Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32351201_OC**
Name of Paper	: **C 3-Real Analysis**
Semester	: **II**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

Q1. Find the infimum and supremum, if they exist, of the following subsets S_i $(i = 1,2,3)$. Justify your answer in each case:

$$S_1 = \left\{1 - \frac{(-1)^n}{n} : n \in \mathbb{N}\right\}$$

$$S_2 = \left\{\frac{\sqrt{2}}{n} : n \in \mathbb{N}\right\}$$

$$S_3 = \left\{\frac{1}{n} + 1 : n \in \mathbb{N}\right\}$$

Let S be a non-empty bounded subset of $\mathbb{R}$. Let $a \in \mathbb{R}$ and define a set $a - S = \{a - s : s \in S\}$. Prove that $a - S$ is a bounded set and $\sup(a - S) = a - \inf S$.

Q2. Let $S = \{s \in \mathbb{R} : 0 \leq s \ \text{ and } s^2 < 3\}$. Show that the set S has a supremum in $\mathbb{R}$. If $x = \sup S$, prove that $x > 0$ and $x^2 = 3$. What is $\inf S$?

Let u and v be real numbers with $u < v$. Show that there exists a rational number r such that

$$u < \sqrt{3}\, r < v.$$

Q3. Discuss convergence or divergence of the following sequences. If convergent, find the limit of the sequence (x_n) using ϵ- definition and if divergent, give reason for the same:

(i) $x_n = \frac{2n+3}{3n+7}$ (ii) $x_n = \frac{n}{(1-n)(1+n)}$ (iii) $x_n = \frac{2^n+4^n}{3^n}$

Are these sequences bounded? Justify your answer in each case.

If (x_n) is a convergent sequence with $x_n \geq 2$ for all $n \in \mathbb{N}$, prove that $\lim(x_n) \geq 2$.

Q4. Using the definition of Cauchy sequence, establish the convergence or divergence of following sequences

(i) $\left(1 + \frac{1}{2!} + \frac{1}{3!} + \cdots - \cdots + \frac{1}{n!}\right)$ (ii) $(\ln n^2)$ (iii) $((-2)^n)$

Show that a sequence (x_n) defined as

$$x_1 = 1$$

$$x_{n+1} = \frac{x_n + 3}{5}, \qquad n \geq 1$$

is convergent and find its limit.

Q5. Check the convergence or divergence of the following series. Clearly specify the result being used:

(i) $\sum_{n=1}^{\infty} e^{-n^2}$

(ii) $\sum_{n=1}^{\infty} \frac{1}{\log n}$

(iii) $\sum_{n=1}^{\infty} \frac{n+1}{2^n}$

(iv) $\sum_{n=1}^{\infty} \left(\frac{n}{n-1}\right)^{n^2}$

Q6. For each of the following, determine whether the series converges absolutely, converges conditionally, or diverges.

(i) $\sum_{n=1}^{\infty} \frac{2^n + n}{2^n - n}$

(ii) $\sum_{n=1}^{\infty} \frac{(\sin n\alpha + cos^2 n\alpha)}{n^2}$

(iii) $\sum_{n=1}^{\infty} \frac{(-1)^n \, 100^n}{n\,!}$

(iv) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n - \log n}$

[This question paper contains 2 printed pages.]

Sr. No. of Question Paper : **Your Roll No………**

Unique Paper Code :32351202_OC

Name of the Course : **B.Sc. (Hons.) Maths-I**

Name of the Paper : C4-Differential Equations

Semester : II

Duration: 3 Hours Maximum Marks: 75

Instruction for Candidates

1) **All questions carry equal marks.**
2) **Attempt any four questions.**

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1. (a) Find a general solution of the following differential equations.

$$\frac{dy}{dx} = \sqrt{x + y + 1}$$

(b) The half -life of a radioactive cobalt is 6.82 years. Suppose that a nuclear accident has left the level of cobalt radiation in a certain region at 100 times the level acceptable for human habitation. How long will it be until the region is habitable?

2. (a) Use the method of variation of parameters to find a particular solution of the differential equation $y'' + 4y = x$.

(b) Find the general solution of $(4xy') + y^3e^{-2x} = 4xy$

3. (a) A water tank has the shape obtained by revolving the curve $y = x^{8/3}$ around y-axis. A plug at the bottom is removed at 1:00 P.M., when the depth of the water in the tank is 18 ft. At 3:00 P.M. the depth of the water is 9ft. When will the tank be empty?

(b) Find the general solution of $xy'' = y'$.

4. (a) Let R(t) denote the number of red army and B(t) denote the number of blue army. Assuming both the armies use aimed fire, formulate the model (a pair of differential equations) and solve them to find the general solution. Also develop a model (a pair of differential equations) for a battle between two armies where both the groups use aimed fire. Assuming that the red army has significant loss due to disease, where the associated death rate (from disease) is proportional to the number of soldiers in that army.

(b) Find the solution of $x^2y'' - 6xy' + 3y = 0, y(2) = 3, y'(2) = 1$.

5. (a) A mass of 5 kg is attached to the end of a spring that is stretched 25 cm by a force of 18 N. It is set in motion with initial position $x_0 = 0$ and initial velocity $v_0 = -15$ m/s. Find the amplitude, period and frequency of the resulting motion.

(b) Use the method of undetermined coefficients to find the general solution of

$y'' - 4y' + 4y = e^{2x}$

6. (a) In a poultry farm, hens are harvested at a constant rate of 700 hens per day. The per-capita birth rate for the hen is 1.4 hens per week per hen, and the per-capita death rate is 4.9 hen per week per hen. Defining each symbol you introduced, write the word equation to describe the rate of change of hen population. Using the above written word equation, obtain the differential equation describing the rate of change of hen population. If the hen population at a given time is 280000. Estimate the number of hen died in one week. Determine if there are any values for which the hen population is in equilibrium.

(b) Show that the differential equation $(4x + 3y^2)dx + 2xy\,dy = 0$ is not exact. Find an integrating factor & solve.

Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32351401_OC**
Name of Paper	: **C8 Partial Differential Equations**
Semester	: **IV**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1 (i) Derive the damped wave equation and telegraph equation of a string.

(ii) Reduce the equation
$u_{xx} + xyu_{yy} = 0 \quad ,x, y < 0$ to canonical form.

(iii) Prove the uniqueness of the solution of the initial boundary-value problem:

$$u_{tt} = 25\, u_{xx}, \quad 0 < x < \pi, t > 0,$$
$$u(x, 0) = f(x), \quad 0 \leq x \leq \pi,$$
$$u_t(x, 0) = g(x)\,, \qquad 0 \leq x \leq \pi,$$
$$u_x(0, t) = 0, \qquad u_x(\pi, t) = 0, \qquad t > 0$$

2 (i) Find the integral surfaces of the equation
$uu_x + u_y = 1$ for the initial data
$x(s, 0) = \frac{s^2}{2},\ y(s, 0) = 2s,\ u(s, 0) = s$

(ii) Apply $e^u = v$ and then $v(x, y) = f(x) + g(y)$ to solve the equation $x^4u_x^2 + y^2u_y^2 = e^{-2u}$.

(iii) Use a separable solution $v(x, y) = g(x) + f(y)$ to solve the equation $v_x^2 + v_y^2 = v$.

3 (i) Find the characteristics and characteristic coordinates, and reduce the equation

$$u_{xx} - (2cosx)u_{xy} + (1 + cos^2x)u_{yy} + u = 0$$

to canonical form.

(ii) Determine the general solutions of the equation

$$u_{xx} + 4u_{xy} + 4u_{yy} = 0$$

(iii) Transform the equation

$$u = -2u_{xx} - 9u_{xy} - 3u_{yy} - u_y$$

to the form $v_{mn} = cv$, where c = constant, by introducing the new variables $v = ue^{-(am+bn)}$, where ‘a’ and ‘b’ are undetermined coefficients.

4 (i) Determine the solution of the Cauchy problem given by

$$u_{tt} = 9u_{xx}, \ \ u(x,0) = x^3, u_t(x,0) = x$$

(ii) Determine the solution of the initial boundary value problem

$$u_{tt} = u_{xx}, \quad 0 < x < 1, t > 0$$
$$u(x,0) = \sin \pi x\,, \quad 0 \le x \le 1$$
$$u_t(x,0) = 0\,, \qquad 0 \le x \le 1$$
$$u(0,t) = 0,\ u(1,t) = 0,\ t \ge 0$$

(iii) Determine the solution of characteristic initial value problem

$$xy^3u_{xx} = x^3 \ \ y\, u_{yy} + y^3u_x - x^3u_y,$$
$$u(x,y) = f(x),\ \ y^{\,2} + x^2 = 4,\ 0 \le x \le 2$$
$$u(x,y) = g(y), \qquad x = 0,\ 0 \le y \le 2,$$
$$f(0) = g(2)$$

5. (i) Solve using method of separation of variables

$$u_{tt} = 9u_{xx}, 0 < x < 2\,, t > 0$$
$$u(x,0) = x(2-x), 0 \le x \le 2$$
$$u_t(x,0) = 0,\ \ 0 \le x \le 2$$
$$u(0,t) = t,\ u(2,t) = 0, t > 0$$

(ii) Determine the solution of

$$u_{tt} = u_{xx} + x^2, 0 < x < 1\,, t > 0$$
$$u(x,0) = 0, 0 \le x \le 1$$
$$u_t(x,0) = 0,\ \ 0 \le x \le 1$$
$$u(0,t) = 0,\ u(1,t) = 0, t \ge 0$$

(iii) Solve the heat conduction problem

$$u_t = 9u_{xx}, 0 < x < 1\,, t > 0$$
$$u(x,0) = x^2(\,1 - x\,), 0 \le x \le 1$$
$$u(0,t) = 0, t > 0$$
$$u(1,t) = 0, t > 0$$

6 (i) Find the partial differential equation arising from the surface

$$2z = (\alpha x + y)^2 + \beta$$

Find the solution of the Cauchy problem:

$uu_x - uu_y = u^2 + (x+y)^2$, with u = 1 on y = 0

(ii) Determine the solution of the initial boundary value problem

$$u_{tt} = 9u_{xx}, 0 < x < \infty, t > 0,$$
$$u(x, 0) = 0, \quad 0 \leq x < \infty,$$
$$u_t(x, 0) = x^3, \quad 0 \leq x < \infty,$$
$$u_x(0, t) = 0, \qquad t \geq 0$$

.

