[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1112 **G** **Your Roll No................**

Unique Paper Code : 235103

Name of the Paper : Analysis – I (MAHT-102)

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each questions.

1. (a) State and prove the Triangle Inequality and show that

$$||a|-|b|| \le |a-b| \quad \forall a, b \in R.$$ (5)

(b) Given $S = \left\{1 - \frac{1}{n} : n \in N\right\}$, show that Sup(S) = 1. (5)

(c) Let $a > 0$ and aS = [as: s ∈ S], Show that Sup(aS) = a Sap(S). (5)

2. (a) Show that arbitrary, intersection of a family of closed sets is a closed set. Is this result true for an arbitrary family of open sets ? Justify your answer. (5)

(b) Define Limit Point of a set. Show that the set of limit points of the set of rational numbers is R, the set of real numbers. (5)

(c) For non-empty bounded subsets A and B of R, show that

$$\inf(A + B) = \inf(A) + \inf(B).$$ (5)

P.T.O.

3. (a) (i) Show that every convergent sequence is bounded but the converse is not true. (5)

(ii) Use the definition of the limit of a sequence to find the following limit :

$$\lim_{n\to\infty}\left(\frac{1}{n^2+1}\right).$$ (2½)

(b) (i) If (x_n) converges to x and (y_n) converges to y then show that $(x_n + y_n)$ converges to (x+y). (5)

(ii) Give an example of two divergent sequences (x_n) and (y_n) such that $(x_n + y_n)$ converges. (2½)

(c) (i) Show that $\lim_{n\to\infty} n^{1/n} = 1$. (5)

(ii) Find the following limit :

$$\lim_{x\to\infty}\frac{n}{b^n}, \quad b > 1$$ (2½)

4. (a) State and prove Cauchy Convergence criterion for sequence of real numbers. (5)

(b) Prove that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$. (5)

(c) Prove that the following sequence is a Cauchy sequence :

$$\left(1+\frac{1}{2!}+\cdots\frac{1}{n!}\right)$$ (5)

5. (a) (i) Find the sets of subsequential limits for the following sequences :

$(a_n) = (0,1,2,1,0,1,2,1,0,1,2,1,0,1,2,1,0 \ldots)$

$(b_n) = (2,1,1,0,2,1,1,0,2,1,0,1,2,1,1,0,2 \ldots)$. (2½)

(ii) Find the Limit inferior and Limit superior for the above defined sequences (a_n) and (b_n). (2½)

(b) Show that the following sequences are divergent :

(i) $a_n = (-1)^n$

(ii) $b_n = \sin\left(\frac{n\pi}{2}\right)$ (5)

(c) Let $x_1 = 8$, $x_{n+1} = \frac{x_n}{2} + 2$. Show that (x_n) is bounded and monotone. Also find its limit. (5)

6. (a) Give examples of the following series with justification :

(i) A divergent series $\sum a_n$ for which $\sum a_n^2$ converges.

(ii) A convergent series $\sum a_n$ for which $\sum a_n^2$ diverges. (5)

(b) Test the convergence of any two of the following series :

(i) $\sum \frac{\cos(n)+1}{3^n}$

(ii) $\sum \frac{1}{\sqrt{n+1}}$

(iii) $\sum \sqrt{n+1} - \sqrt{n}$ (5)

(c) State Ratio test for infinite series and show that $\sum \frac{n!}{3^n}$ diverges. (5)

7. (a) State Integral Test for series of real numbers and show that the series $\sum \frac{1}{n^p}$ is convergent if and only if $p > 1$. (5)

(b) Show that every absolutely convergent series is convergent but the converse is not true. (5)

(c) Examine the convergence of any two of the following series :

(i) $\sum \frac{(-1)^n}{n!}$

(ii) $\sum \frac{(-1)^{n+1} - (-1)^n}{n^2+1}$

(iii) $\sum \frac{(-1)^n}{2n+1}$ (5)

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1113 **G** **Your Roll No...............**

Unique Paper Code : 235104

Name of the Paper : Algebra – I

Name of the Course : **B.Sc. (Hons.) Mathematics credit Course -I**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Do any **two** parts from each questions.

