}{2^n}\right) \\ &> 1 + \frac{1}{2} + \left(\frac{1}{4} + \frac{1}{4}\right) + \dots + \left(\frac{1}{2^n} + \dots + \frac{1}{2^n}\right) \\ &= 1 + \frac{1}{2} + \frac{1}{2} + \dots + \frac{1}{2} \\ &= 1 + \frac{n}{2} \end{aligned}$$

(b) Write the postfix expression for the following mathematical expressions:

(i)
$$\frac{2\sin^2 x + 3x}{2y^3 + 5}$$
 (ii) $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

(c) Write codes in LaTeX to create a **colorful beamer** presentation on solving the quadratic

equation $ax^2 + bx + c = 0$ with first slide being the title slide.

4. Find the errors and write the correct version in the following LaTeX source code (highlight your corrections in the answer). Also, write its output.

 $\label{eq:action} \end{tabular} $$ \en$

- (b) Write codes in LaTeX to draw shaded sector of a circle **using** PSTricks. Also, label the radius and angle in the shaded sector.
- (c) Write codes in HTML to create a web page displaying the solution of the equation

 $x^2 - 4x + 3 = 0$. Keep the font color of the given equation red.

5. (a) Write codes in LaTeX to **plot** graph of the f(x) given below. Also label the graph.

$$f(x) = \begin{cases} x^3 & \text{if } x > 0, \\ x^{-3} & \text{if } x < 0. \end{cases}$$

- (b) Write codes in LaTeX to draw a circle and use HTML to create a web page containing these codes and picture of the circle.
- (c) Write codes in LaTeX to create the following beamer presentation:

Continuous Function Student University of Delhi December, 2021

Introduction

In layman's language, a function is continuous when it's graph is simple unbroken curve i. e. you can draw its graph without lifting pen from the paper. Some examples of continuous functions are

- 1. Polynomial functions
- 2. Exponential functions
- 3. Trigonometric functions in some periodic interval etc.

Definition

Let $A \subseteq \mathbf{R}$, let $f : A \longrightarrow \mathbf{R}$ and let $c \in A$. We say that f is continuous at c if, given any number $\epsilon > 0$ there exists $\delta > 0$ such that if x is any point of A satisfying $|x - c| < \delta$, then $|f(x) - f(c)| < \epsilon$.

Example graph of a continuous function



For the last slide use the picture named "continuous function.png".

6. (a) What is the output of the following command :

 $\left(\sum_{i=0}^{i=0}^{i=0} = \sum_{k=0}^{i=0}^{i=0}^{k} = 0$

(b) Write codes in LaTeX to typeset the following *list* :

$$i. \qquad \prod_{n=0}^{\infty} \left(1 + 2^{-2^n}\right)$$

- ii. Let f be a function from \mathbf{R}^2 to \mathbf{R}^2
- (c) Write codes in LaTeX to draw a picture of 5-12-13 Pythagorean triangle in the picture environment. Also, label the sides.

Unique Paper Code	:	42343307
Name of the Course	:	B. Sc. Programme / B. Sc. Mathematical Science SEC-1
Name of the Paper	:	Data Analysis using Python Programming
Semester	:	III
Year of Admission	:	2019 onwards

Duration: 3 Hours

Maximum Marks: 75

Attempt any **four** questions.

All questions carry equal marks.

1. Consider a list of values:

bag = [25, 26, 21, 22, 31, 29, 33, 34, 26, 30, 31, 46]

- Import the appropriate Python libraries to create a ndarray called bag_weights having 3 rows and 4 columns from the list bag.
- Use Numpy library to display the mean, variance and median of the given data in bag_weights.
- Write a command to display the count of values greater than the median in bag_weights.
- Transpose bag_weights and then split it in two arrays bagA and bagB having 2 rows and three columns each.
- Sort bagA such that it brings the highest value of the row in the first column. Sort bagB such that it brings the lowest value of the row in the first column.
- Find the union and intersection of values in bagA and bagB.

2. Consider a list of values:

rate = [4.23, 3.8, 2.98, 2.56, 3, 114, 3.8, 3.78, 2.98, 4.8, 4.10, 3.65]

- Import the appropriate Python libraries to create a one-dimensional ndarray called growth_rate from the list rate. Create another one-dimensional array named twos having the same number of elements as growth_rate, all set to 2.
- Use Numpy library to find the index of the maximum and the minimum values in the array growth_rate.

