Name of the Course	**:**	**CBCS B.Sc. (Hons.) Mathematics**
Unique Paper Code	**:**	**32351101_OC**
Name of the Paper	**:**	**C1-Calculus**
Semester	**:**	**I**
Duration	**:**	**3 Hours**
Maximum Marks	**:**	**75**

Attempt any four questions out of the following. All questions carry equal marks.

1. Find the n^{th} derivative of $y = \frac{2x}{x^2+a^2}$. Also prove that $y_n = \frac{(-1)^n n!}{r^{n+1}} 2\cos(n+1)\theta$.

If $y = \cos(m\sin^{-1}x)$ then show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0.$$

2. Sketch the graph of $f(x) = x^4 - 4x^3 + 10$ by finding intervals of increase and decrease, critical points, relative extrema, inflection points and concavity for the given function.

Find the horizontal and vertical asymptotes to the graph of the function $f(x) = \frac{2x}{x^2-1}$.

Sketch the graph of the curve in polar coordinates of the curve $r = 1 + 2\cos\theta$.

3. Evaluate the following integrals

(i) $\int_0^{2\pi} \sin m\theta \cos n\theta \; d\theta$, (ii) $\int_0^{\frac{\pi}{3}} \sin^2 6\theta \cos^4 3\theta \; d\theta$.

Give reduction formula for $\int \operatorname{cosec}^n\theta \, d\theta$ and find the value of $\int \operatorname{cosec}^6\theta \, d\theta$.

4. Use the method of cylindrical shells to find the volume of the solid generated when the region bounded by the hyperbola $y = \frac{1}{x}$ and the line $y = \frac{5}{2} - x$ is revolved about the y-axis.

Find the length of the curve $y = (x/2)^{\frac{2}{3}}$ from $x = 0$ to $x = 2$.

The arc of the parabola $y = x^2$ from (1,1) to (2,4) is rotated about y- axis. Find the area of the resulting surface.

5. Sketch the curve $(x-3)^2 = 6(y-2)$.

Sketch the curve $\frac{x^2}{15} - \frac{y^2}{3} = 1$, and label the vertices, foci and asymptotes.

Identify and sketch the curve $x^2 - xy + y^2 - 2 = 0$.

6. Find the limit: $\lim_{t\to 0} \frac{\sin 2t\hat{\imath} + t\hat{\jmath}}{t^2+t-1}$.

A shell is fired from the ground level with muzzle speed of 650 ft/s at an angle of 45°. An enemy gun 21,000 ft. away fires a shot 2 seconds later and the shells collided 45 ft. above the ground at the same speed. What are the muzzle speed (V_0) and angle of elevation (α) of the 2nd gun?

The acceleration vector of a moving particle is $A(t) = 18t^3\hat{\imath} + 3\hat{\jmath}$. Find the particles position vector as a function of t if $R(0) = \hat{\imath} + \hat{\jmath}$ and $V(0) = 1$.

Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32351102_OC**
Name of Paper	: **C2-Algebra**
Semester	: **I**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Find the seventh roots of unity. Use that to solve the equation

$$(z+1)^7 = z^7(\cos 7\theta + i\sin 7\theta)$$

2. Explain the difference between Principle of Mathematical Induction (usual form) and strong form of Principle of Mathematical Induction, if any. Apply Principle of mathematical induction to prove the following:

$$\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+\dots\dots\dots\dots+\frac{1}{(3n+1)(3n+2)}=\frac{n}{6n+4}$$

3. Find the domain and range of the function $f(x) = -2\log_2(x-1)$ of a real variable. Determine whether the function is one-to-one and onto. Find a formula for f^{-1}, if exists.

4. Find the general solution to the linear system of equations

$$\begin{aligned} x_1 + x_2 + 2x_4 - 3x_5 &= 4 \\ x_1 + x_2 + x_3 + 2x_4 - 3x_5 &= 3 \\ 4x_1 + 2x_2 + 4x_3 - 6x_5 &= 8 \\ 4x_1 + 3x_2 + x_3 + x_4 + 9x_5 &= 9 \end{aligned}$$

by row reducing the matrix to Echelon form. Encircle the leading entries; list the basic variables and free variables. Write the general solution in parametric vector form.

5. Let $A = \begin{pmatrix} 1 & 0 & 2 \\ -2 & 1 & -6 \\ 3 & -2 & 4 \end{pmatrix}$; $b = \begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix}$. Define a linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ by $T(x) = Ax$. Find a vector x whose image under T is b. Determine whether the vector x is unique or not.

6. Find the rank of the matrix $A = \begin{pmatrix} 1 & -4 & 7 & 4 \\ 1 & -1 & 3 & -3 \\ 2 & 3 & 0 & -2 \\ 4 & 1 & 5 & -9 \end{pmatrix}$.

Find the basis and dimension for Null space of A.

Name of the Course	:	B.Sc. (Hons.) Mathematics CBCS (LOCF)
Unique Paper Code	:	32351303
Name of the Paper	:	BMATH307 – Multivariate Calculus
Semester	:	III
Duration	:	3 Hours
Maximum Marks	:	75

Attempt any four questions. All questions carry equal marks.

1. Let $f(x,y) = \begin{cases} \frac{y^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

Is the function f continuous at $(0,0)$? Justify your answer.

Find an equation for the tangent plane to the surface $z = f(x,y)$ defined above at the point $P_0\left(1, 2, \frac{8}{5}\right)$.

Also find the directional derivative of $f(x,y)$ at $P_0(1,2)$ in the direction of $\mathbf{v} = 3\boldsymbol{i} - 2\boldsymbol{j}$.

2. Find the critical point and classify each point as a relative minimum, relative maximum, or a saddle point of

$$f(x,y) = xye^{-8(x^2+y^2)}.$$

Find the maximum and minimum values of $f(x,y,z) = xyz$ subject to the constraint

$$x^2 + 2y^2 + 4z^2 = 24.$$

Where is the function $f(x,y) = \sqrt{x^2+y^2}$ differentiable?

3. Compute $\iint_R xe^{xy}\,dA$ where R is the rectangle $0 \leq x \leq 1, 1 \leq y \leq 2$, using iterated integrals in both orders.

Evaluate $\iint_R 6x^2y\,dA$ if R is the region bounded between the curves $y = x, y = 1$ and $4y = x^2$.

Find the area of the region bounded between the curves $r_1(\theta) = 2 + \sin 3\theta$ and $r_2(\theta) = 4 - \cos 3\theta$.

4. Find the mass of the ellipsoid $4x^2 + 4y^2 + z^2 = 16$ lying above the xy-plane if the density is given by $\delta(x,y,z) = z$.

Determine the centroid of the solid bounded above by the sphere $x^2 + y^2 + z^2 = 1$ and below by the xy-plane plane where the density is given by $\delta(x,y,z) = z$.

Compute $\int_0^1 \int_0^1 x^2 y \, dx \, dy$ by changing $u = x$ and $v = xy$.

5. Evaluate $\oint_C (x^2 z dx - y x^2 dy + 3dz)$ where C is the boundary of the triangle with vertices $(0,0,0), (1,1,0)$ and $(1,1,1)$.

Find a non-zero function h for which

$$\boldsymbol{F}(x,y) = h(x)(x \sin y + y \cos y)\boldsymbol{i} + h(x)(x \cos y - y \sin y)\boldsymbol{j}$$

is conservative.

Using line integral, find the area of the region enclosed by the asteroid

$$x = a \cos^3 t, \quad y = a \sin^3 t \ (0 \le t \le 2\pi).$$

6. Find the mass of the lamina that is the portion of the paraboloid $z = x^2 + y^2$ that lies below the plane $z = 2$ with constant density δ_0.

Verify Stokes' Theorem if $\boldsymbol{F}(x,y,z) = (x-y)\boldsymbol{i} + (y-z)\boldsymbol{j} + (z-x)\boldsymbol{k}$ and S be the portion of the plane $x + y + z = 1$ in the first octant assuming that the surface has an upward orientation.

Using the Divergence Theorem, evaluate $\iint_S \boldsymbol{F}.\boldsymbol{N} dS$, where $\boldsymbol{F}(x,y,z) = (z^3\boldsymbol{i} - x^3\boldsymbol{j} + y^3\boldsymbol{k})$ and S is the sphere $x^2 + y^2 + z^2 = a^2$, with outward unit normal vector $\boldsymbol{N}$.

Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32355301_OC**
Name of Paper	: **GE-3 Differential Equations**
Semester	: **III**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Solve the following problems as indicated:

i. Find the orthogonal trajectories of the family of curves: $x^2 - y^2 + 2\rho xy = 1$, where ρ is a parameter.

ii. Find an integrating factor and solve: $(1-x^2)ydy + 2(y^2+4)dx = 0,\ y(3) = 0.$

2. Solve the following problems as specified:

i. Reduce the equation to homogeneous form using the substitution $y = z^2$ and hence solve it:
$2x^2y\frac{d^2y}{dx^2} + 4y^2 = x^2\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx}$.

ii. Find the complimentary functions for the differential equations:
$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = x^2,\ 2\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 12y = e^x,\ 16\frac{d^2y}{dx^2} - 24\frac{dy}{dx} + 9y = \sin x$.

iii. Find a second order homogeneous linear ordinary differential equation having x^{-3} and x^{-5} as its solutions. Also use Wronskian to show linear independence or dependence of these solutions.

3. Using method of undetermined coefficients, solve the differential equations:

i. $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \cos x.$

ii. $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = x^2.$

4. Find the series solution of the differential equations:

i. $\frac{d^2y}{dx^2} + 2xy = 0.$

ii. $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0.$

5. Form the partial differential equations by eliminating the arbitrary constants or arbitrary functions from the following surfaces:

i. $2z = mx^2 + ny^2 + mn$, m and n are arbitrary constants.

ii. $2z = a + (x+by)^2$, a and b are arbitrary constants.

iii. $z = x + y + f_1(cx+y) + f_2(cx-y), c(\neq 0)$ is a fixed constant, f_1 and f_2 are arbitrary functions.

6. Identify the equation which is parabolic by nature. Reduce that equation to canonical form and hence solve that equation.

i. $x^2u_{xx} - y^2u_{yy} - 2yu_y + \sin x\, u_x = 0, x \neq 0, y \neq 0$.

ii. $4y^2u_{xx} - 3xyu_{xy} + x^2u_{yy} + xu_x + yu_y = 0, x \neq 0, y \neq 0$.

iii. $y^2u_{xx} - 2xyu_{xy} + x^2u_{yy} - \frac{y^2}{x}u_x - \frac{x^2}{y}u_y = 0, x \neq 0, y \neq 0$.