(iii) Solve the characteristic initial-value problem

$$x u_{xx} - x^3 u_{yy} - u_x = 0, x \neq 0,$$
$$u(x, y) = f(y) \text{ on } y - \frac{x^2}{2} = 1 \quad \text{for } 0 \leq y \leq 2,$$
$$u(x, y) = g(x) \text{ on } y + \frac{x^2}{2} = 3 \quad \text{for } 2 \leq y \leq 4,$$

with f(2) = g(2).

Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32351403_OC**
Name of Paper	: **C 10-Ring Theory and Linear Algebra-I**
Semester	: **IV**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. For the following vectors in R^3, determine whether the first vector can be expressed as linear combination of the other two.

$$\{(5, 1, -5), (1, -2, -3), (-2, 3, -4)\}$$

Determine whether or not the set

$$\left\{\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\right\}$$

is linearly independent over Z_7.

Determine whether the set $\{1, (x-a), (x-a)^2, ..., (x-a)^n\}$ spans $P_n(F) = \{a_0 + a_1 x + ... + a_n x^n : a_0, a_1, ..., a_n \in F\}$, where a is fixed scalar.

2. Find three different bases of the subspace $W = \{(a_1, a_2, a_3) \in R^3 : a_1 - 4a_2 - a_3 = 0\}$ of R^3.

Let $W_1 = \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_2(R) : a + d = 0\right\}$ and $W_2 = \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_2(R) : d = 0, b + c = 0\right\}$. Find $\dim(W_1 \cap W_2)$ and $\dim(W_1 + W_2)$.

3. Let $\mathcal{M}_2(\mathbb{Z})$ denote the ring of all $2X2$ matrices with integer entries. Show that $\mathcal{M}_2(5\mathbb{Z})$ is an ideal of the ring $\mathcal{M}_2(\mathbb{Z})$.

Write all the elements of the quotient ring $\frac{\mathcal{M}_2(\mathbb{Z})}{\mathcal{M}_2(5\mathbb{Z})}$.

Find all idempotent elements, nilpotent elements, units and zero divisors of the ring $\mathbb{Z}_2 \oplus \mathbb{Z}_4$.

4. Let $\mathbb{Z}[x]$ denote the ring of polynomials with integer coefficients and let $I = \{f(x) \in \mathbb{Z}[x]\ such\ that\ f\ is\ an\ even\ integer\}$. Prove that $I = < x, 2 >$. Further prove that I is a prime ideal as well as a maximal ideal of the ring $\mathbb{Z}[x]$.

Show that $Q[\sqrt{2}]$ and $Q[\sqrt{3}]$ are not isomorphic rings.

5. If $T : R^2 \to R^2$ be a linear transformation for which $T(1,1) = (1,-2)$ and $T(-1,1) = (2,3)$ then what is $T(-1,5)$?

Let $T : R^3 \to R^3$ be a linear transformation defined by

$$T(x_1, x_2, x_3) = (3x_1, x_1 - x_2, 2x_1 + x_2 + x_3),\ x_1, x_2, x_3 \in R.$$

Find a basis for Range space and and a basis for Null space of T and verify Dimension Theorem. Is T one-one? Is T onto? Is T invertible ? If so, find T^{-1}.

6. Let $A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 1 & 3 \\ 0 & -1 & 0 \end{pmatrix}$ be the matrix of linear operator T on R^3, w. r. t. standard ordered basis $\beta = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$. Now if $\gamma = \left\{ \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$ be another basis of R^3, find a matrix B of T w. r. t. γ. Find a non-singular matrix Q such that $B = Q^{-1}AQ$.

Name of the Course : **B.Sc. (Hons.) Mathematics CBCS**

Semester : **IV**

Unique Paper Code : **32351402_OC**

Name of the Paper : **C9 - Riemann Integration and Series of Functions**

Duration: **3 Hours** Maximum Marks: **75**

Attempt any four questions. All questions carry equal marks. All symbols have usual meaning.

1. Calculate the upper Darboux sum and lower Darboux sum of the function

   $f(x) = \frac{1}{x^2}$ *and* $g(x) = xe^x$ on the interval $[1, 2]$ for a partition

   $P = \{1, 1.25, 1.5, 1.75, 2\}$.

   Calculate upper and lower Darboux integral of $g(x) = x^2 + 5$ on $[2,4]$. Is g integrable ?

   Let $f: [a, b] \to R$ be a bounded function. Suppose that there is a partition P of $[a, b]$ such that

   $$L(f, P) = U(f, P).$$

   Show that f is a constant function.

2. Define $f(x) = x[x]$ on $[0, 4]$. Show that f is integrable, and evaluate $\int_0^4 f(x)dx$. Give an example where the functions f and g are not Riemann integrable, but $f.g$ is integrable. Let f be a continuous function on R and define

   $$F(x) = \int_0^{\frac{x^4}{4}} f(t)dt \quad for\ x \in R$$

   show that F is differentiable on R and compute F'.

3. Examine the convergence of following improper integrals

   $$\int_0^\infty e^{-x}(3x+2)dx \qquad and \qquad \int_0^\infty \frac{dx}{\sqrt{3x^4+5x}}$$

   Using the properties of Gamma integral find the value of

   $$\int_0^\infty x^5 e^{-4x^2}\, dx$$

4. Let (f_n) be defined by $f_n(x) = 1 - |1 - x^2|^n\ \forall x \in \left[-\sqrt{2},\ \sqrt{2}\,\right]$

   Find the pointwise limit of (f_n) on $\left[-\sqrt{2},\ \sqrt{2}\,\right]$.

   Does the sequence converge uniformly on this interval? Justify your answer. Show that the sequence (f_n) where $f_n(x) = nxe^{-nx^2}$, $x \geq 0$ is not uniformly convergent on $[0, 2]$.

   Show that the sequence $\left\{\frac{\sin(n^2x^2+1)}{n(n+1)}\right\}$ converges uniformly on R.

5. Examine the convergence of the series of functions $\sum f_n$ where $f_n(x) = \frac{1}{1+x^n}$ and show that convergence is non uniform in $(1, \infty)$ and is uniform in $[a,\ \infty[$, $a > 1$

Show that $\sum_{n=1}^{\infty} \frac{\cos nx}{n^3}$ converges uniformly on R to a continuous function.
Evaluate the integral $\int_0^1 \sum_{n=1}^{\infty} \frac{x}{(n+x^2)^2} dx$

6. Find the radius of convergence and exact interval of convergence for the following power series

$$\sum_0^{\infty} \frac{4^n}{n5^{n+2}} (x-2)^n \qquad and \qquad \sum_0^{\infty} \left[\frac{3+5(-1)^n}{7}\right]^n x^n$$

Write the power series expansion for the integral of the following function:

$$f(x) = \frac{x^2}{3-x^3} \quad ,$$

given that $\frac{1}{1-x} = \sum_0^{\infty} x^n \quad , x \in]-1,1[.$

Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32351601**
Name of Paper	: **C 13- Complex Analysis**
Semester	: **VI**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Determine whether $S = \{z \in \mathbb{C} : |z|^2 > z + \overline{z}\}$ is a domain or not? Justify your answer.

Find the image of line segment joining $z_1 = -i$ to $z_2 = -1$ under the map $f(z) = \overline{iz}$.

Check whether Cauchy-Riemann equations for $f(z) = \sqrt{|z^2 - \bar{z}^2|}$ are satisfied at the origin? Is f analytic at the origin? Justify your answer.

Suppose $f(z) = \cosh(2x)\cos(2y) + i\,v(x,y)$ is analytic everywhere such that $v(0,0) = 0$. Find $f(z)$. Hence find zeros of f.

Solve the equation $e^{z-1} + ie^3 = 0$.

2. Let $S = \{z \in \mathbb{C} : \text{Im } z = 1 \text{ and Re } z \neq 4\}$. Is S open? Is S closed? Justify your answer.

Assume that g is analytic in a region and that at every point either $g = 0$ or $g' = 0$. Show that g is constant.

Suppose $f(z) = \begin{cases} \bar{z}^3/z^2 & if\ z \neq 0 \\ 0 & if\ z = 0 \end{cases}$. Show that f is continuous everywhere on $\mathbb{C}$. Is f analytic at $z = 0$? Justify your answer.

Does there exists an analytic function $f(z) = u(x,y) + i\,v(x,y)$ for which $u(x,y) = y^3 + 5x$? Solve the equation $Log(z) + Log(2z) = 3\pi/2$.

3. Determine whether the following curves are simple, closed, smooth or contour

C_1: $z(t) = |t| + it$, $t \in [-1,1]$

C_2: $z(t) = e^{2it}$, $t \in [0, 2\pi]$,

C_3: $z(t)$ is the positively oriented boundary of the rectangle whose sides lie along $x = \pm 1,\ y = 0,\ y = 1$.

Evaluate $\int_{C_3} |z|dz$. Explain why Cauchy Goursat theorem is not applicable in this case?

Use ML-Inequality to show that

$$\left|\int_C \frac{e^z}{(z+1)} dz\right| \leq 4\pi e^2$$

where C : $z(t) = e^{2it}$, $t \in [-\pi, \pi]$.

4. Evaluate $\int_C ze^{3z}dz$ where C is the parabola $x^2 = y$ from $(0,0)$ to $(1,1)$.

Using Cauchy Integral formula, determine the integral $\int_C \frac{e^z}{z^2(z^2-9)} dz$ where C is positively oriented circle (i) C: $|z| = 1$. (ii) C: $|z - 3| = 1$.

Use Liouville's theorem to establish that $cos\ z$ is not bounded in the complex plane.

Let g be an entire function and suppose that $|g(z)| < 10$ for all values of z on the circle $|z-2| = 3$. Find a bound for $|g'''(2)|$.