- What does a box plot show? Give a command to display a boxplot for growth rate.
- Concatenate the two arrays growth_rate and twos, and reshape the resulting array to have four rows and appropriate number of columns, call it results.
- Find the mean, median, mode and standard deviation of each column in results.
- Write a command to store the array results to a file called result.npy on the disk in the current working directory.

#	Movie_	Director_	Language	Length	Budget	Gross_	User_	Critic_
	title	name				collections	rating	rating
1	AAA	Ram	Urdu	120	90	80	4	7
2	BBB	Eash	Hindi	NULL	65	70	6	6
3	CCC	Anju	Hindi	125	100	150	9	8
4	DDD	Jay	Hindi	150	85	85	6	5
5	EEE	Eash	Hindi	90	60	NULL	7	5
6	FFF	Suraj	French	100	115	120	8	6
7	GGG	Anju	French	NULL	80	81	5	5
8	ННН	Ram	French	115	50	40	3	4
9	JJJ	Anju	French	120	92	75	3	6

3. Consider the following DataFrame (df):

Write suitable Python command(s) in Pandas library:

- Display the number of rows and columns present in the DataFrame df?
- Display the names of columns that have NULL values present in them, along with the count of NULL values. Replace the NULL values present in the column with the lowest value in that column.
- Create a new column in df named Rating, which contains the mean of User_ rating and Critic_rating. Create another column, Profit, which contains the difference of Gross_collections and Budget.
- Find the correlation between Budget and Rating. Based on the correlation values between two variables, what inference(s) can be drawn about the relationship between them?
- Group the movies according to the Director_name. Find the most profitable director.
- What does a contingency table depict? Write commands to display the contingency table between Director_name and Language.

Q4. Consider a dictionary:

```
dict1 = {Chhetri: 80, Shabbir: 23, Gouramangi: 6,
    Subrata: 92, Vijayan: 29, Gawli: NULL, Nabi: 7,
    Renedy: 4, Lalpekhlua: 23, Baichung:41, Surkumar: 2}
```

Write suitable Python command(s) in Pandas library:

- Create a Pandas Series for the dictionary dict1 where the key is name of the footballer and the value is the number of goals scored by him. The Series should have the names of the footballers as its index and values as goals scored.
- Display the names of Footballers who have scored more than 20 goals.
- Due to the good performance of top six footballers, their rankings have increased and the number of goals scored by them need to be increased by 25. Round the resulting value to the nearest integer equal to or more than the computed number of goals. Update the Series to reflect these changes.
- Include a 12th man named 'Mondal' in the above Series whose number of goals scored is not known.
- Display the list of Footballers whose number of goals scored is NOT NULL.
- Due to injury, 'Shabbir' was replaced by 'Sandhu' who number of goals scored is 5. Reflect this change in the Series and display the new Series.

ld	SepalLengthCm	SepalWidthCm	PetalLengthCm	PetalWidthCm	Species
1	5.1	3.5	1.4	0.2	Iris-setosa
2	4.9	3	1.4	0.2	Iris-setosa
3	4.7	3.2	1.3	0.2	Iris-setosa
4	4.6	3.1	1.5	0.2	Iris-setosa
5	5	3.6	1.4	0.2	Iris-setosa
6	5.4	3.9	1.7	0.4	Iris-setosa
7	4.6	3.4	1.4	0.3	Iris-setosa
8	5	3.4	1.5	0.2	Iris-setosa
9	4.4	2.9	1.4	0.2	Iris-setosa
10	4.9	3.1	1.5	0.1	Iris-setosa
11	5.4	3.7	1.5	0.2	Iris-setosa
12	4.8	3.4	1.6	0.2	Iris-setosa
13	4.8	3	1.4	0.1	Iris-setosa
14	4.3	3	1.1	0.1	Iris-setosa
15	5.8	4	1.2	0.2	Iris-setosa
16	5.7	4.4	1.5	0.4	Iris-setosa
17	5.4	3.9	1.3	0.4	Iris-setosa
18	5.1	3.5	1.4	0.3	Iris-setosa
19	5.7	3.8	1.7	0.3	Iris-setosa

Q5. The first few rows of the standard iris dataset in the sklearn library are given below:

• Import the appropriate Python libraries to load the dataset. Create a Pandas DataFrame named iris having all the columns in the dataset.