Name of the Course	**: CBCS B.Sc. (H) Mathematics**
Unique Paper Code	**: 32351301_OC**
Name of the Paper	**: C 5 -Theory of Real Functions**
Semester	**: III**
Duration	**: 3 Hours**
Maximum Marks	**: 75**

Attempt any four questions. All questions carry equal marks.

1. Find the limit

$$\lim_{x\to 1} \frac{x^2 - x + 1}{x + 1}$$

and establish it by using ε-δ definition of the limit of a function.

Suppose that $\lim_{x\to c} f(x) = L$ where $L > 0$ and $\lim_{x\to c} g(x) = \infty$. Show that $\lim_{x\to c} f(x)\, g(x) = \infty$. If $L = 0$ then show by example that the conclusion may fail.

Let $x > 0$ and let $[x]$ denotes the greatest integer less than equal to x, then find

$$\lim_{x\to 0^+} \left\{x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \cdots + \left[\frac{7}{x}\right]\right)\right\}.$$

2. Use sequential criterion of continuity to prove that the function

$$f(x) = \begin{cases} \frac{\sin}{x}, & \text{if } x \neq 0, \\ 1, & \text{if } x = 0, \end{cases}$$

is continuous at 0.

Let $S = \{x \in \mathbb{R} : f(x) = 0\}$ be the zero set of a function f. If $\{x_n\}$ is a sequence in S and $\lim_{n\to\infty} x_n = x$, Show that $x \in S$.

Let $I = [0, \pi/2]$ and let $f: I \to \mathbb{R}$ be defined by $f(x) = \sup\{x^2, \cos x\}, x \in I$. Show that there exists an absolute minimum point $x_0 \in I$ for f on I. Also show that x_0 is a solution of the equation $\cos x = x^2$.

3. Prove that a continuous real valued function defined on a closed and bounded interval is uniformly continuous therein.

Prove the inequality $\frac{x-1}{x} < \log x < x - 1$ for $x > 1$, by using mean valve theorem.

Show that the function $f(x) = \frac{1}{1+x^2}, x \in \mathbb{R}$ is uniformly continuous on $\mathbb{R}$.

4. Suppose that f is a real valued function on $\mathbb{R}$ and that $f(a).f(b) < 0$ for some $a, b \in \mathbb{R}$. Prove that there exists x between a and b such that $f(x) = 0$.

Show that a continuous function $f: [0,1] \to [0,1]$, has a fixed point.

State and prove the chain rule of differentiation and use it differentiate the function $\sin\left(\sqrt{1 + \cos 2x}\right)$.

5. If f is continuous in $[a, b]$and differentiable in (a, b) then prove that there exists at least one $c \in (a, b)$ such that

$$\frac{f'(c)}{3c^2} = \frac{f(b)-f(a)}{b^3 - a^3}.$$

Prove that $\sin^2\theta < \theta.\sin(\sin\theta)$ for $0 < \theta < (\pi/2)$.

Determine the interval in which the function $f(x) = e^{\sqrt{x}}$ is convex.

6. Use Taylor's theorem to approximate sin(0.4) by fourth degree polynomial and determine the accuracy of the approximation.

Obtain the Maclaurin series expansion of the function $\cos^2 2x$.

Show that $e^{\pi} > \pi^{e}$.

Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32351302_OC**
Name of Paper	: **C6-Group Theory-I**
Semester	: **III**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Let $G = \mathrm{GL}(n, \mathbb{R})$. Let $H = \{A \in G \mid \det A \text{ is a power of } 5\}$. Then prove or disprove that H is a subgroup of G. Find the elements in $U(10)$ and $U(12)$ that satisfy the equation $x^2 = 1$.

2. List all the elements of order 3 in $\mathbb{Z}_{24}$. Find the smallest subgroup of $\mathbb{Z}$ containing 12 and 18. Determine the subgroup lattice for $\mathbb{Z}_{24}$.

3. Let S_n be the symmetric group of degree n. Suppose that $\alpha \in S_n$ can be written as a product of disjoint cyclic permutations of lengths $m_1, m_2, \dots, m_r, (r \in \mathbb{N})$, respectively. Then prove that the order of α is $\mathrm{lcm}(m_1, m_2, \dots, m_r)$. Find the orders of (13)(27)(456)(8)(1237)(648)(5) and (124) (345). Furthermore, show that if H is a subgroup of S_n then either every member of H is an even permutation or exactly half of them are even. Also, find $Z(S_n)$ for $n \geq 3$.

4. Show that for a finite group G, the index of a subgroup H in G is $|G|/|H|$. Prove that every subgroup of index 2 of a group G is normal. Give an example of a subgroup H of index 3 in a group G which is not normal in G. Also, determine the index of $3\mathbb{Z}$ in $\mathbb{Z}$.

5. Let $H = \{\beta \in S_5 : \beta(1) = 1\}$ and $K = \{\beta \in S_5 : \beta(2) = 2\}$. Prove that H is isomorphic to K. Is the same true if S_5 is replaced by S_n, where $n \geq 3$? Further prove or disprove that S_4 is isomorphic to D_{12}.

6. If H is a subgroup of G and K is a normal subgroup of G, then prove that $H/(H \cap K)$ is isomorphic to HK/K. Also determine all homomorphisms from $\mathbb{Z}_n$ to itself.

Name of Course	: **CBCS B.Sc. Hons Mathematics**
Unique Paper Code	: **32351302**
Name of Paper	: **BMATH306-Group Theory-1**
Semester	: **III**
Duration	: **3 hours**
Maximum Marks	: **75 marks**

Attempt any four questions. All questions carry equal marks.

1. Let A be a non-empty set and $\langle G, . \rangle$ be a group. Let F be the set of all functions from A to G. Define an operation $*$ on F as follows:

For $f, g \in F$, let $f * g : A \to G$ as $(f * g)(x) = f(x).g(x)\ \forall\ x \in A$.

Prove that $\langle F, * \rangle$ is a group.

Find the inverse of $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$ in $GL(2, \mathbb{Z}_5)$, the group of 2×2 non-singular matrices over $\mathbb{Z}_5$. Verify the answer by direct calculation.

Describe the group of symmetries of a non-square rectangle and draw its Cayley's table.

2. Let a be an element of a group such that $|a| = 3$, prove that $C(a) = C(a^2)$. Give an example to show that the conclusion fails if $|a| = 4$.

Find the orders of each of the elements of U(14). Show that it is cyclic and find all its generators.

3. Define Centre $Z(G)$ of a group G and prove that $Z(S_4) = \{e\}$.

For $n > 2$, show that every even permutation in S_n is a product of 3-cycles.

Let σ = (1,5,7)(2,5,3)(1,6). Express σ^{17} as a cycle.

4. Prove or disprove any six, stating the results used

(i) $\langle \mathbb{R}, + \rangle \approx \langle \mathbb{Q}, + \rangle$, (ii) $\langle \mathbb{Q}, + \rangle \approx \langle \mathbb{Z}, + \rangle$, (iii) $\langle \mathbb{R}, + \rangle \approx \langle \mathbb{R}+, . \rangle$,

(iv) $D_4 \approx$ Group Q of Quaternions, (v) $U(20) \approx D_4$,

(vi) $U(8) \approx U(12)$, (vii) $U(10) \approx \mathbb{Z}_4$, (viii) $\frac{GL(2,\mathbb{R})}{SL(2,\mathbb{R})} \approx \mathbb{R}^*$.

5. Let H be a subgroup of a group G. Prove that $aH \mapsto Ha^{-1}$ is a bijective mapping from the set of all left cosets of H in G to the set of all right cosets of H in G. Can the same be said for $aH \mapsto Ha$?

If G is a non-abelian group of order 8 with $Z(G) \neq \{e\}$, prove that $|Z(G)| = 2$.

6. Let *N* be a normal subgroup of *G* and *M* be a normal subgroup of *N*. If *N* is cyclic, prove that *M* is a normal subgroup of *G*. Show by an example that the conclusion fails to hold if *N* is not cyclic.

If φ is a homomorphism from a finite group G to a finite group G', prove that $|\varphi(G)|$ divides the gcd of $|G|$ and $|G'|$.

Name of the Course	: **B.Sc. (H) Mathematics**
Unique Paper Code	: **32357505**
Name of the Paper	: **DSE-II Discrete Mathematics**
Semester	: **V Semester**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Show that the set $A = \{2, 3, 4, 6, 8, 24, 48\}$ with divisibility as the relation is a partial ordered set. Draw the Hasse diagram of A. Is it a chain? Justify your answer. Find out the maximal and minimal elements of A. Also, determine the least and greatest elements of A. Give subset of A which is a chain and a subset of A which is an antichain with regard to the same partial order relation.

Consider the partial ordered sets $(L, \subseteq)$ where $L = P(S)$ is the power set of a non-empty set S and $(Q, \leq)$ where $Q = \{0,1\}$ and $0 < 1$.

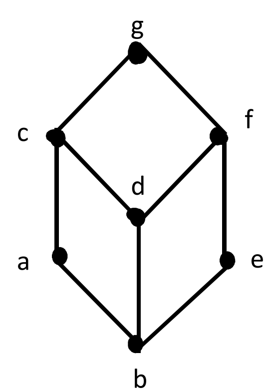
Consider $\varphi : L \to Q$ defined by

$$\varphi(U) = \begin{cases} 1, & \text{if } U = S \\ 0, & \text{if } U \neq S \end{cases}.$$

Is φ order preserving? Justify your answer.

2. Consider lattices $L_1 = \{1, 5\}$ and $L_2 = \{1, 3, 9\}$ with divisibility as the partial order relation. Is $L_1 \times L_2$ a lattice? If yes, then state the partial order relation on $L_1 \times L_2$ and draw its Hasse diagram. Prove that a finite lattice always has the greatest element and least element.

Consider lattice L_3 represented by the Hasse diagram shown below



Find out 5 sublattices of L_3. Is union of any two sublattices of L_3 a sublattice of L_3? Justify your answer by providing examples.

3. Prove or disprove the following two statements

i) sublattice of a modular lattice is modular

ii) sublattice of a distributive lattice is distributive.