5. Determine the radius of convergence of the series $\sum_{k=0}^{\infty} \frac{z^k}{k!}$ and $\sum_{k=0}^{\infty} k^k z^k$. Also discuss the convergence of the series.

Obtain the Maclaurin series of the function (z) $= \frac{1}{z^2} sinh\left(\frac{1}{z}\right)$. Specify the region in which the series is valid.

Find the Laurent series of the function $f(z) = \frac{1}{(z+1)(z+3)}$ valid for $0 < |z+1| < 2$.

6. Determine the residue and singularities of the function $g(z) = \frac{z+1}{z^2+4}$. Also evaluate $\int_C g(z)dz$ where C is the positively oriented circle $|z-i| = 2$.

Using a single residue, evaluate the integral $\int_{C'} \frac{3z-1}{z(z+1)} dz$ where C' is the positively oriented circle $|z-1| = 4$.

Use residue to evaluate the integral $\int_0^{2\pi} \frac{dt}{3+cost}$

Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32351602**
Name of Paper	: **C14-Ring Theory and Linear Algebra-II**
Semester	: **VI**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Prove that $\mathbb{Z}[x]$ is not a principal ideal domain. Also show that $2x^2 + x + 1$ is irreducible over $\mathbb{Z}_3$. Construct a field of order 16.
2. Prove that in a unique factorization domain, an element is irreducible if and only if it is prime. Prove or disprove that a subdomain of a principal ideal domain is a principal ideal domain. Show that $x^4 + 1$ is irreducible over $\mathbb{Q}$ but reducible over $\mathbb{R}$.
3. Let $T: P_2(\mathbb{R}) \to P_2(\mathbb{R})$ be a linear operator such that
$$T\big(f(x)\big) = f'(x) + f''(x).$$
Find the eigenvalues of T and their corresponding eigenspaces. Is T a diagonalizable linear operator? Find the minimal polynomial of T. Now suppose that $V = \mathbb{R}^3$ and$\beta = \{(1,0,2), (0,1,1), (1,1,0)\}$ be an ordered basis for V.Find an ordered basis $\beta^*$of V^* which is the dual basis corresponding to β.
4. Find an ordered basis for the T-cyclic subspace W of $\mathbb{R}^4$ generated by the vector z where $T: \mathbb{R}^4 \to \mathbb{R}^4$ is a linear operator such that
$$T(a, b, c, d) = (c + d, -b, a + b, 2a + b)$$
and $z = e_1$.Is W a T-invariant subspace of $\mathbb{R}^4$?Find the characteristic polynomial of T_W.Show that the characteristic polynomial of T_W obtained above divides the characteristic polynomial of T.Verify Cayley-Hamilton Theorem for T_W.
5. Apply the Gram-Schmidt process to the subset
$$S = \{f_1, f_2, f_3\}$$
of the inner product space $V = C[-\pi, \pi]$ with the inner product given by
$$\langle f, g\rangle = \int_{-\pi}^{\pi} f(t)g(t)\, dt$$
to obtain an orthogonal basis forspan(S), i.e., the subspace of V spanned by the functions in S, where $f_1(x) = 1$, $f_2(x) = \sin x$ and $f_3(x) = \cos x$. Then normalize the vectors in this basis to obtain an orthonormal basis β for span(S).

6. Use the least squares approximation to find the best fit quadratic function for the set

$$\{(-1,5),(1,1),(2,1),(3,-3)\}.$$

Also compute the corresponding error E. Also find the minimal solution to the following system of linear equations:

$$x+y+z-w=1$$

$$2x-y+w=1$$

Name of Course : **CBCS B.Sc. (H) Mathematics**

Unique Paper Code : **32353401_OC**

Name of Paper : **SEC-2 Computer Algebra Systems and Related Softwares**

Semester : **IV**

Duration : **3 hours**

Maximum Marks : **38 Marks**

Attempt any four questions. All questions carry equal marks.

1. (a) Write the Command to define function

$$f(x) = x, \quad \text{if } x \le 1$$
$$= x - 2, \quad \text{if } x > 1$$

.

and write command to evaluate f(x) at x = 3 and first derivative of f(x) at x = -3.
Also write command to integrate f(x) over interval [-5,5].

(b) Write the command to obtain a square matrix A of order 4 with random entries between 1 and 20 and write commands to find the determinant, rank and transpose of matrix A.

2. (a) Write a command to find x, y, z, t for the following system of equations:

$$-2x - 2y + 3z + t = 8$$
$$-3x + 0y - 6z + t = -19$$
$$6x - 8y + 6z + 5t = 47$$
$$x + 3y - 3z - t = -9.$$

(b) Write a command to find the basis for the space spanned by the vectors

$\{v1, v2, v3, v4\}$, where

$$v1 = (2, 1, 15, 10, 6),$$

$$v2 = (2, -5, -3, -2, \ 6),$$

$$v3 = (0, 5, 15, 10, 0),$$

$$v4 = (2, 6, 18, 8, 6).$$

Also, write a command to find basis for the column space and to check whether the given set is linearly independent.

3. Write a command to plot the function $f(x) = x^2 \frac{1}{\sin x}$ over the domain $-20 \le x \le 20$ with a two dimensional slider with the label "Move the axes" by assuming minimum and maximum value for the axes as -20 and 20, respectively. Place the control to the right.

4. Explain the following in software-R
   - (i) Explain stem and leaf plot with example.
   - (ii) Write difference between as.data.frame() and data.frame() commands.
   - (iii) Write command to create a 3x3 square matrix "A" and add row and column names to this matrix "A" after creating.
   - (iv) What is difference between matrix and data fame.
   - (v) Write difference between order() and rank() commands.
   - (vi) What is difference between Cleveland dot charts and bar charts.
   - (viii) Write difference between vector and list.

5. Write possible R commands for the following questions:

−	$C1$	$C2$	$C3$	$C4$	$C5$
$R1$	38	31	7	15	25
$R2$	56	89	56	NA	11
$R3$	17	95	23	89	75
$R4$	77	55	11	45	99
$R5$	65	NA	26	10	28
$R6$	91	8	70	77	65

   - (i) Change row name "R2" to "Row2" of this matrix.
   - (ii) Extract rows and columns; find the mean and standard deviation of each row.
   - (iii) Write code to draw histogram of "C2".
   - (iv) Covert this matrix into data frame.
   - (v) Find mean of the vector "Row 2" of the converted data frame.

6. (a) Write a R program to create a data frames which contain details of 5 employees and display the details: Name, Gender, Age, Designation and SSN No.

   (b) Write a R program to create a simple bar plot of ten subjects marks.

   (c) Write a R program to create a vector which contains 20 random integer values between -100 and +100

Name of the Course : CBCS B.Sc. (H) Mathematics

Unique Paper Code : 32357607

Name of the Paper : DSE - III Probability Theory and Statistics

Semester : VI

Duration : 3 hours

Maximum Marks : 75

Attempt any four questions. All questions carry equal marks.

1. If the random variable T is the time to failure of a commercial product and the values of its probability density and distribution function at time t are $f(t)$ and $F(t)$, then its failure rate at time t is given by $\frac{f(t)}{1-F(t)}$. Thus, the failure rate at time t is the probability density of failure at time t given that failure does not occur prior to time t.

   Show that if T has the exponential distribution, the failure rate is constant.
   Show the random variable $\boldsymbol{X}$ has probability density function $f(x)$ if it is defined by

$$f(x) = \begin{cases} \frac{\alpha}{x^{\alpha+1}}, & for\ x > 1 \\ 0, & elsewhere \end{cases}$$

   where $\alpha > 0$. Also show that μ_r' exists only if $r < \alpha$.

2. Let $\boldsymbol{X}$ be binomially distributed with parameters $\boldsymbol{n}$ and $\boldsymbol{\theta}$. Show that as $\boldsymbol{k}$ goes from $\boldsymbol{0}$ to $\boldsymbol{n}$, $P(\boldsymbol{X} = \boldsymbol{k})$ increases monotonically, then decreases monotonically reaching its largest value in the case that $(\boldsymbol{n} + 1)\,\boldsymbol{\theta}$ is an integer, when k equals either $(\boldsymbol{n} + 1)\,\boldsymbol{\theta} - 1$ or $(\boldsymbol{n} + 1)\,\boldsymbol{\theta}$.

   An airline knows that 5 percent of the people making reservations on a certain flight will not show up. Consequently, their policy is to sell 52 tickets for a flight that can hold only 50 passengers. What is the probability that there will be a seat available for every passenger who shows up?

3. The joint probability density function of X & Y is:

$$f(x,y) = \begin{cases} \frac{2}{3}\,(x+y) & ,0 < x < 1\,, 0 < y < 1 \\ 0 & ,\text{otherwise} \end{cases}$$

   Find **(a)** the marginal density functions of X and Y **(b)** conditional density of X given y **(c)** evaluate $P(X \leq 1/2 | \, Y = 1/2\ )$ (d) conditional mean and variance of X given $Y = \frac{1}{2}$.

4. The joint probability density function of (X, Y) is given to be

$$f(x,y) = \begin{cases} k(y-x)e^{-y} &, \quad -y < x < y \\ 0 &, \quad 0 < \mathrm{y} < \infty \end{cases}$$

Find **(a)** the constant k **(b)** mean of X **(c)** mean of Y **(d)** Covariance (X,Y)

5. Variates X and Y have zero means and standard deviations σ_1, σ_2 are normally correlated with correlation coefficient ρ. Show that

$$U = \frac{X}{\sigma_1} + \frac{Y}{\sigma_2}, \qquad V = \frac{X}{\sigma_1} - \frac{Y}{\sigma_2}$$

are independent random variables and follow the normal distribution.

Let the Markov chain consisting of the states 1, 2, 3, 4, 5, 6 and have the transition probability matrix

$$P = \begin{bmatrix} 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.4 & 0.6 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.3 & 0.0 & 0.4 & 0.2 & 0.1 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.3 & 0.7 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.5 & 0.0 & 0.5 \\ 0.0 & 0.0 & 0.0 & 0.3 & 0.0 & 0.7 \end{bmatrix}$$

Determine which states are transient and which are recurrent.