- Use an appropriate command to display a summary of the vital statistics of all numerical and categorical attributes in iris.
- What is the role of pre-processing in data analysis? Discuss how will you choose between (a) deleting the rows containing missing values or (b) replacing the missing values in a column with the mean or (c) replacing them with the mode of the column.
- Give a Pandas command to convert the categorical attribute, species into dummy variables. Display all the columns of the DataFrame including the dummy variables. Give a command to drop the column species from the DataFrame.
- Draw a scatterplot between the columns sepal length and petal length for the species setosa in iris.
- Create 5 equal length bins for each of the two columns sepal length and sepal width. Draw two histograms, one each for the values of sepal length and sepal width in these bins in a single figure. Save this image in a file on the hard disk.

Pc No	carat	cut	color	clarity	depth	table	Price in
							(thousand
							INR)
1	0.23	Ideal	Ε	SI2	61.5	55	326
2	0.21	Premium	E	SI1	59.8	61	326
3	0.23	Good	Ε	VS1	56.9	65	327
4	0.29	Premium	I	VS2	62.4	58	nan
5	0.31	Good	J	SI2	63.3	58	335
6	0.24	Very Good	J	VVS2	nan	57	336
7	0.24	Very Good	I	VVS1	62.3	57	336
8	0.26	Very Good	Н	SI1	61.9	55	337
9	0.22	Fair	Ε	VS2	65.1	61	nan
10	0.23	Very Good	Н	VS1	59.4	61	338
11	0.3	Good	J	SI1	64	55	339
12	0.23	Ideal	J	VS1	62.8	56	340
13	0.23	Ideal	J	VS1	nan	56	340
14	0.22	Premium	F	SI1	60.4	61	342
15	0.31	Ideal	J	SI2	62.2	54	344

Q6. Consider the details 15 rubies as follows:

- Import the appropriate Python libraries to create a Pandas DataFrame named rubies having the above columns. The columns and rows of the DataFrame should have appropriate names.
- Draw box plots for all numerical columns of the dataset in the same chart. Display the median of all numerical attributes in rubies for each type of cut.
- Display the per carat average price of all rubies grouped by the two attributes clarity and color.
- Normalize all quantitative features in range of [0,1].
- Draw word cloud for attribute cut.

Unique Paper Code	:	42353327
Name of the Paper	:	SEC-Mathematical Typesetting System
Name of the Course	:	BSc. (Prog.) CBCS Skill Enhancement Course
Semester	:	III
Duration	:	3 Hours
Maximum Marks	:	38Marks

Instructions for Candidates

Attempt any four questions. All questions carry equal marks.

- 1(i) What is the difference between the commands \equarray and \ equarray*
- (ii) What is wrong with the following LaTeX input? What is the correct way to do it?

If m=1 and n=2, then m+n=3

- (iii) What is the output of the command $\lim_{x\to\infty} x^{\pm}$.
- (iv) Which command is used to draw a circle with center(x, y) and radius r.
- (v) Which package is used for plotting a function with PSTricks.
- (vi) What is the output of the command

 $\frac{d}{dx} = \frac{0}{x}f(t), dt = f(x)$.

(vii) Typeset the following in LaTeX: $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$

- 2(a) Write a code in LaTeX to get the following: if x < y 1 then x < y
- (b) Write the command in LaTeX to obtain the expression: $\left(\frac{a+b}{x+y}\right)^{\frac{2}{3}}$

(c) Write the command in LaTeX to obtain the expression: $(a + b + a^2b + ab^2)^2$

3. (a) Write a code in LaTeX to get the following:

$$\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \\ \mathbf{b}_1 & \mathbf{b}_2 & \mathbf{b}_3 \end{bmatrix}$$

(b) Write a code in LaTeX to get the following: $|x-2| = \begin{cases} x-2 & x \ge 2 \\ -x+2 & x < 2 \end{cases}$