Verify whether or not the lattice $L = (\{1, 2, 3, 6, 10, 12, 24, 60, 120\}$, GCD, LCM) is modular.

Find the disjunctive normal form of the given polynomial $p = x(y+z)' + (xy + z')x$.

Also, find the conjunctive normal form of above polynomial p.

4. Does the expression $x'z'$ imply the expression $xy'z' + x'y + x'y'z' + x'yz$. Give reasons for your answer.

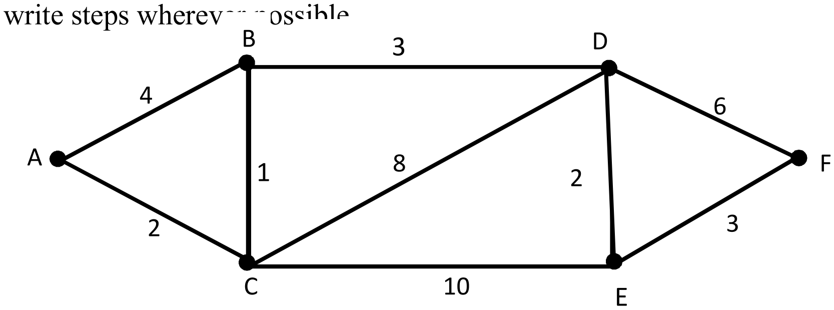
Find the prime implicants of $p = xyz + xyz' + x'yz' + x'y'z$.

Also reduce the above polynomial p into minimal sum of products form using Quine−McCluskey method or Karnaugh maps.

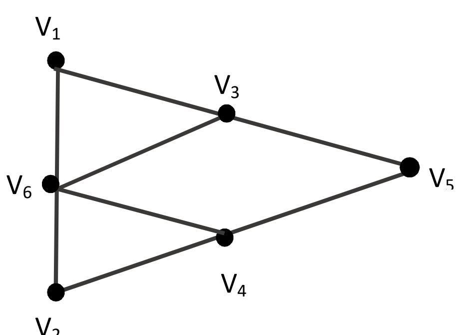
Draw the contact diagram of the circuit $q = (xy')' + (x' + y + z)' + xz$.

Further, give the symbolic representation of the above circuit q.

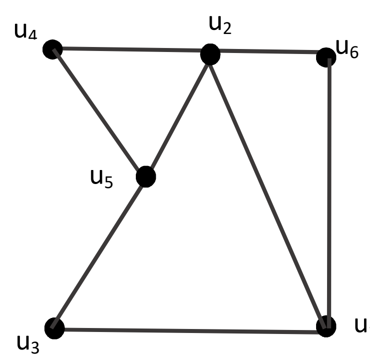
5. Apply Dijkstra's Algorithm OR improved version of Dijkstra's Algorithm to find a shortest path from A to F, also write steps wherever possible.



Find the adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 shown below.



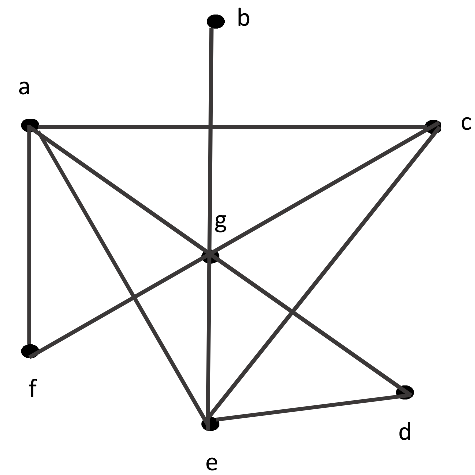
G_1



G_2

Find a permutation matrix P such that $A_2 = PA_1P^T$.

6. Explain the Königsberg bridge problem and discuss the solution provided by graph theory to this problem. In the graph given below either describe an Eulerian circuit or explain why no Eulerian circuit exists. Is the graph Hamiltonian? Display a Hamiltonian cycle or explain clearly why no such cycle exists.



Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32357506**
Name of Paper	: **DSE-II Cryptography and Network Security**
Semester	: **V**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Consider a Playfair Cipher where the 5×5 matrix consists of letters A to Z excluding J. The plaintext

THIS FOUR LETTER WORD DUCK IS USED TO SHOW SOME SCORED ZERO

is encrypted twice using the playfair cipher, first with use of key MOZART and then with an unknown key. The final cipher is

FNQYFAVRHSLZSRVZVIMTSTSGQYZKASCRQHVWQMIROSVIASMBIV

Find the unknown key used in the second playfair matrix.

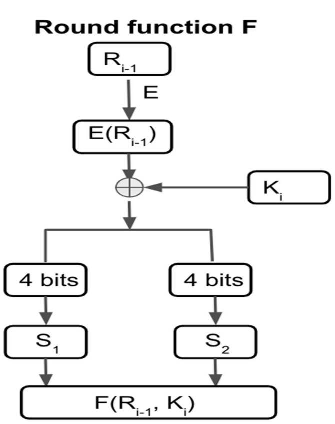
2. Consider the map $\phi : \mathbb{Z}_{60} \to \mathbb{Z}_3 \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_5$ defined as

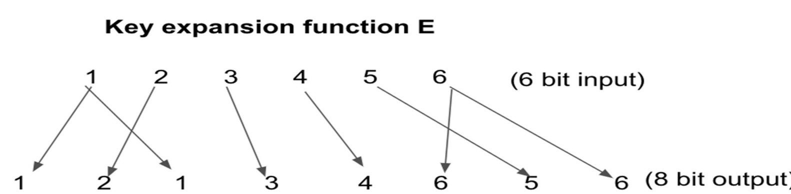
$$\phi(x) = \big(x \ (\mathrm{mod}\ 3), x \ (\mathrm{mod}\ 4), x \ (\mathrm{mod}\ 5)\big), 0 \le x \le \ 59.$$

Show that ϕ is one-one and onto. Find the pre-image of the point $(2, 3, 3)$ under ϕ.

3. Consider a cryptosystem based on the Feistel structure, where plaintext m is divided into two equal parts say $m = L_0R_0$ and L_iR_i are generated in various rounds during encryption process as follows:

$L_i = R_{i-1}$ and $R_i = F(R_{i-1}) \oplus L_{i-1}$, where F is the round function defined as





S_1	010	100	011	101	110	001	111	000
	001	100	000	110	111	101	010	011

S_2	011	110	000	001	101	010	111	100
	010	000	011	111	110	100	001	101

E is a key expansion function that takes 6 bit input and produces 8 bit output. For example $(100101) = 10101101$. S_i are S-boxes that take 4 bit input and produce 3 bit output. The first bit of the input gives the row number and the last three bits of the input gives the column number. For example, to calculate $S_1(0101)$, we will take the cell value at 0^{th} row and 5^{th} column of S_1, so $S_1(0101) = 001$. Similarly $S_1(1010) = 000$ (cell value at 1^{st} row and 2^{nd} column).

Suppose the plaintext $m = 101101001101$ (12 bits) and the first two rounds keys are $K_1 = 10011010$ and $K_2 = 01101101$. Perform two iterations of the above described encryption scheme.

4. Define the basic components of a Public key cryptosystem and its encryption and decryption algorithms. What are the advantages of a Public key cryptography over Symmetric key cryptography? Define a trapdoor function and discuss its significance in the Public key cryptography. Mention the trapdoor function on which the security of RSA cryptosystem relies.

5. Let $p = 29$, $a = 4$ and $b = 20$, consider Elliptic curve $E_p(a, b): y^2 = x^3 + a\,x + b$. Show that $P = (5, 22)$ and $Q = (16, 27)$ lie on $E(F_p)$. Also, prove that $E_p(a, b)$ is nonsingular and find R & $3R$, where $R = P + Q$.

6. What components are prescribed in PGP to ensure confidentiality and integrity of an e-mail? Discuss the roles of ZIP compression and radix 64 expansion in PGP.

Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32357504**
Name of Paper	: **DSE-II Mathematical Finance**
Semester	: **V**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Consider a stream of cash flows $(-5,2,2,2,2,2)$ received at time points t = 0,1 2, 3, 4, 5 where the time is measured in years. Let the rate of interest be 10% per annum compounding annually. Determine present and future values of the cash flow stream and verify the relationship between them. Also find the internal rate of return (up to two decimal places) of the cash flow stream.

2. Consider two 12-year bonds: one has 8% coupon, face value \$100 and yield 8% per annum compounded annually; the other has 10% coupon, face value \$100 and yield 9% per annum compounded annually. Find the price of these bonds assuming that the coupon is paid at the end of each year. Identify the bond which is more sensitive among the two and give reasons for your answer. Also find the price of a 12-year zero coupon bond.

3. Assume that there are three assets having mean rates of return $\bar{r}_1 = 8\%$, $\bar{r}_2 = 10\%$, $\bar{r}_3 = 6\%$, standard deviations $\sigma_1 = 1.5$, $\sigma_2 = 0.5$, $\sigma_3 = 1.2$ and correlations $\rho_{12} = 0.3$, $\rho_{23} = 0$, $\rho_{13} = -0.2$.

(a) Find the covariance matrix for these three assets.

(b) Find the mean rate of return and the standard deviation of portfolio P_1 consisting of the above three assets with respective weights $w_1 = 40\%$, $w_2 = -20\%$, $w_3 = 80\%$.

(c) Find the mean rate of return and the standard deviation of portfolio P_2 consisting of the above three assets with respective weights $w_1 = 30\%$, $w_2 = -10\%$, $w_3 = 80\%$.

(d) Find the mean rate of return and the standard deviation of portfolio P_3 consisting of the above three assets with respective weights $w_1 = 20\%$, $w_2 = 10\%$, $w_3 = 70\%$.

(e) Plot the points representing portfolios P_1, P_2, P_3 on the $\bar{r}$-σ diagram.

(f) Identify the least risky portfolio and the portfolio with maximum risk.

4. Assume that the risk-free rate of interest is 7%, the mean rate of return and the standard deviation of the market portfolio are 12% and 17% respectively. Assume that the market portfolio is efficient.

(a) Find the equation of the capital market line.

(b) If the standard deviation of the rate of return of a risky asset A is 30%, then find the mean rate of return predicted by the capital market line.

(c) How will you allocate \$2000 to achieve the mean rate of return obtained in part (b)?

(d) If you invest \$500 in the risk-free asset and \$1500 in the market portfolio, how much money do you expect to have at the end of the year.