6. Let X has the probability density function

$$f(x) = \begin{cases} \dfrac{1}{2\sqrt{5}}, & -\sqrt{5} < x < \sqrt{5} \\ 0, & elsewhere \end{cases}$$

Find the actual probability $P\left[|X - E[X]| \geq \frac{3}{2}\sigma\right]$ and compare it with the upper bound obtained by Chebyshev's inequality. Further, if the variate X has the probability density function $f(x) = e^{-x}, \quad x \geq 0$. Use Chebyshev's inequality to show that

$$P[|X-1| > 2] < \frac{1}{4}$$

and show that the actual probability is e^{-3} .

Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32357611**
Name of Paper	: **DSE-4 Linear Programming and Theory of Games**
Semester	: **VI**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

Q.1 Solve the following LPP by Big-M method and verify your answer by finding all the existing basic feasible solutions:

Maximize $Z = x_1 - x_2 - x_3$

Subject to $x_1 + x_2 + x_3 \geq 2$

$2x_1 - x_2 + x_3 = 3$

$x_1, x_2, x_3 \geq 0$

Q.2 Obtain the inverse of the following matrix by using simplex method

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

Verify your answer by matrix multiplication.

Q.3 Verify for the following Linear Programming Problem that dual of dual is primal. Also using complementary slackness theorem solve both primal and dual problems.

Maximize $Z = x_1 + x_2$

Subject to

$x_1 + 2x_2 \leq 5$

$2x_1 + x_2 \geq 0$

$x_2 \leq 3$

$x_1, x_2 \geq 0.$

Q.4 For the following cost minimization transportation problem find initial basic feasible solutions by using North West Corner rule, Least cost method and Vogel's approximation method. Compare the three solutions:

Source \ Destination	A	B	C	D	E	Supply
I	16	16	13	22	17	50
II	14	14	13	19	15	60
III	19	19	20	23	15	50
IV	12	10	15	8	12	50
Demand	30	20	70	30	60	

Also find the optimal basic feasible solution of above problem using UV- method.

Q.5 Solve the cost minimization assignment problem:

Job \ Man	I	II	III	IV	V
A	2	3	5	5	6
B	4	5	7	7	8
C	7	8	8	10	9
D	3	5	3	6	5
E	4	3	5	2	1

Does this problem has more than one solution? If yes, then find any FOUR possible solutions.

Q. 6 Show that the following rectangular game does not have any saddle point.

$$\begin{bmatrix} 2 & 3 & 4 & 5 \\ 12 & -6 & 3 & 0 \\ 4 & 0 & 2 & 1 \\ 0 & 4 & 3 & 4 \\ 6 & -1 & 3 & -2 \end{bmatrix}$$

Solve it by graphical method by reducing its size using dominance principle.

Name of the Course	**: B.A. (Prog)**
Unique Paper Code	**: 62351201_OC**
Name of the Paper	**: Algebra**
Semester	**: II**
Duration	**: 3 Hours**
Maximum Marks	**: 75**

Attempt any four questions. All questions carry equal marks.

1. Consider the vector space $\mathbb{R}^3$ and its subset S

$$S = \{(a,b,c) : 3a - 4b + c = 0, a + 2b - c = 0, \;\; a,b,c \in \mathbb{R}\}$$

Show that S is a subset of $\mathbb{R}^3$ and also find $\dim S$.

Determine whether or not the vectors $(1,-3,2), (2,4,1)$ and $(1,1,1)$ form a basis of $\mathbb{R}^3$

Are the following sets linearly independent or linearly dependent?

$$A = \{(3 - i, 2 + 2i, 4), (2,2 + 4i, 3), (1 - i, -2i, 1)\} \; ; \quad B = \{(1,2,3), (1,3,2), (3,7,8)\}$$

2. Solve the system of equations:

$$3x + 4y - 6z + w = 7$$
$$x - 2y + 3z - 2w = -1$$
$$x - 3y + 4z - w = -2$$
$$5x - y + z - 2w = 4$$

Find the rank of the matrix

$$A = \begin{pmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{pmatrix}$$

Verify that the matrix B satisfies its own characteristic equation and hence find its inverse

$$B = \begin{pmatrix} 12 & -1 & 4 \\ -1 & 12 & -1 \\ 4 & -1 & 12 \end{pmatrix}.$$

3. Show that

$$2^7 \sin^4 2\theta \sin^4 \theta = \cos 12\theta - 4\cos 10\theta + 2\cos 8\theta + 12\cos 6\theta - 17\cos 4\theta - 8\cos 2\theta + 14$$

Sum the series

$$\sin\theta \sin 2\theta + \sin 2\theta \sin 3\theta + \sin 3\theta \sin 4\theta + \cdots \text{upto n terms}, \qquad n \in \mathbb{N}$$

Solve the equation

$$(z+1)^7 = z^7[\cos 7\alpha + i \sin 7\alpha] \, .$$

4. Solve the equation $\quad x^3 - 9x^2 + 23x - 15 = 0 \quad$; the roots being in A.P.

If α, β, γ are roots of the equation $x^3 - mx^2 + nx - s = 0$, find the value of

$$\sum \alpha^2\beta \quad , \quad \sum \frac{\alpha^2}{\beta\gamma}$$

Find all the roots of the equation $x^4 + 2x^3 - 16x^2 - 22x + 7 = 0$, where it is given that $2 + \sqrt{3}$ is one of the roots.

5. Find the multiplicative inverses of the given elements if they exist:

$$[11]_{16} \; in \; \mathbb{Z}_{16} \quad and \quad [38]_{83} \; in \; \mathbb{Z}_{83} \; .$$

Let a be a fixed element of a group G and let

$$N(t) = \{x \in G : xt = tx\}.$$

Show that $N(t)$ is a subgroup of G.

Find the order and inverse of the permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 4 & 3 & 6 & 1 \end{pmatrix}.$$

6. In permutation group S_6, if $\alpha = (2\ 3\ 4)$ and $\beta = (2\ 5\ 6)$, then find a permutation ϕ such that $\phi\alpha\phi^{-1} = \beta$.

Let a be a fixed element of a ring R, and let

$$I_s = \{x \in R : sx = 0\}.$$

Show that I_s is a subring of R.

Show that $\mathbb{Z}_8 = \{0,1,2,3,4,5,6,7,8\}$ is a commutative ring w.r.t. addition modulo 8 and multiplication modulo 8.

Name of Course	: **CBCS B.Sc. Mathematical Sciences**
Unique Paper Code	: **42351201_OC**
Name of Paper	: **C 3-Calculus & Geometry**
Semester	: **II**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Use (ε, δ) $definition\ to$ find δ such that $\lim_{x\to 3}(5x-2) = 13$; $\varepsilon = 0.01$.

Examine the continuity of the function $f(x) = \begin{cases} \cos x\,, & x \neq \frac{\pi}{2} \\ 1 \quad, & x = \frac{\pi}{2} \end{cases}$

at $x = \frac{\pi}{2}$.

Show that the function

$g(x) = \begin{cases} x\ sin\frac{1}{x}, & when\ x \neq 0 \\ 0 \quad, & when\ x = 0 \end{cases}$ is continuous everywhere.

2. Discuss the continuity and differentiablity of the function $f(x) = (2x-3)^{\frac{5}{2}}\ at\ x = \frac{3}{2}$.

Show that the function $f(x) = e^{-|x|}\ and\ g(x) = x + |x|$ are not differentiable at $x = 0$.

Verify the lagrange's mean value theorem for the function $f(x) = \sqrt{25 - x^2}$ in the interval [1,5].

3. Find the asymptotes of the curve $xy^2 - x^2y - 3x^2 - 2xy + y^2 + x - 2y + 1 = 0$.

Find the integration of the function $f(x) = \sqrt{x^2 + 4x - 5}$.

Derive the formula for the volume of a sphere of radius r.

4. Describe the graph of the equation $x^2 - 4y^2 + 2x + 8y - 7 = 0$.

Find the equation for the ellipse with foci $(0, \pm 6)$ and length of minor axis 16.

5. Trace the conic $16x^2 - 24xy + 9y^2 + 110x - 20y + 100 = 0$ by rotating the coordinate axes to remove the xy -term.

Let $\boldsymbol{r}(t) = 2t\,\boldsymbol{i} + 3t^2\boldsymbol{j} + t^3\boldsymbol{k}$. Find $\lim_{t\to 2} \boldsymbol{r}(t).\left(\boldsymbol{r}'(t) \times \boldsymbol{r}''(t)\right)$.

Find a vector of length $\sqrt{17}$ that makes an angle of $\pi/6$ with the positive x -axis.

6. Sketch the ellipsoid $x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$.

For $\boldsymbol{A} = 2x^2\boldsymbol{i} - 3yz\,\boldsymbol{j} + xz^2\boldsymbol{k}$ and $\varphi = 2z - x^3y$, find $\boldsymbol{A}.\boldsymbol{\nabla}\varphi$ and $\boldsymbol{A} \times \boldsymbol{\nabla}\varphi$ at the point (2, -1, 1).

If $\boldsymbol{A} = 2yz\,\boldsymbol{i} - x^2y\,\boldsymbol{j} + xz^2\boldsymbol{k}$ and $\varphi = 2x^2yz^3$, find $(\boldsymbol{A}.\boldsymbol{\nabla})\varphi$ and $\boldsymbol{A}.(\boldsymbol{\nabla}\varphi)$.

Unique Paper Code : 62354443_OC··

Name of the Paper : Analysis

Name of the Course : B.A.(Prog.)

Semester : IV

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

Attempt any four questions. All questions carry equal marks.