- 4. (a) Write the command in PSTricks to plot the function y = cos(x).
 - (b) Write the command in PSTricks to draw the following picture



5(a) Write the code to typeset the following: $\begin{bmatrix} 5 \\ -1 \end{bmatrix}$

a	b	p	q	_	ap +br	aq + bs
c	d	r	s		$_{cp} + dr$	cq + ds

_

(b) Write the code in PSTricks to draw the following picture





(b) Write a code in LaTeX to get the following:

The general solution to the differential equation y'' - 3y' + 2y = 0 is $y = c_1 e^x + c_2 e^{2x}$

6. Using beamer produce a presentation with the following content:

Slide 1: Title of the presentation with authors name and date

Slide 2 : Some trigonometry identities : $\sin^2 \theta + \cos^2 \theta = 1$ $2\sin \theta \cos \theta = \sin 2\theta$

Slide 3: The area of the triangle with the sides a,b,c is given by *Heron's formula* : $A = \sqrt{s(s-a)(s-b)(s-c)}$ where s is the semiperimeter (a+b+c)/2

Slide 4 : Thank you

Unique Paper Code	:	42343306-OC
Name of the Course	:	B.Sc. (Prog.)/B.Sc. Mathematical Sciences
Name of the Paper	:	Office Automation Tools (SEC)
Semester	:	III
Year of Admission	:	Admissions of 2015 -2018

Duration of Examination: 3 Hours

Maximum Marks: 75

Attempt any **four** questions. All questions carry equal marks.

1. The worksheet given below contains first name, last name, age, D.O.B. and segment column. Show all the steps performed for determining the value of age segment column using VLOOKUP function. Also explain the functionality of VLOOKUP function and briefly describe all the parameters it takes.

	А	В	С	D	E	F	G	Н	1
1	First Name	Last Name	DOB	Age	Segment			Age	Label
2	Smith	Jone	23/2/1976	67				19	Young
3	Jones	Marry	8/6/1788	88				45	Mature
4	Rafel	Wadera	15/01/1959	90				90	Senior
5	Harry	Potter	27/09/1991	45					
6	Jim	Mathew	18/05/1968	37					
7	Trek	Lim	5/3/1990	19					

If you add either new rows or new columns to the pivot table source data, the pivot table is not updated even when you click on 'refresh data'. Why and what is the solution? Is it possible to make pivot table from multiple data sources?

- 2. Explain the layout components of PowerPoint. What is the purpose of having ribbons and explain the steps to customize or adding your own commands to the ribbon with the help of an example? How can a user add speaker notes to the slides? Also explain the functions of Auto content Wizard and Template.
- 3. You are a manager of ABC Ltd. and you want to send a newsletter to all the employees of your organization, in order to accomplish this task which option of MS Word will you use to merge the employees contact information stored in the excel file with word document? Explain the entire process by giving steps followed in sequence. Also write the steps to add a graphic or logo to my labels in MS Word?
- 4. Explain the steps to create a 10x10 multiplication table in a spreadsheet as shown below. The cells inside the table should contain only formulas and not numbers. Generate the table

with only one formula that you can drag to fill rest of the table. Can we use relative addressing or absolute addressing in this case? If not which addressing can be used and why? Explain different types of addressing supported by excel with suitable example.

1	А	В	С	D	E	F	G	Н	I	J
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	5	12	14	16	18	20
3	3	6	9	12	10	18	21	24	27	30
4	4	8	12	16	15	24	28	32	36	40
5	5	10	15	20	20	30	35	40	45	50
6	6	12	18	24	25	36	42	48	54	60
7	7	14	21	28	30	42	49	56	63	70
8	8	16	24	32	35	48	56	64	72	80
9	9	18	27	36	40	54	63	72	81	90
10	10	20	30	40	45	60	70	80	90	100

- 5. Explain the components of a computer system with the help of logical diagram? Explain different types of Memory. What are the advantages of cache memory? Explain in detail the technological developments in each generation of computers?
- 6. Explain the term Macro used in MS-Word? What are its features? Write steps to create a Macro in a document and also the steps to record it? How many ways are there in MS Word to view the document? Explain these in detail.