5. A 1-year long forward contract on a non-dividend paying stock is entered into when the stock price is \$300 and the risk-free rate is 10% per annum compounding continuously. What are the forward price and initial value of a forward contract? Six months later the price of the stock is \$350 and the risk-free rate is still 10%. Obtain the forward price and the value of forward contract.

6. The current price of a non-dividend paying stock is \$29 and the price of a six-month European call option on the above stock with strike price \$30 is \$2. The risk-free rate of interest is 4% per annum compounding continuously. Find the price of a 6-month European put option written on the same stock with strike price \$30. If the above European put and call options are selling at \$3 and \$4 respectively, then identify an arbitrage opportunity if it exists.

Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32357503**
Name of Paper	: **DSE- I, C++ Programming**
Semester	: **V**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Write a function, *printgrid*, that takes two parameters, (one parameter is a two-dimensional array and the second parameter is for the number of rows) and print the following grid structure using repetition and controlled statements. After that call the function, *printgrid*, in the main program.

```
#
 #  #
 #  1#
 #  1  2  #
 #  1  2  3  #
 #  1  2  3  4  #
 #  1  2  3  4  5  #
 #  1  2  3  4  5  6  #
 #  1  2  3  4  5  6  7  #
 #  #  #  #  #  #  #  #  #  #
```

2. Write a function in C++ using the one dimensional array to calculate the following quantity:

$$\sqrt{\frac{\left|\sum_{i=1}^{n}(x_i-\bar{x})^3\right|}{n(n-1)}}$$

where,
x_i denotes the data stored in the cells of array
$\bar{x}$ denotes the average of the data stored in the array
n denotes the number of data stored in the array and $n > 1$

3. Write a program to find the inverse of a 3×3 matrix over the field Z_{23} entered by the user, if the determinant of the matrix is non-zero. The program also finds the sum of the square of the diagonal elements under modulo 23 of the inverse of the given matrix.

4. Write a program which finds the solution of the following system of equations by matrix inversion method.

$$2x + y + 2z = 0,$$
$$2x - y + z = 10,$$
$$x + 3y - z = 5.$$

5. Find the all errors of given program

```
            #include<iostream>;
            Using Namespace Std :
void Swap(int *x, int *y):
Intmain()
             {
        Int a, b, d;
             a = 4;
             b = 2;
        c = c + a;
        j = 1:
        for(k = 1; k<=n, k++)
                {
cout<<setw(4)<< j;
             j = j + 3
        d = d +Pow(k, 2);
        cout<<"the Value of l in the" <<k<<"th iteration is  "<<l<<Endl;
                 }
        Double p;
        p = Sqrt{c, 3};
        cout<<p<<endl;
        Cout<<"Square root of "<< c<< "is  "<<p<<Endl;
int x = 5; y = 10;
cout<< "Before swap, x: " << x << " y: " << y <<Endl;
swap(&x, &y);
Cout<< "Main. After swap, x: " << x << " y: " << y <<Endl;
              Return 0:
             }
       void swap (int *px, int *py)
            {
       int temp;
       cout<< "Swap. Before swap, *px: " << *px<<" *py: " << *py<<endl;
       temp = *pX
            *pX = *pY
            *pY = temp
       cout<< "Swap. After swap, *px: " << *px<<" *py: " << *py<<endl;
            }
```

Write the correct program of part (a)

Write the equivalent programs by using **while** and **do... while** loops.

6. There are 20 students in a class. Write an appropriate code in C++ using arrays to generate the Internal Assessment Marks of a particular paper based on the following information:

i. Maximum Marks of the paper=100
ii. Roll Number, Student Name, Test Marks, Assignment Marks and Attendance Percentage are stored in arrays.
iii. Maximum Marks for Test = 10% of Maximum Marks of the paper
Maximum Marks for Assignment = 10% of Maximum Marks of the paper
Maximum Marks for Attendance = 5% of Maximum Marks of the paper

Attendance Marks based on Attendance percentage is given in following table:

67 % ≤ Attendance < 70 %	1 % of Maximum Marks for Attendance
70 % ≤ Attendance < 75 %	2 % of Maximum Marks for Attendance
75 % ≤ Attendance < 80 %	3 % of Maximum Marks for Attendance
80 % ≤ Attendance < 85 %	4 % of Maximum Marks for Attendance
85 ≤ Attendance	5 % of Maximum Marks for Attendance

iv. Internal Assessment Marks = Test Marks + Assignment Marks + Attendance Marks
Print the following details for each student-

Roll Number	Student Name	Test Marks	Assignment Marks	Attendance Marks	Internal Assessment Marks

Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32357501**
Name of Paper	: **DSE-I NUMERICAL METHODS**
Semester	: **V**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Consider the equation $2x - \log_{10} x - 7 = 0$ on $]3.78, 3.79[$. Apply bisection method to find an approximate root of the equation. Do two iterations.

Find an approximate root of the equation $f(x) = x \log_{10} x - 1.2 = 0$ using Regula-Falsi method. Do two iterations.

Approximate the second order derivative of $f(x) = e^x$ at $x_0 = 0$, taking $h = 1, 0.1$ and 0.01 by using the formula

$$f''(x_0) \approx \frac{f(x_0-h) - 2f(x_0) + f(x_0+h)}{h^2}.$$

2. Find the maximum value of the step size h that can be used in the tabulation of $f(x) = \sin 5x$ on $[1,2]$ so that the error in the linear interpolation of $f(x)$ is less than 5×10^{-4}.

The equation $f(x) = x^4 - 4x^2 + 4 = 0$ has a double root at $x = \sqrt{2}$. Starting with $x_0 = 1.5$, compute two successive approximations to the root by Newton-Raphson method.

Show that the equation $log_e x = x^2 - 1$ has exactly two real roots, $\alpha = 0.45$ and $\beta = 1$. Determine for which initial approximation x_0, the iteration $x_{i+1} = \sqrt{1 + \log_e(x_n)}$ converges to α or β.

3. Compute T_{jac} for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 4 & 2 \\ 0 & 1 & 6 \end{bmatrix}$. Also determine the spectral radius of the above iteration matrix.

Solve the following system of equations using SOR iteration method

$$\begin{aligned} 2x_1 - x_2 + x_3 &= 0 \\ -x_1 + 4x_2 + 2x_3 &= 4 \\ x_1 + 2x_2 + 6x_3 &= 5. \end{aligned}$$

Take $w = 0.9$ with $X^{(0)} = [0, 0, 0]^T$ and iterate three times.

4. Find an LU decomposition of the matrix $A = \begin{bmatrix} 6 & -2 & 3 \\ 3 & -1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ and use it to solve the system $AX = [5, 6, 5]^T$.

Construct the divided difference table for the following data set and then write out the Newton form of the interpolating polynomial. Hence, estimate the value of y when $x = -3$.

x	-1	0	1	3
y	-121	-60	-29	-33

Hence, estimate the value of y when $x = 2$.

5. Approximate the derivative of $f(x) = \sin 2x$ at $x_0 = \pi$, taking $h = 1, 0.1$ *and* 0.01. Using the formula

$$f'(x_0) \approx \frac{f(x_0) - f(x_0 - h)}{h}$$

Find the order of approximation.

Solve the following system of equations using Gauss Seidel iteration method

$$2x_1 - x_2 + x_3 = -1$$
$$-x_1 + 4x_2 + 2x_3 = 3$$
$$x_1 + 2x_2 + 6x_3 = 5.$$

Take $X^{(0)} = [0, 0, 0]^T$ and iterate three times.

6. Find the order of approximation. The following are the five successive iterations obtained by secant method to find the root

$\alpha = -2$ of the equation $x^3 - 3x + 2 = 0$,

$x_1 = -2.6, x_2 = -2.4, x_3 = -2.106598985, x_4 = -2.022641412, x_5 = -2.000022537.$

Compute the asymptotic error constant and show that $\varepsilon_5 \approx \frac{2}{3}\varepsilon_4$.

Approximate the solution of the initial value problem in 5 steps using Euler's method

$$\frac{dy}{dx} = \frac{x}{y}, y(0) = 1, 0 \le x \le 5.$$

Also find the absolute error at each step given that the exact solution is $y(x) = \sqrt{x^2 + 1}$.

Name of the course : **CBCS B.Sc. (H) Mathematics**

Unique Paper Code : **32357502**

Name of Paper : **DSE-I: Mathematical modelling and graph theory**

Semester : **V**

Duration : **3 hours**

Maximum Marks : **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Find the power series solution of the equation $(x^2+1)\,y'' + x\,y' - x\,y = 0$ in powers of x. Also discuss the singularities of the equation $x^2(x+1)^2\,y'' + (x^2-1)\,y' + 2y = 0$.

2. Consider a spring-mass system governed by $m\,y'' + \beta\,y' + k\,y = f(t)$, $y(0) = y'(0) = 0$. Suppose we apply a unit step force $f(t) = u(t)$ to the mass, initially at equilibrium and you observe the system respond as $y(t) = -½\,e^{-t}\cos t - ½\,e^{-t}\sin t + ½$. What are the physical parameters $m,\ \beta$ and k? Find $\mathscr{L}^{-1}\left(\frac{s^2+1}{s(s+1)^2}\right)$, $\mathscr{L}^{-1}\left(\frac{s}{s^2+9} + \frac{5}{s^2+9}\right)$. Is the function $-6\,te^{\frac{3t}{2}}\sin t,\ t \geq 0$ of exponential order? Justify your claim.

3. Using Monte Carlo simulation, write an algorithm to calculate the area under the curve $y - 2x^2 = 2$, and above x-axis on the interval $0 \leq x \leq 4$.

Consider a small harbor with unloading facilities for ships, where only one ship can be unloaded at any time. The unloading time required for a ship depends on the type and the amount of cargo. Below is given a situation with 6 ships

	Ship 1	Ship 2	Ship 3	Ship 4	Ship 5	Ship 6
Time between successive ships	10	25	15	30	115	55
Unload time	15	50	25	70	60	80

Draw the timeline diagram depicting clearly the situation for each ship. Also determine length of longest queue, average waiting time for ships and total time in which docking facilities are idle.