1. If a, b are real numbers, prove the following:

(i) If $a + b = 0$, then $b = -a$

(ii) $-(-a) = a$

(iii) $(-a).(-b) = a.b$

Which of the following sets are bounded below, which are bounded above and which are neither bounded below nor bounded above:

(i) $\{-1,\ -2,\ -3,\cdots -n,\cdots\}$

(ii) $\{-1,\ 2,\ -3,\ 4,\cdots (-1)^n n,\cdots\}$

(iii) $\{2,\ \frac{3}{2},\ \frac{4}{3},\cdots (\frac{n+1}{n})\}$

(iv) $\{3,\ 3^2,\ \ 3^3,\cdots 3^n,\cdots\}$

(v) $\{1,\ \frac{1}{2},\ \frac{1}{2^2},\cdots,\ \frac{1}{2^n},\cdots\}$

(vi) $\{-1,\ \frac{1}{2},\ -\frac{1}{3},\ \frac{1}{4},,\cdots,\ \frac{(-1)^n}{n},\cdots\}$.

Show that the set of natural numbers has no limit points.

2. Show that the function defined below is not continuous anywhere on the real line.

$$f(x) = \begin{cases} 1 & \text{if x is irrational} \\ -1 & \text{if x is rational} \end{cases}$$

Show that the function $(x) = \frac{1}{x}$, is uniformly continuous on the set $[a, \infty)$, where a is a positive constant.

Show that the set of rational numbers is neither open nor closed.

3. Use the definition of limit of a sequence to establish the following limits as n tends to ∞:

(i) $\lim \left(\frac{n}{n^2+1}\right) = 0$

(ii) $\lim \left(\frac{3n+1}{2n+5}\right) = \frac{3}{2}$

Discuss the convergence of the sequence $\left(\frac{11^{2n}}{7^{3n}}\right)$.

Show that the sequence (x_n) defined by

$$x_1 = a > 0,\ x_{n+1} = \frac{2\,x_n}{1+x_n}\ ,\ \ n > 1.$$

Is bounded and monotone. Also find the limit of the sequence.

4. Give an example of an unbounded sequence that has a convergent subsequence.

Show that the sequence (x_n) defined by

$$x_n = \frac{1}{9} + \frac{1}{13} + \frac{1}{17} \cdots + \frac{1}{4n+5}$$

is not Cauchy.

Calculate the value of $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^{2n}$

5. Discuss the convergence the series Σx_n where x_n is defined as $x_n = \sin \frac{n\pi}{2}$.

Test for convergence of the series $\sum \frac{(2n)!}{(n!)^2}$

Discuss the convergence of the series $\frac{log2}{2^2} - \frac{log3}{3^2} + \frac{log4}{4^2} - + \cdots$

6. Establish the convergence or the divergence of the series whose nth term is:

(i) $\frac{1}{(n+1)(n+2)}$ (ii) $\left(n(n+1)\right)^{-\frac{1}{2}}$ (iii) $\frac{2,\ 4,\ 6,\dots\ (2n)}{3\ .\ 5\ .\ 7\cdots\ (2n+1)}$

Let $f(x) = |x|$ for $-1 \leq x \leq 2$. Calculate $L(f;\ \mathcal{P})$ and $U(f;\ \mathcal{P})$ for the following partitions:

(i) $\mathcal{P} =: (-1,\ 0,\ 1,\ 2)$ (ii) $\mathcal{P} =: (-1, -\frac{1}{2},\ 0,\ \frac{1}{2},\ 1,\ \frac{3}{2},\ 2)$

Is the function Riemann integrable? Justify.

Name of Course	: **CBCS B.Sc. Mathematical Sciences**
Unique Paper Code	: **42354401_OC**
Name of Paper	: **Real Analysis**
Semester	: **IV**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. State and prove the condition for a sequence of real numbers to have a convergent subsequence. Is the converse true? Give example. Is every Cauchy sequence bounded? If yes, prove it.

2. Using the definition prove the convergence of the following sequences:

(i) $\lim \frac{\text{Cos } n\alpha}{n+1} = 0$ (ii) $\lim \frac{n^3-1}{3+2n^3} = \frac{1}{2}$ (iii) $\lim \frac{1}{(n+1)^2+1} = 0$

Further, find all $x \in \mathbb{R}$ that satisfy the inequality: $|x| + |x+1| < 2$

3. Check the convergence or divergence of the following series

(i) $\sum_{n=1}^{\infty} \sqrt{n^3+1} - \sqrt{n^3-1}$ (ii) $\sum_{n=1}^{\infty} \frac{7^{n+1}}{9^n}$

(iii) $\sum_{n=1}^{\infty} \frac{n^3}{3^n}$ (iv) $\frac{1}{e} + \frac{4}{e^2} + \frac{27}{e^3}$ ....

4. For each of the following, determine whether the series converges absolutely, converges conditionally, or diverges.

(i) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{4n+5}$ (ii) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n}{n+5}$`

(iii) $\sum_{n=1}^{\infty} \frac{\sin n\pi/2}{n^{5/2}}$ (iv) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n \log n}$.

5. Check pointwise and uniform convergence of the sequence $< f_n >$ defined as $f_n = nxe^{-nx^2}$ on [0,1] . Show that the series $\sum_{n=1}^{\infty} \frac{\sin(x^2+nx^2)}{n(n+1)}$ is uniformly convergent for all real values of x. Further find the radius of convergence and interval of convergence for the power series

(i) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)}$ (ii) $\sum_{n=0}^{\infty} \frac{n!\,(x+2)^n}{n^n}$.

6. Let f be a function on $[a, b]$ and P be a partition of $[a, b]$ and Q be its refinement, then show that

$$U[Q, f] - L[Q, f] \leq U[P, f] - L[P, f]$$

If f is defined on $[0,1]$ by $f(x) = 2x^2 \ \forall \ x \in [0,1]$, then prove that f is Reimann integrable. Also find the value of integral.

Further show that $\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, -1 \leq x \leq 1$ and hence deduce that $\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \ldots$

Name of Course	: **CBCS B.Sc. Mathematical Sciences**
Unique Paper Code	: **42353604**
Name of Paper	: **SEC-4: Transportation and Network Flow Problems**
Semester	: **VI**
Duration	: **3 hours**
Maximum Marks	: **55 Marks**

Attempt any four questions. All questions carry equal marks. All Symbols have usual meaning.

1. Consider the transportation model is given in the table. Use Vogel Approximation Method (VAM) to find the starting basic feasible solution. Hence find optimal solution by the method of multipliers.

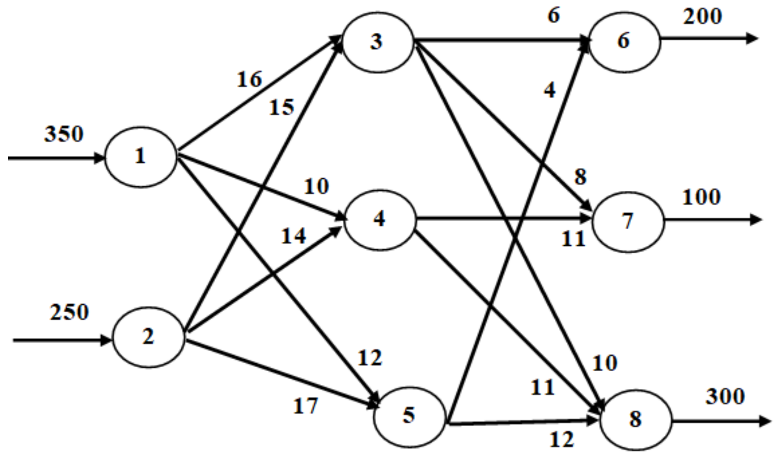
		Destinations						
		D_1	D_2	D_3	D_4	D_5	D_6	Availability
Sources	S_1	5	3	7	3	8	5	3
	S_2	5	6	12	5	7	11	4
	S_3	2	8	3	4	8	2	2
	S_4	9	6	10	5	10	9	8
	Requirement	3	3	6	2	1	2	

2. Consider the following cost matrix of assigning five jobs to four persons:

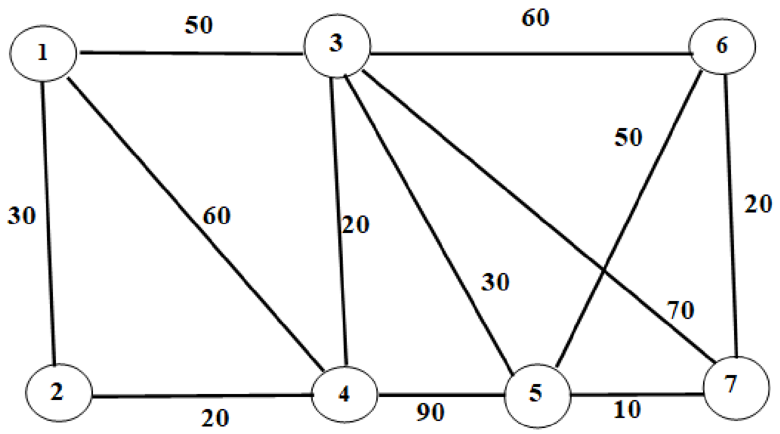
		Jobs				
		J_1	J_2	J_3	J_4	J_5
Persons	P_1	8	9	12	11	8
	P_2	4	3	6	7	5
	P_3	13	20	17	18	12
	P_4	23	26	25	33	20

Use Hungarian method to find an optimal assignment of the above problem.

3. Develop the transshipment model for the following network. Also identify pure supply nodes, pure demand nodes, transshipment nodes and the buffer amount.