Name of Course	: CBCS (LOCF) B.Sc. (Math Sci II; Phy Sci-II)
Unique Paper Code	: 42353504
Name of Paper	: SEC- Transportation and Network Flow Problems
Semester	: V
Duration	: 3 hours
Maximum Marks	: 55 Marks

Attempt any four questions. All questions carry equal marks.

1. Consider the transportation model in the given table:

	D1	D2	D3	D4	D5	Supply
S1	11	13	15	20	18	80
S2	18	16	21	14	12	100
S3	17	23	9	13	16	120
S4	16	22	10	8	15	70
Demand	70	60	90	90	60	

Determine the initial basic feasible solution of the transportation model using Vogel's Approximation Method (VAM) and North West Corner Method. Find the optimal solution using the initial basic feasible solution obtained from Vogel's Approximation Method. Does this model have alternate optimal solution? Justify your answer.

2. A company has four jobs and five machines. The cost of each job on each machine is given in the following cost table:

Machines Jobs	M1	M2	M3	M4	М5
J1	22	20	15	10	17
J2	21	18	10	7	16
J3	16	12	5	5	12
J4	20	20	17	15	16

Determine the job assignment to the machines so as to minimize the total cost using Hungarian Method with explanation.

3. In the network shown below, find the flow pattern that gives the maximal flow from node **A** (source) to node **E** (sink) where the arc capacities are mentioned on respective arcs. Also, compute the optimum flow in each arc along with the direction of flow.



4. In the following network the distance (in miles) between different stations is shown on each link. Determine the shortest route from Station **O** to Station **T** using Dijkstra's algorithm.



5. A, B, C and D are four different cities. A salesman must travel from city to city to maintain his accounts. The cost of travelling between the various cities is shown in the matrix given below:

		To city				
		А	В	С	D	
	А	∞	4	7	3	
From city	В	4	∞	6	3	
	С	7	6	∞	7	
	D	3	3	7	∞	

The home city is city B. Determine the tour that will minimize the total cost of visiting all cities and returning home.

6. A small project involves 7 activities, and their time estimates are listed in the following table.

Activity	Estimated Duration (weeks)						
(i-j)	Optimistic	Most Likely	Pessimistic				
1-2	1	1	7				
1 – 3	1	4	7				
1 - 4	2	2	8				
2 - 5	1	1	1				
3 - 5	2	5	14				
4 - 6	2	5	8				
5 - 6	3	6	15				

Activities are identified by their beginning (i) and ending (j) node numbers

(a) Draw the network diagram of the activities in the project.

(b) Find the expected duration and variance for each activity. What is the expected project length?

(c) Calculate the variance and standard deviation of the project length.

Unique Paper Code	:	42343503
Name of the Paper	:	Web Designing Using HTML5 (SEC)
Name of the Course	:	B.Sc. Programme/B.Sc. Mathematical Science
Semester	:	V
Year of Admission	:	2019 onwards

Duration: 3 Hours

Maximum Marks: 75

Attempt any FOUR questions. All questions carry equal marks.

- Q1. Ramnik wants to design a website for his new travelling business that will contain 10 web pages. He wants that the basic look of 7 to 8 web pages of the website remains same. On the basis of this requirement, answer the following questions:
 - Which type of Cascading style sheet is appropriate for this website and how to define it in the web pages?
 - > Write the CSS properties for changing the following on the web page:
 - Background color
 - Headings in any light color and font
 - Highlight one offer using marquee
 - Given a web page of a restaurant website for providing feedback form to the customers. Write CSS properties for making following changes to the feedback form:
 - Setting the background color of the textboxes.
 - > Creating black color border around SUBMIT and RESET button.
 - Manisha is an alumnus of Delhi University. She wants to create a website that will resolve the queries of the students who wishes to take admission in Delhi University. Which type of website (Static or Dynamic) she should create and why? Explain any three web development issues that Manisha can face and needs to tackle. Also explain how the approach selected by you (Static or Dynamic) is processed.
 - Write an HTML program for displaying following pattern on a web page:



• Write an HTML code for displaying the following equation on a web page: $(x + y)^2 = x^2 + y^2 + 2xy$