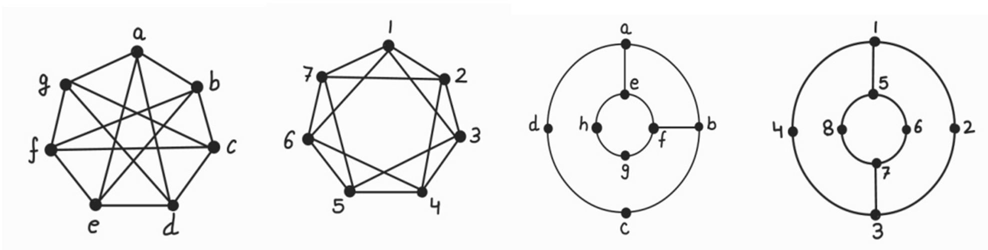
4. Using simplex method, solve the following linear programming problem

$$\begin{aligned} \text{Maximize} \quad & 8x_1 + 12x_2 \\ \text{subject to} \quad & -10x_1 - 20x_2 \geq -140 \\ & 6x_1 + 8x_2 \leq 72 \\ & x_1,\ x_2 \geq 0. \end{aligned}$$

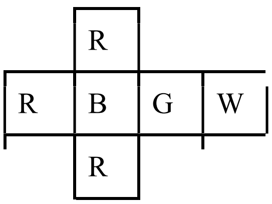
Determine the sensitivity of the optimal solution to a change c_1 using the objective function $c_1x_1 + 12x_2$.

5. Are the following pairs of graphs isomorphic? If so, find a suitable one-one correspondence between the vertices of graphs: if not, explain why no such correspondence exists. Determine which of the given four graphs are Eulerian and/or Hamiltonian. Write down the corresponding Eulerian trail and/or Hamiltonian cycle, where possible.

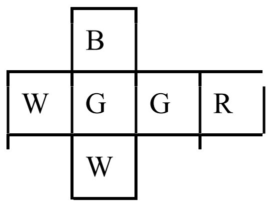
Pair: 1 Pair: 2



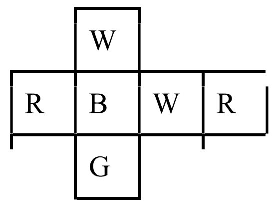
6. Find a solution to the four- cube problem for the following set of cubes:



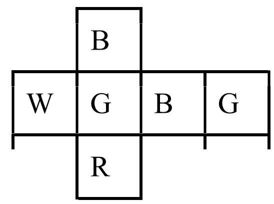
Cube - 1



Cube - 2



Cube - 3



Cube - 4

Draw a graph representing predator behavior in the following problem: Snakes eat frogs and birds eat spiders; birds and spiders both eat insects; frogs eat snails, spiders and insects. Is this graph bipartite? Is this a regular graph? Justify your answers.

Name of Course	: **CBCS B.A. (Prog.)**
Unique Paper Code	: **62351101_OC**
Name of Paper	: **Calculus**
Semester	: **I**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Examine the continuity of the following functions

(i). $f:(0,\infty)\to\mathbb{R}$ defined as

$$f(x)=\begin{cases} 7x, & \text{if } 0<x\le 1 \\ 2-x, & \text{if } 1<x\le 2 \\ x^2-2x, & \text{if } 2<x\le 4 \\ x+4, & \text{if } x>4 \end{cases}$$

at $x=1,2$ and 4.

(ii). $f:\mathbb{R}\to\mathbb{R}$ defined as

$$f(x)=\begin{cases} \dfrac{5xe^{5/x}}{1+e^{5/x}}, & \text{if } x\ne 0 \\ 0, & \text{if } x=0 \end{cases}$$

at $x=0$.

(iii). $f:\mathbb{R}\to\mathbb{R}$ defined as

$$f(x)=\begin{cases} -2x^3, & \text{if } x\le 0 \\ 5x-8, & \text{if } 0<x\le 1 \\ x^2-3x, & \text{if } 1<x<2 \\ 3x+4, & \text{if } x\ge 2 \end{cases}$$

at $x=0,1$ and $x=2$.

2. If $y=\left[2x+2\sqrt{1+x^2}\right]^m$, then prove that

$$(1+x^2)y_{n+2}+(2n+1)xy_{n+1}+(n^2-m^2)y_n=0.$$

Also, verify Euler's Theorem for

$$z=2^n x^n \log\frac{y}{4x}.$$

3. Find the asymptotes of the following curves

(i). $(x^2-y^2)^2-4x^2+x=0$

(ii). $x^2y+xy^2+xy+y^2+3x=0.$

Also, trace the curve given by

$$y^2(2+x)=6x^2-x^3.$$

4. Verify Lagrange's Mean Value Theorem for the function given by

$$f(x) = (2x - 1)(x - 3)(2x - 5) \text{ for } \quad x \in [0,4]$$

and apply it to prove that

$$\sqrt{1+x} < 1 + \frac{1}{2}x \qquad \text{if } \; x \in (-1, \infty), x \neq 0.$$

Also, verify Cauchy's Mean Value Theorem for following functions

$$f(x) = x^2 - 3x - 5, \qquad g(x) = x^2 + 2x - 1 \text{ in } [0,1].$$

5. Find the equation of the tangent and normal at the point 'θ' to the curve

$$x = 6\cos^3\theta\,, \quad y = 6\sin^3\theta.$$

Find the radius of curvature at the origin for the following curves:

(i). $x^4 - 4x^3 - 18x^2 - y = 0.$

(ii). $x = 5(\theta + \sin\theta)$, $y = 5(1 - \cos\theta)$.

6. Evaluate

$$\lim_{x\to 0} \frac{1 - \cos x^2}{x^2 \sin x^2} \qquad \text{and} \qquad \lim_{x\to 0^+} (\cot x)^{\sin x}.$$

Find the maximum and minimum values of the function $f(x) = 3x^5 - 15x^4 + 15x^3 - 1$ and separate the intervals in which the function $g(x) = 2x^3 - 9x^2 + 12x - 5$ is increasing or decreasing.

Unique Paper Code	**:**	**62351101_OC**
Name of the Paper	**:**	**Calculus**
Name of the Course	**:**	**B.A. (Prog.) I Year**
Semester	**:**	**I**
Duration	**:**	**3 Hours**
Maximum Marks	**:**	**75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Find the limit of the given functions

(i) $\lim\limits_{x\to 0}\frac{x\cos x-\sin}{x^2\sin x}$ (ii) $\lim\limits_{x\to 0}\frac{1}{1+e^{1/x}}$.

Discuss the continuity and discontinuity of following functions,

(i) $f(x)=|x-1|+|x-2|$ at $x=1$ & $x=2$

(ii) $f(x)=\begin{cases}\frac{xe^{1/x}}{1+e^{1/x}}, & \text{if } x\neq 0,\\ 0, & \text{if } x=0,\end{cases}$ at $x=0$.

Can a function have more than one limit? Explain.

2. If $y=\sin mx+\cos mx$, then show that $y_n=m^n[1+(-1)^n sin\, 2mx]^{1/2}$.

State Leibnitz's theorem. If $y=\sin^{-1}x$ then prove that

$$(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0.$$

If $u=x^2\tan^{-1}\left(\frac{x}{y}\right)-y^2\tan^{-1}\left(\frac{x}{y}\right)$ then prove that

$$\frac{\partial^2u}{\partial x\partial y}=\frac{x^2-y^2}{x^2+y^2}.$$

3. Prove that the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2021$ touches the straight line $\frac{x}{a}+\frac{y}{b}=2$ at the point (a,b), whatever be the value of n.

Show that normal at any point of the curve

$$x=a\cos\theta+a\,\theta\sin\theta,\ y=a\sin\theta-a\,\theta\cos\theta$$

is at constant distance from origin.

Find the curvature at any point θ on the curve

$$y=a\left(\cos\theta+\log\left(\tan\left(\frac{\theta}{2}\right)\right)\right),\ y=a\sin\theta.$$

4. Find asymptotes of the curve $(y-3)^2(x^2-9) = x^4 + 81$.

Determine the position and nature of double points on the curve

$$x^4 - 2y^3 - 3y^2 - 2x^2 + 1 = 0$$

Trace the curve $x^2y^2 = x^2 - 25a^2$.

5. State Lagrange's mean value theorem and give its geometrical interpretation. Applying Lagrange's mean value to the function defined by $f(x) = \log(1+x)\ \forall\ x > 0$ show that

$$0 < [\log(1+x)]^{-1} - x^{-1} < 1 \text{ whenever } x > 0.$$

Separate the interval in which the function defined on $\mathbb{R}$

$$f(x) = 2x^3 - 15x^2 + 36x - 1$$

is increasing or decreasing.

Prove that $\tan x > x$ whenever $0 < x < \pi/2$.

6. Obtain Maclaurin's series expansion of $\cos 2x$.

Evaluate $\lim\limits_{x\to 0} \frac{sinx - x}{x\, sinx}$

Find the maximum value of $\left(\frac{1}{x}\right)^x$.

Unique Paper Code	: **42351101**
Name of the Paper	: **Calculus and Matrices**
Name of the Course	: **B.Sc. (Math. Sci.)-I, B.Sc. (Phy. Sci.)-I, B.Sc. (Life. Sci.)-I**
Semester	: **I**
Duration	: **3 Hours**
Maximum Marks	: **75**

Attempt any four questions. All questions carry equal marks.

1. Check whether the set $\{(0,2,0), (3,0,-1), (-1,1,0)\}$ is linear independent or not. Is the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined as $T(x,y) = (2x+y, -y)$ linear? Sketch the image of the unit square with vertices $(0,0), (0,1), (1,1), (1,0)$ under the given transformation. Find a matrix representation for T.

2. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation satisfying $T(1,1) = (1,7)$ and $T(1,2) = (2,6)$. Find a matrix representation for T and determine $T(1,3)$.

Find the eigenvalues and the corresponding eigenvectors of the matrix $A = \begin{bmatrix} 5 & -1 \\ 0 & 2 \end{bmatrix}$.

An amount of 20 ml of a medicine is injected into a patient's body. Half the amount of the medicine is absorbed by the patient body in 12 hours. How long will it take for the patient to absorb 70% of the medicine?

3. Solve, if consistent, the following system of linear equations using elementary row operations

$$x + y + z = 5$$
$$2x + 3y + 4z = 16$$
$$y - 4z = -12.$$

Reduce the following matrix A to triangular form using elementary row operations and also determine its rank

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 5 & -17 \\ 2 & 11 & -8 \end{bmatrix}.$$

Is $W = \{(2x, y+1, 0) : x, y \in \mathbb{R}\}$ a subspace of $\mathbb{R}^3$? Justify your answer.