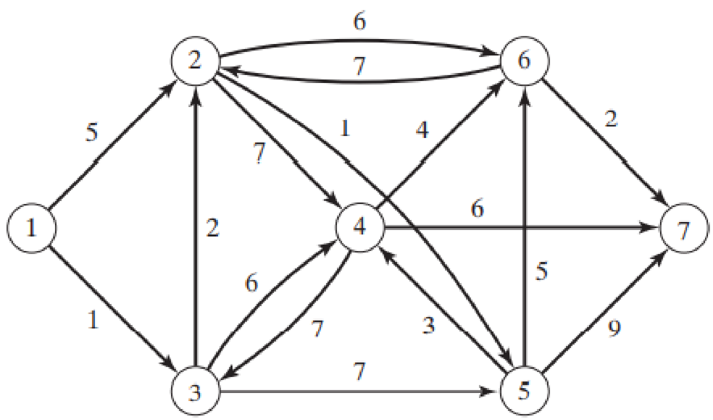


4. Consider the following network:

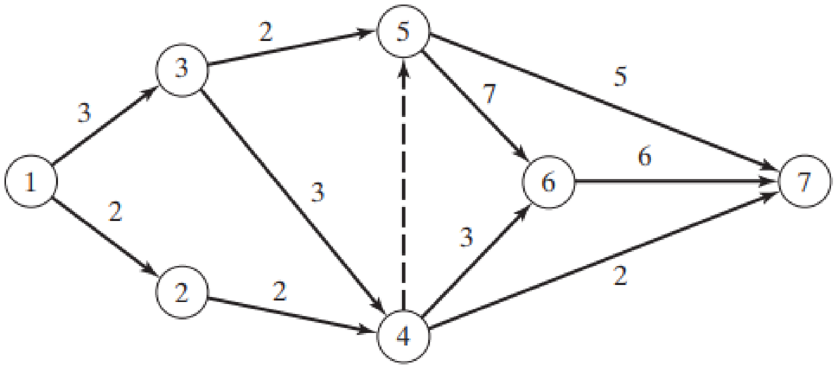


In the above network, Find 2 paths, 2 tree, a spanning tree and the minimal spanning tree.

5. Find the shortest route between node 1 to node 7 using Dijkstra's algorithm.



6. Determine the critical path for the project network:



Unique paper Code : 42343601
Name of the Course : B.Sc. (Prog.) Physical Science/ Mathematical Science (SEC)
Name of the Paper : Android Programming
Semester : VI
Year of Admission : 2015, 2016, 2017 & 2018
Maximum Marks : 75
Duration : Three hours

Instructions for candidates:
Attempt any four questions. All questions carry equal marks.

<table>
<tr><td>Q1.</td><td>Create an Android App with two edit boxes (to take two numbers from user) and four buttons. The details of four buttons are:
<table>
<tr><th>Label</th><th>On Click Functionality</th></tr>
<tr><td>1. Show Double</td><td>Double of the first number is shown in second edit box.</td></tr>
<tr><td>2. Show Half</td><td>Half of the second number is shown in first edit box.</td></tr>
<tr><td>3. Reset</td><td>Clears the contents of both the edit boxes</td></tr>
<tr><td>4. Close</td><td>A dialog box appears, asking user “Are you sure to close the app?” The app is closed only if user chooses yes on dialog box.</td></tr>
</table>
Write the XML and the Java code.</td></tr>
<tr><td>Q2.</td><td>How is Dynamic method dispatch useful for resolving call to Overridden methods in Java, explain with suitable example?
What are intents? What are they used for?
A main activity has an edit box and a button, when user clicks the button the contents of the edit box are sent to the second activity where the reverse of this text is shown. Both the activities have ‘Close’ button. Write the XML and the Java code for the same.</td></tr>
<tr><td>Q3.</td><td>What is Android Runtime?
How is multiple inheritance implemented in Java? How is it used in Android programming?
Write XML and Java code for a menu-driven application for changing the background color of the application. The menu should have 5 items corresponding to different color names.</td></tr>
</table>

<table>
<tr><td>Q4.</td><td>Is it necessary to include multiple radio buttons in a radio group? Justify your answer.
Differentiate between AndroidManifest.xml and activity_main.xml files.
Write the steps to deploy an application on USB-connected android device and steps to create Android Virtual Device.
Draw Android activity life cycle and give callback methods associated with all activities involved for the following user actions:
(i) When you move from Activity A to previous Activity B.
(ii) When you move from Activity B to some other Activity X in another application.
(iii) When you move from Activity B to Main Activity.</td></tr>
<tr><td>Q5.</td><td>Name the class which represents the SQLiteDatabase results in a table format. Discuss any 2 methods of this class.

Write an application to create a Database ABC_School with Table Student (StudentId INTEGER AUTOINCREMENT, StudentName TEXT, DepartmentId INTEGER) using SQLite Database. Insert the following 3 records into the table.
<table>
<tr><th>StudentName</th><th>DepartmentId</th></tr>
<tr><td>ABC</td><td>4</td></tr>
<tr><td>DEF</td><td>2</td></tr>
<tr><td>XYZ</td><td>6</td></tr>
</table>
Delete the row where DepartmentId =2.</td></tr>
<tr><td>Q6.</td><td>Write and explain the code to display text notification (toast) on screen with the message “OBE Examination Submitted Successfully!” for a long period of time.

What is the difference between Broadcast Receivers and Content Providers?

What are Linear, Table, Frame and Relative layouts in android? Also differentiate between List View and Grid View layouts.</td></tr>
</table>

Name of Course	: **CBCS B.Sc. Mathematical Sciences**
Unique Paper Code	: **42353404_OC**
Name of Paper	: **SEC-2 Computer Algebra Systems**
Semester	: **IV**
Duration	: **2 hours**
Maximum Marks	: **38 Marks**

Attempt any four questions. All questions carry equal marks

1. State Collatz conjecture. Find the value of **Collatz(**5). Write the command to define and integrate the function $f(x) = x^2 + 3cos\ x$. Write the command to extract the second element from the list $\{a, b, c, d\}$. Write the difference between = and ==. Write the command to compute

$$\sum_{i=1}^{n} (i+1)^2.$$

2. What is the use of **Exclusion, Mesh, AspectRatio,** options while plotting? Write the command to plot the function $f(x) = 2(x-1)^2 + 1$ in the range $[0,5]$, axes origin should be at (1,0), frame should be true and gridlines should be automatic. Explain the **Manipulate** command with example.

3. Write the commands to find the null space, eigen values, eigen vectors and rank of given matrix **A.** Also write commands to produce the following matrix using **ArrayFlatten**

$$\begin{pmatrix} 3 & 2 & 0 & 0 & 0 \\ 3 & 3 & 0 & 0 & 0 \\ 0 & 0 & 5 & 12 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -3 \end{pmatrix}.$$

4. Write the command to factor the expression $f(x) = 1 + 5x + 2x^3 + 10x^4$. Write the command to find the approximate numerical roots of $f(x)$. What is the output of

$\boldsymbol{Reduce}[(a^2 + 5a + 6)x == 2, x]$. Write the command to find a root of the equation $\sin x = 2 - x^2$ near $x = 1.2$. What is **Epilog, ReplaceAll** commands?

5. Define the function $f(x) = \frac{Sin\ (\pi(x-2))}{x-2}$. Write the command to plot the function into the range $[-3,3]$. Write the commands to find the first, second and n^{th}derivative of $f(x)$. Write the command to integrate the function $f(x)$ in the range $[-3,3]$.

6. Write the command to plot the surface $z = e^{x^2+y^2}$, for $-2 \leq x, y \leq 2$. Write all the commands to find the extrema of the function $f(x) = x^3 - 3x + 6$ on the interval $[-3,3]$ using second derivative test.

Name of Course	: **CBCS B.Sc. Mathematical Sciences**
Unique Paper Code	: **42353604**
Name of Paper	: **SEC-4: Transportation and Network Flow Problems**
Semester	: **VI**
Duration	: **3 hours**
Maximum Marks	: **55 Marks**

Attempt any four questions. All questions carry equal marks. All Symbols have usual meaning.

1. Consider the transportation model is given in the table. Use Vogel Approximation Method (VAM) to find the starting basic feasible solution. Hence find optimal solution by the method of multipliers.

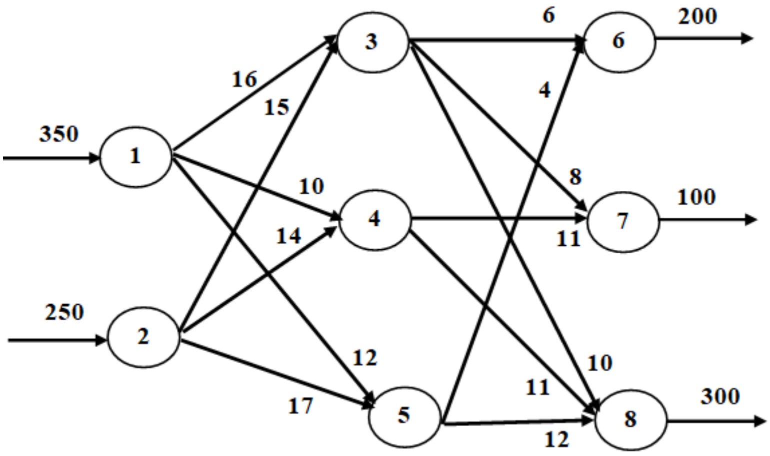
		Destinations						
		D_1	D_2	D_3	D_4	D_5	D_6	Availability
Sources	S_1	5	3	7	3	8	5	3
	S_2	5	6	12	5	7	11	4
	S_3	2	8	3	4	8	2	2
	S_4	9	6	10	5	10	9	8
	Requirement	3	3	6	2	1	2	

2. Consider the following cost matrix of assigning five jobs to four persons:

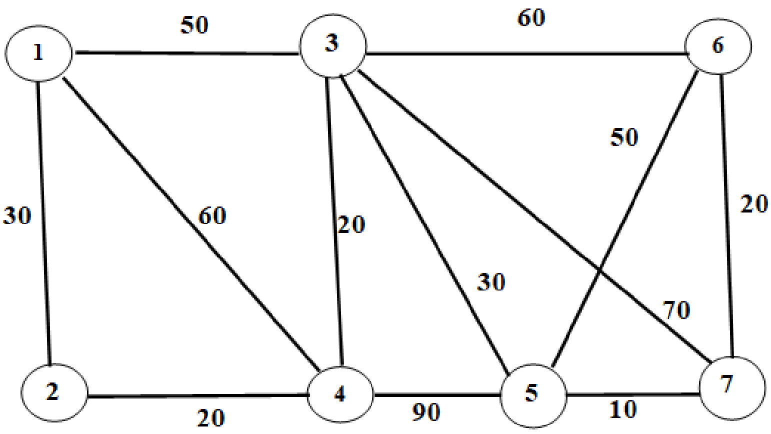
		Jobs				
		J_1	J_2	J_3	J_4	J_5
Persons	P_1	8	9	12	11	8
	P_2	4	3	6	7	5
	P_3	13	20	17	18	12
	P_4	23	26	25	33	20

Use Hungarian method to find an optimal assignment of the above problem.