- Q2. Write an HTML program for creating a nested list for Grocery shop. The main items of the list should be created as unordered list and should contain 5 items and each item should contain 3 sub items. The sub-items list should be ordered list.
 - Create a web page with three different headings H1, H2 and H3 and define the following three different classes in CSS for these headings: Normal – it should change the heading color as green. Important – it should change the font size of the heading. Extra – it should change the background color as grey.
 - Create a web page that contains information about the varieties of Cake recipes. Write the HTML code for the following:
 - > Add the quotation marks in the start and end of the Cake recipe title.
 - > Add the details of the web page author in the footer and add the copyright symbol.
 - Add an image of the Cake along with alternative text.
 - > Add the description of the Cake recipe using description list.
 - ➢ Add a line in between two Cake recipes.
- Q3. Write an HTML program for creating the following frame structure for displaying the information about a website that offer various online courses:

1. <u>About Us</u>	This frame should display the detailed
2. <u>Course 1: C</u>	information about the option coming in the
3. <u>Course 2: C++</u>	left pane, when user clicks on that option.
4. <u>Course 3: Java</u>	
5. <u>Course 4: Python</u>	

- Divi wants to add images along with the text that describe those images in her website. Write CSS properties for the following requirements:
 - > Float the image on the webpage to the right and set its top, bottom and left margins to create adequate space around it.
 - Add a thick black border around the image.
 - \blacktriangleright Make this image as clickable so that when user clicks on the image, its description should be displayed.
- Write an HTML program for displaying following C++ code on a web page:

```
# include<iostream.h>
# include<conio.h>
int main()
ſ
cout<<"Hello User";
return 0;
}
```

• Write an HTML program for creating the following table for an organization who wants to display the information about interns on their website:

Organization Logo			Internship Data					
S.No.	Candidate Name	DOB	Job Code	I Job Title	nternship Date of Joining	Details Duration	Stipend	Qualification

The table should contain at least 5 entries i.e., information about 5 candidates.

- Create a web page for an online book store. Write the CSS properties for the following settings that website owner wants to set on the web page:
 - Add a border around the body of the web page
 - Add a heading to the page
 - > Add top and bottom border to the heading.
 - Add the email of the website owner as the footer.
 - Set the background color of the footer as red, font color of the email as white and add the top border to this footer.
- Write an HTML program for creating a web page displaying information about six tools available for conducting online meetings. Use the internal linking to navigate the information available on the page.
 - Mayand is starting a coaching center for teaching various programming languages. He wants to design a website for it. The website will display the information about the following:
 - > The programming languages taught in coaching center.
 - > The profile of faculties teaching in the coaching center.
 - > Time table of all the batches currently running by the center.
 - Links or sources for downloading open source softwares.
 - > Enquiry form if a new student wants to join the coaching center.
 - > Feedback form to be filled by the currently enrolled students.
 - Contact details of the coaching center.

Create the SITE MAP for the website of the coaching center.

Also tell which domain name extension is suitable for this website and why.

- Riya wants to change textual appearance of her fashion website. Write CSS properties for making the following changes in the website:
 - > Adding a colourful shadow to the heading "Latest Collection".
 - > For keeping the description of a Kurta to the right of the web page.
 - For showing the discount on a Kurta by crossing the original price and displaying discounted price.
 - ➢ For setting the font that is easily readable.

- Write an HTML program for designing an application form to apply for a course in the Delhi University. The web page should display the heading "Application Form to Apply in Delhi University". The form should contain the controls for the following information: Username and Password of the candidate, Name, Gender (radio control), Marks scored in XII standard in various subjects (number control), Courses applied in Delhi University (checkbox) and Contact no. Use regular expression to validate the contact no.
 - A website contains headings, links and paragraphs. Write CSS properties using id selector for the following:
 - ▶ For H2 headings, set the font size to 150%.
 - ➢ For H4 headings, set the color to #800000.
 - ▶ For H6 headings, set the font type to Times New Roman.
 - > Italicize the links that has the mouse hovering over it.
 - > Centre align the paragraph in the footer.
 - A web page contains information about environment. Write CSS properties for the following:
 - > For the main element, add 40 pixels of padding to the right and left.
 - Add a linear gradient as the background for the heading "Save Environment" on the web page.
 - > Add the top and bottom padding to the heading.
 - > Add an image with 3em bottom padding.