4. Discuss the convergence of the sequences $a_n = \sqrt{1 + (-\frac{1}{2})^n}$ and $b_n = 5^{n/(n+1)}$.

Find the n^{th} derivative of $y = 2e^x \sin x \cos 2x$.

Also sketch the graph of the functions

$$f(x) = 5 - |x+3| \text{ and } f(x) = 1 + 3e^{-2x}$$

mentioning the transformations used at each step.

5. Find the Taylor's polynomial of order 4 generated by the function $f(x) = \sin 3x$ at $x = \frac{\pi}{3}$.

If $u = f(r)$ where $r = \sqrt{x^2 + y^2}$ show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$.

Verify that the function $u(x, t) = \sin(x + 3t) + \cos(x + 3t)$ satisfies the Wave Equation.

6. Find the polar representation of the points $z_1 = -1 - i,\ z_2 = -1 + i\sqrt{3}$ and $z_1 z_2$.

Form an equation in lowest degree with real coefficients which has $3 - i,\ 1 + 3i$ as two of its roots.

Solve the equation $z^3 - 1 + i = 0$.

Find the equation of the straight line joining the points

$$z_1 = -1 - i \text{ and } z_2 = -1 + i\sqrt{3}.$$

Name of Course	: **B.A. (Prog.) Mathematics**
Unique Paper Code	: **62354343**
Name of Paper	**: Analytical Geometry and Applied Algebra**
Semester	: **III**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Describe, sketch and label the focus, vertex and directrix of the parabola

$$y^2 + 2y + 12x - 23 = 0.$$

Describe, sketch and label the centre, vertices, foci and asymptotes of the hyperbola

$$9x^2 - 4y^2 - 54x + 8y + 41 = 0.$$

2. Find the centre, vertices, foci and ends of minor axis of the ellipse

$$4x^2 + 9y^2 - 16x - 54y + 61 = 0$$

and sketch it.

Find equation of the parabola that has vertex at (1, 1) and focus (–3,1). What is its directrix?

Find equation of hyperbola having vertices $(\pm 3,0)$ and foci $(\pm 5,0)$.

3. Find a vector oppositely directed to $3i - 4j$ and $\frac{1}{3}rd$ the length of it. Also express $\mathbf{v} = 2i - 4j$ as sum of vector parallel and orthogonal to $\boldsymbol{a} = i + j$.

Rotate the coordinate axes to remove the '*xy*' term and identify the curve

$$x^2 - x\,y + y^2 - 2 = 0 \ .$$

Sketch $z = y^2$ in 3- space.

4. Find the equation of the sphere that is inscribed in the cube that has sides of length 4 and is parallel to coordinate planes also the sphere is centered at the point $(-1, 0, 2)$.

Find the direction cosines of $\mathbf{v} = 2i + j - k$ and use them to find the direction angles. Also find the vector component of $\mathbf{v}$ along to $\mathbf{b} = j - k$.

Find the distance between the point $P(-4,0,1)$ and the line through the points $A(0, 0, -1)$ and $B(-3, 2, -3)$.

5. Find the parametric equation of line L passing through the points $(2,\ 4, -1)$, and $(5,\ 0,\ 7)$. Where does the line intersect the xy-plane?

Show that the line L: $x = 3 + 8t,\ \ y = 4 + 5t,\ \ z = -3 + t$ and the plane

$x - 3y + 7z = 12$ are parallel.

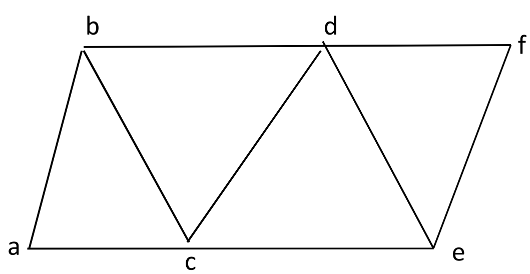
Show that the lines

$$L_1:\ x = 1 + 7t, \qquad y = 3 + t, \qquad z = 5 - 3t$$
$$L_2:\ x = 4 - t, \qquad y = 6, \qquad z = 7 + 2t$$

are skew. Also find the distance between them.

6. Define a Latin square. Give an example of a Latin square of order 6.

Find a minimal edge cover for the following graph. Give a detailed logical analysis.



Three pitchers of sizes 10 litres, 4 litres and 7 litres are given. If initially 10 litres pitcher is full and the other two empty, find a minimal sequence of pouring so as to have exactly 3 litres of water in two pitchers.

Unique Paper Code	: 42347902
Name of the Course	: B.Sc. Programme / B.Sc. Mathematical Science
Name of the Paper	: Analysis of Algorithms and Data Structures
Semester	: V
Year of Admission	: Upto 2018

Duration: 3 Hours　　　　　　　　　　　　　　　　　　　　　Maximum Marks: 75

<u>Instructions for candidates:</u>

1. All questions carry equal marks.
2. Attempt any **FOUR** Questions.

1. Consider the pseudocode given below to sort a set of numbers in increasing order:

```
my_sort(lst)
    for i = 0 to length(lst) - 2
        k = i
        for j = i to length(lst)-1
            if lst[j] < lst[k]
                 k = j
        swap(lst[i], lst[k])
    print(lst)
```

Specify the best and worst case runtime complexity of my_sort(). Perform my_sort() on the input list [3, 2, 4, 6, 0, 7, 5, 1] and show the results obtained after every iteration.
If you have insertion sort also to work with, which one of the two (insertion/my_sort) will you prefer for an input list which is almost sorted? Justify with reasons.
Consider a scenario where insertion sort and merge sort are available to sort an array of integers in ascending order. For each algorithm, determine the time complexity if the input is described as: Case 1 - already sorted, Case 2 - sorted in descending order, and Case 3 - contains all elements with equal values. Justify with reasons.

2. Construct any possible Binary Search Tree for the following values and perform the inorder, postorder, preorder, and level order traversals.

8, 5, 1, 7, 10, 12, 14, 3, 6

Next, consider the pseudocode below:

```
FindMe(node)
    if (node != Null)
        FindMe(node.right)
        if (node.key % 2 != 0)
            print(node.key)
        FindMe(node.left)
```

For the tree constructed, what is the output of the above piece of code? In what ways does the result relate to any of the above-mentioned traversals?
Starting form the original code, find the output for the following scenarios and write how the result relates to any of the above-mentioned traversals: Case 1: What will be the output if lines 2 and 5 are swapped? Case 2: What will be the output if line 3 is removed?

3. Given two non-empty doubly linked lists each representing a non-negative large integer such that each node represents one digit of the integer, write an algorithm to add these two integers. The head node represents the most significant digit of the integer. The result should be stored in a third linked list. Derive the time complexity of the proposed solution.

For the resulting list, the i-th list node (i≥1) is to be swapped with the (n-i+1)-th list node where n is the number of nodes in the list. Write an algorithm to achieve this functionality. Also, derive the time complexity.

4. Convert the infix expression (A + B)*(C $ (D - E) + F) - G to postfix using a stack. Draw a table to show the contents of the stack at every character read (Here $ is the exponentiation symbol). Also, evaluate the generated postfix expression for A = 2, B = 3, C = 4, D = 5, E = 1, F = 2, and G=6. Which data structure will you use for evaluating the postfix expression? For the given infix expression, create an expression tree. Use the expression tree to find the prefix notation. Further, reverse the given string (ignoring the parenthesis) using a stack and a queue without using any extra variable.

5. Consider the following recursive function:

```
Mystery(Integer) {
        if (Integer == 0)
                return 0
        if (Integer == 1)
                return  1
         else {
                x = Integer % 2 + 10 * (Mystery(Integer / 2))
                return x
        }
}
```

What function is **Mystery()** performing? What is the value of **Mystery(15)**? How many times will **Mystery()** be called to compute **Mystery(15)**? Also, draw a tree to show all generated calls. Show the activation record at every recursive call.
Write an iterative version of the function **Mystery** with the same functionality. Also, derive its computational complexity.

6. A 2-dimensional array **tray[m][n]** is used to represent an egg tray in which eggs are stored in m rows and n columns. The variants of egg trays available in the market have rows ranging from 1 to 6 and columns ranging from 1 to 9. Consider a scenario where a person buys 25 eggs. Identify dimensions of the two-dimensional array required to store the eggs. Also, develop a row-major and column-major mapping function that maps the resulting two-dimensional array into one-dimension.

Further, suppose 20 out of the 25 eggs break due to mishandling. Will you still prefer to use the tray of dimensions identified above? If no, determine the new dimensions required and justify with reasons.

Consider the arrangement of eggs in the **tray** to store 5 eggs at positions - tray[0][0], tray[1][1], tray[2][2], tray[3][3], and tray[4][4]. Modify the mappings developed above to represent it in one-dimension.

Name of Course : **CBCS-2 Generic Elective**

Unique Paper Code : **32355112_OC**

Name of Paper : **GE-1 Analytical Geometry and Theory of Equations**

Semester : **I**

Duration : **3 hours**

Maximum Marks : **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Find an equation of the sphere that is circumscribed in the cube that is centered at the point (–2, 1, 3) and has sides of length 1 that are parallel to the coordinate planes.

Give the geometrical interpretation of the Rolle's theorem and verify it for the function $f(x) = \frac{1}{(1+x^2)}$ in the interval $[-3, 3]$.

2. Rotate the axes of coordinates to remove the xy-term from the equation and then name the conic and sketch the graph $x^2 + xy + y^2 - 2 = 0$.

Find an equation of the parabola that is symmetric about the y-axis, has its vertex at the origin and passes through the point (5, 2).

3. Form a polynomial equation of minimum degree in rational coefficients whose one of the roots is $\sqrt{2} - \sqrt{-7}$.

Given that the equations $x^3 - 2x^2 - 2x + 1 = 0$ and $x^4 - 7x^2 + 1 = 0$ have two roots in common. Find them.

4. Find a vector of length 2 that makes an angle $\pi/4$ with the positive x-axis and describe the surface $x^2 + y^2 + z^2 + 10x + 4y + 2z - 19 = 0$.

Find equation of the sphere with centre $(2, -1, -3)$ and has tangent to the xy-plane.

5. Remove the second term from the equation $x^3 - 6x^2 + 4x - 7 = 0$. Also, if α, β, γ be the roots (all non-zero) of the equation $x^3 - px^2 + qx - r = 0$, find the value of $\sum \alpha/\beta$ and $\sum 1/\alpha$.