3. Develop the transshipment model for the following network. Also identify pure supply nodes, pure demand nodes, transshipment nodes and the buffer amount.

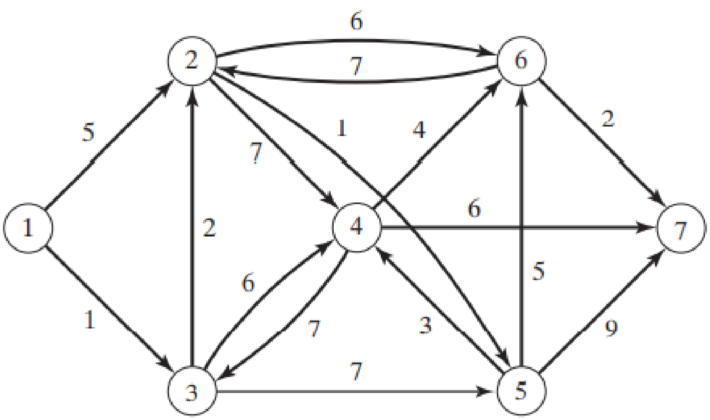


4. Consider the following network:

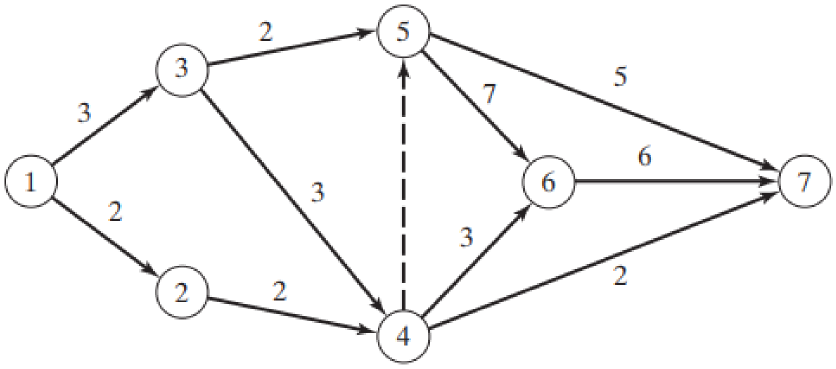


In the above network, Find 2 paths, 2 tree, a spanning tree and the minimal spanning tree.

5. Find the shortest route between node 1 to node 7 using Dijkstra's algorithm.



6. Determine the critical path for the project network:



Unique paper Code	: 42343601
Name of the Course	: B.Sc. (Prog.) Physical Science/ Mathematical Science (SEC)
Name of the Paper	: Android Programming
Semester	: VI
Year of Admission	: 2015, 2016, 2017 & 2018
Maximum Marks	: 75
Duration	: Three hours

Instructions for candidates:
Attempt any four questions. All questions carry equal marks.

<table>
<tr><td>Q1.</td><td>Create an Android App with two edit boxes (to take two numbers from user) and four buttons. The details of four buttons are:
<table>
<tr><td>Label</td><td>On Click Functionality</td></tr>
<tr><td>1. Show Double</td><td>Double of the first number is shown in second edit box.</td></tr>
<tr><td>2. Show Half</td><td>Half of the second number is shown in first edit box.</td></tr>
<tr><td>3. Reset</td><td>Clears the contents of both the edit boxes</td></tr>
<tr><td>4. Close</td><td>A dialog box appears, asking user “Are you sure to close the app?” The app is closed only if user chooses yes on dialog box.</td></tr>
</table>
Write the XML and the Java code.</td></tr>
<tr><td>Q2.</td><td>How is Dynamic method dispatch useful for resolving call to Overridden methods in Java, explain with suitable example?
What are intents? What are they used for?
A main activity has an edit box and a button, when user clicks the button the contents of the edit box are sent to the second activity where the reverse of this text is shown. Both the activities have ‘Close’ button. Write the XML and the Java code for the same.</td></tr>
<tr><td>Q3.</td><td>What is Android Runtime?
How is multiple inheritance implemented in Java? How is it used in Android programming?
Write XML and Java code for a menu-driven application for changing the background color of the application. The menu should have 5 items corresponding to different color names.</td></tr>
</table>

Q4.

Is it necessary to include multiple radio buttons in a radio group? Justify your answer.

Differentiate between **AndroidManifest.xml** and **activity_main.xml** files.

Write the steps to deploy an application on USB-connected android device and steps to create Android Virtual Device.

Draw Android activity life cycle and give callback methods associated with all activities involved for the following user actions:

(i) When you move from Activity **A** to previous Activity **B**.

(ii) When you move from Activity **B** to some other Activity **X** in another application.

(iii) When you move from Activity **B** to Main Activity.

Q5.

Name the class which represents the SQLiteDatabase results in a table format. Discuss any 2 methods of this class.

Write an application to create a Database *ABC_School* with Table Student (*StudentId* INTEGER AUTOINCREMENT, *StudentName* TEXT, *DepartmentId* INTEGER) using SQLite Database. Insert the following 3 records into the table.

StudentName	*DepartmentId*
ABC	4
DEF	2
XYZ	6

Delete the row where *DepartmentId* =2.

Q6.

Write and explain the code to display text notification (toast) on screen with the message “OBE Examination Submitted Successfully!” for a long period of time.

What is the difference between **Broadcast Receivers** and **Content Providers**?

What are Linear, Table, Frame and Relative layouts in android? Also differentiate between List View and Grid View layouts.

Name of Course	: **CBCS B.A. (Prog.)**
Unique Paper Code	: **62355604**
Name of Paper	: **GE-2: General Mathematics-II**
Semester	: **VI**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Attempt all parts.
   (i) What was the first scientific achievement of Henri Poincare?
   (ii) Discuss the life of Cantor as a teacher and researcher.
   (iii) What do you know about Ramanujan's lost notebook?
   (iv) Name any four honours/ awards received by Neumann.

2. State which of the following function(s) is/are even, odd or neither:

$$\sin(|x|), x - \cos(x), x^3 + 2x^2 + x + 1, x|x|, x^2 + \cos(x), x + \tan(x).$$

   What are Fractals? Give two examples and discuss its importance in nature.

   Let $f(x) = \sin(|x|)$ be a real valued function. Then
   (i) Find a set of extreme points of $f(x)$.
   (ii) Find an interval on which $f(x)$ increases.
   (iii) Find an interval on which $f(x)$ decreases.

3. If $\text{cosec}(x) = \frac{-1}{5}$, then determine the values of all other basic trigonometric functions in third and fourth quadrant.

   Is it true that extremum points and inflection points are same? Justify your answer.

   Explain platonic solids through diagrams and illustrate how they are related with four basic elements (fire, earth, air and water)?

4. Use the Gauss Elimination method to solve each of the following systems of the linear equations. In each case, indicate whether the system is consistent or inconsistent. Give the complete solution set, and if the solution set is infinite, specify three particular solutions.

(i)
$$\begin{aligned} 3x_1 - 3x_2 - 2x_3 &= 23 \\ -6x_1 + 4x_2 + 3x_3 &= -38 \\ -2x_1 + x_2 + x_3 &= -11 \end{aligned}$$

(ii)
$$\begin{aligned} 5x_1 - 5x_2 - 15x_3 - 3x_4 &= -34 \\ -2x_1 + 2x_2 + 6x_3 + x_4 &= 12 \end{aligned}$$

5. If $A = \begin{bmatrix} -5 & -2 & 2 \\ 3 & 1 & -1 \\ 2 & 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 & 4 \\ -1 & 1 & -2 \\ 5 & -4 & 8 \end{bmatrix}$, then find the rank of AB.

Without row reduction, find the inverse of the coefficient matrix and hence find the solution of the following system of equations

$$\begin{aligned} x + 4y &= -3 \\ -2x + 3y &= -6 \end{aligned}$$

Illustrate, using diagrams, that a connected network can be drawn without lifting the pen and without repeating the lines only if it has at most two vertices with odd valence. Draw an Euler path and count the valences of each vertices.

6. Express the vector $x = [2,2,3]$ as a linear combination of the vectors

$a_1 = [6,-2,3], a_2 = [0,-5,-1], a_3 = [-2,1,2]$, if possible.

Find the reduced row echelon form of A^2 , where $A = \begin{bmatrix} -5 & 3 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$.

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Name of Course	: **B.A. (Prog.)**
Unique Paper Code	: **62357602**
Name of Paper	: **DSE: Numerical Analysis**
Semester	: **VI**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Find an interval of unit length which contains the smallest positive root of the equation $f(x) = x^4 - x - 10 = 0$. Taking the end points of this interval as initial approximations, do two iterations each of the Secant method and Regula Falsi Method.

2. Find the number of significant digits for the following:

2410, 2.41,0.00241, 2.410$\times 10^4$, 2.4100$\times 10^4$, 2.41000$\times 10^6$.

If $x = 0.278143 \times 10^4$ and $y = 0.278456 \times 10^4$, find the number of significant digits in $x + y, x - y, xy$.

3. Given the following system of equations

$$x_1 + x_2 + x_3 = 1$$
$$4x_1 + 3x_2 - x_3 = 6$$
$$3x_1 + 5x_2 + 3x_3 = 4$$

(i) Find the solution using the Gauss- Jordan method.

(ii) Perform two iterations of the Gauss-Seidel method starting with $X^{(0)} = (1, 1, 1)$.

4. Generate the forward and backward difference table for the data

x	0	0.2	0.4	0.6	0.8
f(x)	0.12	0.46	0.74	0.9	1.2

Hence interpolate the values of $f(0.1)$ and $f(0.7)$ by using Gregory Newton forward and backward differences Interpolation formulae respectively.