Find grad[grad V. grad W], if $V = 3x^2y$ and $W = xz^2 - 2y$.

6. Find the sine and cosine of an angle through which the co-ordinate axes can be rotated to eliminate the cross-product term from the equation $4x^2 - 4xy + y^2 - 8x - 6y + 5 = 0$ and identify the conic.

Find an equation for the hyperbola that passes through the origin and whose asymptotes are $y = 2x + 1$ and $y = -2x + 3$. Also, sketch the graph.

Name of the course	:	**Generic Elective**
Unique Paper Code	:	**32355101_OC**
Name of the Paper	:	**GE-1 Calculus**
Semester	:	**I**
Duration	:	**3 hours**
Maximum Marks	:	**75**

Attempt any four questions. All questions carry equal marks.

1. Let $f(x)$ be a function defined by $f(x) = x^5 + 5x^4$. Determine the intervals in which this function is increasing or decreasing. Further, determine the points of local maxima and local minima. Find the open intervals in which $f(x)$ is concave up and concave down. Also, determine the point of inflexion, if any.

2. Find the following limits

(i) $\lim_{x\to 0}(\operatorname{cosec} x - \cot x)$, (ii) $\lim_{x\to 0}\left(\frac{1}{e^x - 1} - \frac{1}{x}\right)$, (iii) $\lim_{x\to 1}(1-x)\tan\frac{\pi x}{2}$.

3. Use cylindrical shells to find the volume of the solid generated when the region bounded by curves $y = 4x - x^2,\ y = 3$ is revolved about line $x = 1$.

4. Find the area of surface generated by the revolving the curves

(i) $x = \sqrt{16 - y^2}$, $0 \le y \le 2$ about y-axis, (ii) $= \sqrt{x-1}$, $2 \le x \le 3$ about x-axis.

5. Identify and sketch the conic $4y^2 + x^2 + 8y - 10x + 13 = 0$. Mark the coordinates of foci.

Find the equation of the ellipse whose foci are (2, 2) and (2, 4) and major axis of length 2.

6. If $r(t)$ is the position of a particle in plane at time t, find the time in the given interval when the velocity and acceleration are orthogonal, where

$$r(t) = (\,t - \sin t)i \;+ (1 - \cos t)j\,, 0 \le t \le 2\pi.$$

Name of Course	: **Generic Elective**
Unique Paper Code	: **32355301**
Name of Paper	: **GE-3 Differential Equations**
Semester	: **III**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Determine the constant A such that the differential equation

$$(Ax^2y + 2y^2)dx + (x^3 + 4xy)dy = 0$$

is exact and solve the resulting exact equation.

Solve the following initial value problems:

i) $\frac{dy}{dx} + \frac{4}{x}y = x^3y^2, \quad y(2) = -1, \; x > 0.$

ii) $(3xy + y^2)dx + (x^2 + xy)\mathrm{d}y = 0, \; y(1) = -1$.

2. Find the orthogonal trajectory of the family $y = c\sin x$ that passes through the point $(2\pi, 2)$. Also find the family of oblique trajectory that intersects the family of circles $x^2 + y^2 = c^2$ at an angle $\frac{\pi}{4}$.

Show that the family of confocal conics $\frac{x^2}{\lambda+a^2} + \frac{y^2}{\lambda+b^2} = 1$, where a and b are fixed constants and λ is the parameter, is self orthogonal.

3. Show that the set $\{1, x, x^2\}$ of functions forms a basis for the solution set of a differential equation. Also, find such a differential equation.

Find the general solution of the second order equation $t^2y'' + 2ty' - 2y = 0$ given that $y_1(t) = t$ is a solution. Also solve the initial value problem

$$y''' + 3y'' - 4y = 0, \; y(0) = 1, \; y'(0) = 0, \; y''(0) = \frac{1}{2}.$$

4. Find the general solution of the following differential equations:

i) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2 + 3e^{2x}.$

ii) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \log x.$

iii) $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 \log x.$

5. Find the partial differential equation arising from the equation $ax^2 + by^2 + z^2 = 1$, where $z = z(x, y)$.

Find the general solution of the linear partial differential equation

$$(y + xu)p - (x + yu)q = x^2 - y^2.$$

Using $v = \ln u$ and $v = f(x) + g(y)$, solve the equation

$$y^2 u_x^2 + x^2 u_y^2 = (xyu)^2, \; u(x, 0) = e^{x^2}.$$

6. Reduce the following equations to canonical form and hence find their solutions

i) $u_x - yu_y = 1 + u.$

ii) $y\, u_{xx} + (x + y)u_{xy} + x\, u_{yy} = 0, \; y \neq x.$

iii) $u_{xx} - 4u_{xy} + 4\, u_{yy} = 0.$

Unique Paper Code : **32355345**

Name of Paper : **Linear Programming and Game Theory (NC)**

Name of Course : **Mathematics: Generic Elective CBCS (LOCF) GE-3**

Semester : **III**

Duration : **3 hours**

Maximum Marks : **75**

Attempt any four questions. All questions carry equal marks.

1. Find all the basic feasible solutions of the following equations

$$x_1 + x_2 + 2x_3 = 9$$
$$3x_1 + 2x_2 + 5x_3 = 22$$

and also show that the set $S = \{(x, y)|\ xy \geq 1,\ x \geq 0, y \geq 0\}$ is convex.

2. Use two-phase method to solve the following linear programming problem

$$\text{Maximize } z = 2x_1 - x_2 + x_3$$
subject to
$$x_1 + x_2 - 3x_3 \leq 8$$
$$4x_1 - x_2 + x_3 \geq 2$$
$$2x_1 + 3x_2 - x_3 \geq 4$$
$$x_1, x_2, x_3 \geq 0.$$

3. Find the dual of the following problem and by solving the dual using big M-method show that the following linear programming problem has unbounded solution.

$$\text{Maximize } z = 4x_1 + 3x_2$$
subject to
$$5x_1 - 2x_2 \leq 6$$
$$3x_1 + x_2 \geq 1$$
$$x_1, x_2 \geq 0.$$

4. Solve the following cost minimization problem

	D_1	D_2	D_3	D_4	D_5	
O_1	9	11	13	18	16	110
O_2	15	13	18	11	9	150
O_3	15	21	7	11	14	190
O_4	14	20	8	6	13	200
	100	180	60	120	190	

5. A company has six jobs to be processed by six machines. The following table gives the Number of hours taken by the machines for the different jobs. If any job can be done by any machine, assign the machines to jobs so as to minimize the total machine hours.

Jobs

Machines	I	II	III	IV	V	VI
1	10	23	59	12	20	28
2	44	79	73	51	64	49
3	42	29	92	38	46	34
4	75	43	28	50	40	33
5	37	12	58	23	26	19
6	4	57	54	32	18	29

6. By the notion of dominance, reduce the following game to 2×4 game and then solve it graphically

$$\begin{bmatrix} 8 & 15 & -4 & -2 \\ 19 & 15 & 17 & 16 \\ 0 & 20 & 15 & 5 \end{bmatrix}.$$

Name of the Course	: B.A.(Prog.)
Unique Paper Code	: 62354343_OC
Name of the Paper	: DSC- Analytical Geometry and Applied Algebra
Semester	: III
Duration	: 3 Hours
Maximum Marks	: 75 Marks

Attempt any four questions. All questions carry equal marks.

1. Find the vertex, focus and the equation to the directrix of the parabola $x^2 - 6x - 6y + 6 = 0$. Sketch the conic $2x^2 + 5y^2 = 20$. Find an angle through which the rectangular coordinate axes must be rotated to eliminate the xy term from the equation $2x^2 + xy + 2y^2 = 1$.

2. Sketch the parabola $4(y-1)^2 = -7(x-3)$. Find the equation to the hyperbola with directrix as the straight line $x + 2y = 1$, focus as the point (2,1) and eccentricity 2. If the tangent line to an ellipse at a point P makes an angle $\pi/4$ with the line joining P to one focus S_1 of the ellipse, then find the angle that the tangent line to the ellipse at the point P makes with the line joining P to the other focus S_2 of the ellipse.

3. Find the equation of a sphere that is centered at (1,1,1) and is tangent to the sphere $x^2 + y^2 + z^2 - 8x + 2y - 10z + 38 = 0$. How many such spheres exist? Find the equation of any other such sphere, if it exists.

4. Find if the points A(2,0,2), B(6, −8, −6) and C(8, −12, −10) are collinear. Further, find the vector component of **AB** along **AC** and the vector component of **AB** orthogonal to **AC**.

5. Show that the plane whose intercepts with the coordinate axes are $x = 2, y = \frac{3}{2}, z = 5$ is given by the equation $\frac{x}{2} + \frac{2y}{3} + \frac{z}{5} = 1$.

6. Construct a Latin Square of order 5 on $\{1, 2, 3, 4, 5\}$. Is it unique? Justify. Show that the given Latin Square cannot be a group table of a finite group.

A	B	C	D	E
B	A	E	C	D
C	D	A	E	B
D	E	B	A	C
E	C	D	B	A

Name of the Course	: B.A.(Prog.)
Unique Paper Code	: 62354343_OC
Name of the Paper	: DSC- Analytical Geometry and Applied Algebra
Semester	: III
Duration	: 3 Hours
Maximum Marks	: 75 Marks

Attempt any four questions. All questions carry equal marks.

1. Find the vertex, focus and the equation to the directrix of the parabola $y^2 - 4x - 4y = 0$. Sketch the conic $4x^2 + 3y^2 = 48$. Find an angle through which the rectangular coordinate axes must be rotated to eliminate the xy term from the equation $3x^2 + \sqrt{3}xy + 2y^2 + 2 = 0$.

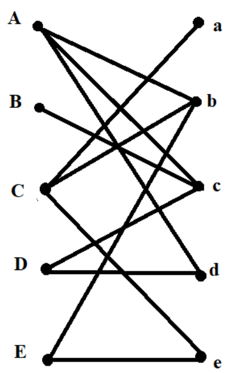
2. Sketch the parabola $(y - 2)^2 = 8(x + 1)$. Find the equation to the hyperbola referred to its axes as coordinate axes, the distance between the foci is 16 and the eccentricity is $\sqrt{2}$. If the tangent line to an ellipse at a point P makes an angle α with the line joining P to one focus S_1 of the ellipse, then find the angle that the tangent line to the ellipse at the point P makes with the line joining P to the other focus S_2 of the ellipse.

3. Describe the surface S whose equation is given by
   $3x^2 + 3y^2 + 3z^2 + 30x + 12y + 6z - 102 = 0$.
   Find the equation of the sphere with center same as that of S and tangent to the xy-plane.

4. Define skew lines. Find if the following lines L_1 and L_2 are skew lines.
   L_1: $x = -1 + 4t$, $y = 3 + t$, $z = 1$
   L_2: $x = -13 + 12t$, $y = 1 + 6t$, $z = 2 + 3t$.
   Further, find a vector orthogonal to both L_1 and L_2.

5. Find the equation to the plane through the points $P_1(1, 2, -1)$ and $P_2(0, 1, 4)$ and perpendicular to the plane $2x + y - z + 1 = 0$.

6. Does there exist a feasible matching for the following graph? Find if any.



Name of Course	: **B.A. (Prog.) DSE: Mathematics**
Unique Paper Code	: **62357502**
Name of Paper	: **Differential Equations**
Semester	: **V**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Solve the differential equation

$$(x^2 + y^2 + 1)\,dx - 2xy\,dy = 0.$$

Also, solve the differential equation

$$(D^2 + 4)y = \sin 3\,x + e^x$$

where $D \equiv \frac{d}{dx}$.

2. Show that $y_1(x) = \sin x$ and $y_2(x) = \sin x - \cos x$ are linearly independent solutions of

$$y'' + y = 0.$$

Determine c_1 and c_2 so that the solution

$$\sin x + 3\cos x \equiv c_1 y_1(x) + c_2 y_2(x).$$

Also, solve

$$x^2 y'' - 2xy' + 2y = 6, y(0) = 3, y'(0) = 1.$$

3. Using the method of variation of parameters ,solve

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2 e^x.$$

Also, solve

$$yz^2 dx - xz^2 dy - (2xyz + x^2)dz = 0.$$

4. Solve the equations:

$$\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{x^2 + y^2}.$$

Also, solve

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = x + \sin x.$$

5. Form a partial differential equation by eliminating the function f from $z = f\left(\frac{y}{x}\right)$.

Also solve the partial differential equation

$$\left(\frac{y^2 z}{x}\right) p + xzq = y^2.$$

6. Classify the partial differential equation into elliptic, parabolic or hyperbolic

$$x(xy-1)r - (x^2y^2-1)s + y(xy-1)t + (x-1)p + (y-1)q = 0$$

where $r = \frac{\partial^2 z}{\partial x^2}$, $s = \frac{\partial^2 z}{\partial x \partial y}$, $t = \frac{\partial^2 z}{\partial y^2}$, $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.

Also, find a complete integral of partial differential equation

$$z = px + qy + p^2 + q^2.$$

Name of Course	: **B.A. (Prog.) DSE: Mathematics**
Unique Paper Code	: **62357502**
Name of Paper	: **Differential Equations**
Semester	: **V**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Solve the differential equation

$$(4x + 3y^2)\,dx + 2xy\,dy = 0.$$

Also, show that the solutions e^x, e^{-x}, e^{2x} of

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0$$

are linearly independent and hence or otherwise find the general solution.

2. Solve the following differential equation

$$y'' - 3y' + 2y = 3e^{-x} - 10\cos x\,, y(0) = 1, y'(0) = 2.$$

Also, solve the differential equation

$$x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 20y = (x+1)^2.$$

3. Using the method of variation of parameters, solve

$$\frac{d^2y}{dx^2} + 4y = \sec^2 2x.$$

Also, solve

$$(y^2 + z^2 - x^2)dx - 2xydy - 2xzdz = 0.$$

4. Solve the differential equations

$$\frac{dx}{y + zx} = \frac{dy}{-(x + zy)} = \frac{dz}{x^2 - y^2}.$$

Also, solve

$$\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = e^x + \cos x.$$

5. Classify the following partial differential equation into elliptic, parabolic or hyperbolic

$$\sin^2 x \frac{\partial^2 z}{\partial x^2} + 4\cos x \frac{\partial^2 z}{\partial x \partial y} - 4\frac{\partial^2 z}{\partial y^2} = 0.$$

Also, solve the following partial differential equation

$$(mz - ny)p + (nx - lz)q = ly - mx.$$

6. Form a partial differential equation by eliminating the arbitrary function f from the equation

$$x + y + z = f(x^2 + y^2 + z^2).$$

Also, find a complete integral of partial differential equation

$$px + qy = pq.$$

Name of Course	: **CBCS B.Sc. (Math Sci)- II / B.Sc. (Phy Sci)-II / B.Sc. (Life Sci)-II /Applied Sciences-II**
Unique Paper Code	: **42357502**
Name of Paper	: **DSC- Mechanics and Discrete Mathematics**
Semester	: **V**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. (a) Define the motion along a curved path. Explain all physical quantities of the below equation:

$$\ddot{r} = \frac{dv}{dt}\,\hat{t} + v\,\omega\,\hat{n}$$

By using Equation (1), prove that

$$\ddot{r} = \frac{dv}{dt}\,\hat{t} + \frac{v^2}{\rho}\,\hat{n}$$

where ρ is the radius of curvature.

For what values of n, the following graphs are bipartite. Justify your answer.

(i) $\mathbf{K_n}$ (ii) $\mathbf{C_n}$ (iii) $\mathbf{W_n}$.

2. A weight of mass m is suspended by a spring and stretched from its natural length by x_0. Show that its motion is simple harmonic motion. (Take k as spring constant).

Draw all possible types of graphs, whether directed or indirected for the given adjacency matrices. If the drawn graph is indirected then find degree of each vertex and in case drawn graph is directed then find in-degree & out-degree of each vertex.

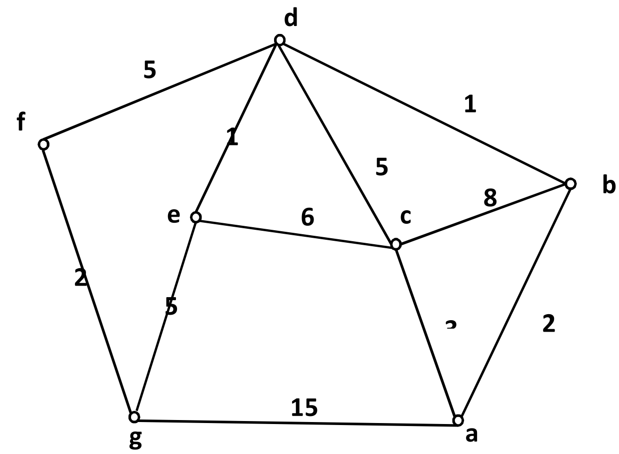
$$\begin{pmatrix} 0 & 2 & 1 & 1 \\ 2 & 0 & 2 & 0 \\ 1 & 2 & 0 & 1 \\ 1 & 0 & 1 & 2 \end{pmatrix}$$

3. Let β be the angle of the inclined plane and α be the angle of projection, then show that range R up a plane:

$$R = \frac{V^2}{g}\,\frac{\sin(2\alpha - \beta) - \sin\beta}{\cos^2\beta}$$

where V is the magnitude of velocity at $t = 0$, g is the gravitational acceleration constant.

Using Dijkstra’s algorithm, find the length of shortest path between the vertices a and g in the following graph.



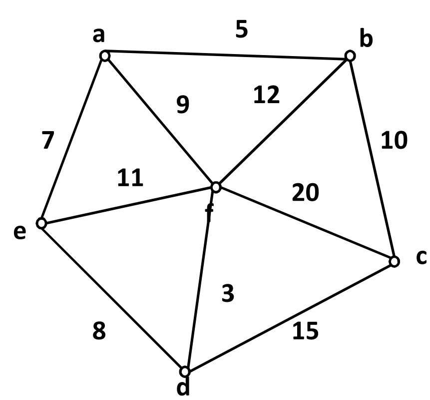
4. Three forces P, Q and R act along the sides BC, CA, AB of triangle ABC, and forces P', Q' and R' act along OA, OB, OC, O is the centre of circumscribing circle. Prove that if the six forces are in equilibrium,

$$P\cos A + Q\cos B + R\cos C = 0,$$

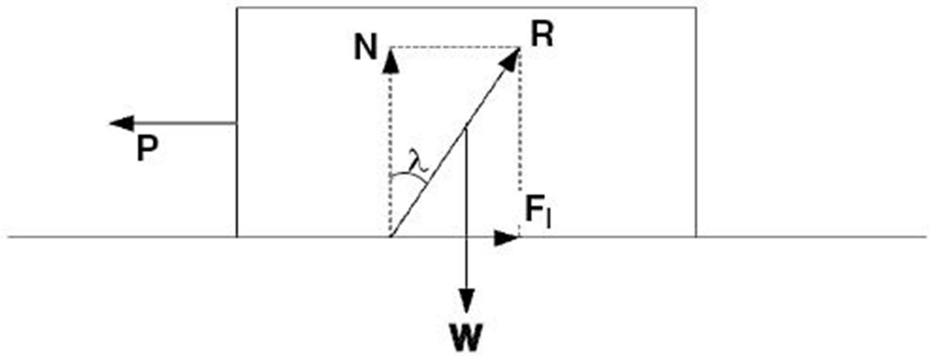
and

$$\frac{PP'}{BC} + \frac{QQ'}{CA} + \frac{RR'}{AB} = 0.$$

Use Kruskal’s algorithm to find a spanning tree with minimum weight from the graph given below. Also calculate the total weight of spanning tree.



5. Describe the angle of friction. In the below figure, let the pull P be removed but let the surface on which the block rests be tilted until the block is on the verge of slipping. Find the relation between the angle of tilt of the surface



and the angle of friction.

Either draw a graph with the given specifications or explain why no such graph exists.

(i) Tree with nine vertices and nine edges.

(ii) Tree with six vertices and having total degree 14.

(iii) Tree with five vertices and having total degree 8.

(iv) Tree with six vertices having degrees 1, 1, 1, 1, 3, 3.

6. Describe work done by a particle along the curve path and also explain conservative field of force.

For the graph given below, determine which of the following sequences are paths, simple paths, cycle and simple cycle. Explain

(a) b e_7 b

(b) d e_3 c e_2 b e_5 e e_4 d

(c) a e_6 d e_3 c e_2 b e_5 e.