5. Approximate the second order derivative of $f(x) = e^x$ at $x_0 = 0$, taking $h = 1, 0.1, 0.01$ by using the formula

$$f"(x) \approx \frac{f(x_0 - h) - 2f(x_0) + f(x_0 + h)}{h^2}$$

Also approximate the derivative of $f(x) = 1 + x + x^2 + x^3$ at $x_0 = 0$, taking $h = 1,\ 0.1,\ and\ 0.01$ by using the formula

$$f'(x_0) \approx \frac{f(x_0 + h) - f(x_0)}{h}$$

Find the order of approximations in both the cases.

6. Find the approximate value of $I = \int_0^1 \frac{dx}{1+x^3}$ using the Trapezoidal rule with 2, 4 and 8 equal subintervals. Improve the result by Romberg integration.

Name of the Course **: CBCS B.Sc. Mathematical Sciences / B.Sc. (Prog.)**

Unique Paper Code **: 42357602**

Name of the Paper **: DSE- Probability and Statistics**

Semester **: VI**

Duration **: 3 Hours**

Maximum Marks **: 75**

Attempt any four questions. All questions carry equal marks.

1. Let the random variable X be defined as the sum of faces in throwing two unbiased dice. Write the probability distribution of X. Find the cumulative distribution function of X. Also find mean deviation about mean and variance. Further find the probability that the sum is
(i) greater than 9 (ii) neither 5 nor 7.

   Let X have the probability density function

$$f(x) = \begin{cases} \frac{3x^2}{8} & 0 < x < 2 \\ 0 & \text{elsewhere.} \end{cases}$$

   Find the distribution function of X and the probability density function of $Y = X^3$.

2. Let

$$f(x) = \begin{cases} ke^{-3x} & \text{for } 0 < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

   be the probability density function of a random variable X. Evaluate k. Find 25^{th} percentile, first four moments, mode and the moment generating function of X.

3. Suppose that the CGPA scores of a large population of first-year college students are approximately normally distributed with mean 2.5 and standard deviation 0.7. What fraction of the students will have a CGPA more than 3.0?
If students possessing a CGPA score less than 1.8 are not promoted to next year, what percentage of the students will not be promoted to next year?
Suppose that three students are randomly selected from the first-year student body. What is the probability that all three will possess a CGPA score more than 3.0?

   Let X be a geometric random variable with parameter $p = 0.4$ and let $Y = 2X$-1. Find $E(Y), Var(Y)$ and the moment generating function of Y.

4. Let X and Y be the random variables with the joint probability density function

$$f(x,y) = \begin{cases} e^{-(x+y)} & x > 0, y > 0 \\ 0 & \text{elsewhere.} \end{cases}$$

i) Evaluate $P(X < 1, Y > 5)$

ii) Evaluate $P(X + Y < 3)$

iii) Find the marginal densities of X and Y.

iv) For any $y > 0$, find the conditional density function of X given that $Y = y$.

v) Evaluate $E(X - Y)$ and $Var(X - Y)$.

5. Let the random variables X and Y have the joint probability density function

$$f(x,y) = \begin{cases} 3x & 0 < y < x < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Are X and Y independent? If not, then find $\mu_{Y|x}$ and $\mu_{X|y}$. Also, compute the correlation coefficient between X and Y.

6. Let X be a random variable such that $P(X \leq 0) = 0$ and let $\mu = E(X)$ exist. Show that $P(X \geq 2\mu) \leq \frac{1}{2}$.

Calculate the rank correlation coefficient r_S for the following data representing the statistics grades, x, and psychology grades, y, of 18 students:

x: 78, 86, 49, 94, 53, 89, 94, 71, 70, 97, 74, 53, 58, 62, 74, 74, 70, 74

y: 80, 74, 63, 85, 55, 86, 90, 84, 71, 90, 85, 71, 67, 64, 69, 71, 67, 71

Name of the Course	**: CBCS B.Sc. Mathematical Sciences / B.Sc. (Prog.)**
Unique Paper Code	**: 42357618**
Name of the Paper	**: DSE- Numerical Methods**
Semester	**: VI**
Duration	**: 3 Hours**
Maximum Marks	**: 75 Marks**

Attempt any four questions. All questions carry equal marks. All symbols have usual meanings.

1. Let $f(x) = x^4 - 18x^2 + 45$ and $g(x) = x^3 + x^2 - 3x - 3$.
   - (i) Verify that both the equations $f(x) = 0$ and $g(x) = 0$ have a root on the interval (1, 2).
   - (ii) By performing three iterations of Newton-Raphson method, with $x_0 = 1$, find an approximation of the root of $f(x) = 0$.
   - (iii) By performing three iterations of Bisection method, find an approximation of the root of $g(x) = 0$.
   - (iv) Given that the exact value of the root in both cases is $x = \sqrt{3}$, compute the absolute error in the approximations obtained.

2. Find the inverse of A, using Gauss-Jordan elimination

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 7 & 13 \\ 3 & 11 & 22 \end{bmatrix}.$$

   Using Gauss-Seidel iteration method, solve the system of equations given by

$$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1.1 \\ 1.2 \\ 1.3 \end{bmatrix}.$$

3. Use the Lagrange's interpolation to find a polynomial that passes through the points $(0, 2), (1, 3)$, $(2, 12)$ and $(5, 147)$.

   Using the following data

x	1.0	1.5	2.0	2.5
$f(x)$	2.7183	4.4817	7.3891	12.1825

   estimate the value of $f(2.15)$ using
   - (i) Newton's forward difference interpolation
   - (ii) Newton's backward difference interpolation.

Compare the errors and find which of the methods gave a better approximation of $f(2.15)$.

4. The following data gives the velocity of a particle for 20 seconds at an interval of 5 seconds. Find the initial acceleration using the entire data:

Time t (sec)	0	5	10	15	20
Velocity v(m/sec)	0	2	13	68	227

A boundary value problem is defined by

$$\frac{d^2y}{dx^2} + y + 1 = 0, \quad 0 \le x \le 1$$

where $y(0) = 0$ and $y(1) = 0$ with $h = 0.5$. Use the finite difference method to determine the value of $y(0.5)$. Its exact solution is

$$y(x) = \cos x + \frac{1 - \cos 1}{\sin 1} \sin x - 1.$$

Calculate the error.

5. Use the formula

$$f'(x_0) \approx \frac{f(x_0) - f(x_0 - h)}{h}$$

to approximate the derivative of $f(x) = \sin x$ at $x_0 = \pi$ taking $h = 1, 0.1, 0.01$.

Use Euler's method to approximate the solution of the initial value problem

$$\frac{dy}{dt} = \frac{(1+y)^2}{t}, \qquad y(1) = 0, 1 \le t \le 4$$

taking 5 steps.

6. Find approximate value of the integral $I = \int_0^2 e^x dx$ using
   (i) Trapezoidal Rule;
   (ii) Simpson's 1/3 rule;
   (iii) Simpson's 3/8 rule.

Name of the course : **CBCS B.Sc(H)Mathematics**

Unique Paper Code : **32357609**

Name of Paper : **DSE-3 : Bio-Mathematics**

Semester : **VI**

Duration : **3 hours**

Maximum Marks : **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Describe the mathematical model of population growth for any species changing by birth only. Draw and describe the graph of the behavior of the population with an increase in time. What would be the change in your model if you consider deaths of individuals also along with births. In a research on population dynamics of mosquitoes, it was estimated that the initial population is 2000. Over the time period of one month, 300 births and 100 deaths were recorded in the population. Predict the population size at the end of 10 months.
2. Find and draw the trajectories in the phase plane of
   a) $\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = 0.$
   b) $\frac{dx}{dt} = 5x + 2y$; $\frac{dy}{dt} = -2x + 5y$.

3. What is the threshold effect in a heartbeat model? Why is it an important feature to be included in the model? Modify the following model for heartbeat to reflect the threshold effect.

$$\epsilon\frac{dx}{dt} = -a\,(x - x_0) - (b - b_0)$$
$$\frac{db}{dt} = x - x_0$$

   where $(b_0, x_0)$is the unique rest state. Present a phase plane analysis of the model obtained above and explain how it includes the physiological considerations of the heartbeat cycle?

4. Define Hopf bifurcation. Show that the system

$$x' = -y + x(\beta - x^2 - y^2)$$

$$y' = x + y(\beta - x^2 - y^2)$$

   has Hopf bifurcation. Consider the iteration scheme for points on a Poincare Plane:

$$x_{n+1} = \frac{1}{2}y_n$$

$$y_{n+1} = -x_n + \frac{1}{2}\alpha y_n - y_n^3$$

Show that there is a bifurcation at $\alpha = 3$. Show that the limit cycle in $0 < \alpha < 3$is stable and becomes of saddle type when αexceeds 3.

5. Consider ancestral and descendent sequences of 400 bases which were simulated according to Kimura 2-parameter model with $\gamma = \frac{\beta}{5}$.A comparison of aligned sites gave the frequency data in Table-1 below:

Table -1

S_1/S_0	A	G	C	T
A	92	15	2	2
G	13	84	4	4
C	0	1	77	16
T	4	2	14	70

Compute the Jukes-Cantor distance and Kimura 2-parameter distance to 10 decimal digits.

In what cases Jukes-Cantor distance is zero? Derive the formula $d_{JC} = -\frac{3}{4}ln(\frac{4q-1}{3})$ where q is the proportion of bases that are the same in the before and after sequences.

6. Draw the single topologically distinct unrooted bifurcating tree that could describe the relationship between 3 taxa. Draw Punnett square for $Dd\ X\ Dd$ and $DdWw\ X\ ddWw$, where D denotes allele for dominant plant, d allele for dwarf plant, W dominant allele for round seeds and w recessive allele for wrinkled seeds. What is the genotypic ratio and phenotypic ratio in $Dd\ X\ Dd$? What percentage is the progeny of $DdWw\ X\ ddWw$ dwarf with round seeds? Now consider the distance data in the following Table-2, which is exactly fit by the following tree, use UPGMA to construct a tree from this data. Also use Neighbor-Joining method to compute R_1 , R_2 , R_3 and R_4 , and then a table of values for M for the taxa $S1$, $S2$, $S3$ and $S4$.

Table-2

	$S1$	$S2$	$S3$	$S4$
$S1$		0.3	0.4	0.5
$S2$			0.5	0.4
$S3$				0.7