1. (a) Find a cubic equation whose roots are the squares of the roots of the equation

$$x^3 - x^2 + 3x - 10 = 0.$$ (6)

(b) State Descartes' rule of signs. Using this rule verify that (6)

$$t^{11} + t^8 - 3t^5 + t^4 + t^3 - 2t^2 + t - 2$$

has at most 5 positive and 2 negative zeros. Deduce that it has at least 4 non-real zeros.

(i) Consider the polynomial equation $x^4 + px^3 + qx^2 + rx + s = 0$.

Prove that if the product of two of its roots is equal to the product of the other two, then $r^2 = p^2s$. (4)

(ii) Let z_1, z_2, z_3 be non-zero complex coordinates of the vertices of the triangle $A_1A_2A_3$. If $z_1^2 = z_2z_3$ and $z_2^2 = z_1z_3$, show that triangle $A_1A_2A_3$ is equilateral. (2)

2. (a) (i) Find the polar representation of the number $z = 2 + 2i$. (2)

(ii) Prove that (4½)

$\sin 5t = 16 \sin^5 t - 20 \sin^3 t + 5 \sin t$;

$\cos 5t = 16 \cos^5 t - 20 \cos^3 t + 5 \cos t$.

(b) Solve the equation : (6½)

$$z^7 - 2iz^4 - iz^3 - 2 = 0.$$

(c) On the sides AB, BC, CD, DA of quadrilateral ABCD and exterior to the quadrilateral, we construct squares of centers O_1, O_2, O_3 and O_4 respectively. Prove that O_1O_3 is perpendicular to O_2O_4 and $O_1O_3 = O_2O_4$. (6½)

3. (a) For $a, b \in N$, define $a \sim b$ if and only if $a^2 + b$ is even.

(i) Prove that '~' defines an equivalence relation on N.

(ii) What are the equivalence classes of 0 and 1 ?

(iii) Find the quotient set determined by this equivalence relation ? (5)

(b) Let '~' denote an equivalence relation on a set A, let $a \in A$, then for any $x \in A$, prove that $x \sim a$ if and only if $\bar{x} = \bar{a}$. (5)

(c) Let $n > 1$ be a fixed natural number, prove that congruence mod n is an equivalence relation on Z. (5)

4. (a) Define $f: Z \to Z$ by $f(x) = 2x^2 + 7x$. Determine whether f is one-to-one and\or onto. (5)

(b) Define Countable set. Show that intervals (0,1) and (3,5) have the same cardinality. (5)

(c) Prove, using Principle of Mathematical Induction that every integer greater than 1 is a prime or a product of primes. (5)

5. (a) Write the following system as a vector equation and as a matrix equation. Row reduce the augmented matrix into reduced echelon form. Describe the general solution in parametric form.

$$x_1 + 3x_2 + x_3 = 1$$

$$-4x_1 - 9x_2 + 2x_3 = -1$$

$$-3x_2 - 6x_3 = -3 \qquad (7½)$$

(b) Boron sulphide reacts violently with water to form boric acid and hydrogen sulphide gas (the smell of rotten egg). The unbalanced equation is

$$B_2S_3 + H_2O \rightarrow H_3BO_3 + H_3S$$

For each compound, construct a vector that lists the numbers of atoms of boron, sulfur, hydrogen, and oxygen. Balance the chemical equation using vector equation approach. (7½)

(c) (i) Find the value(s) of h for which the vectors are linearly dependent. Justify your answer.

$$\begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ 7 \\ -3 \end{bmatrix}, \begin{bmatrix} 8 \\ h \\ 4 \end{bmatrix}$$

(ii) Give a geometric description of Span $\{v_1, v_2\}$ for the vectors

$$v_1 = \begin{bmatrix} 8 \\ 2 \\ -6 \end{bmatrix} \text{ and } v_2 = \begin{bmatrix} 12 \\ 3 \\ -9 \end{bmatrix}$$ (4½,3)

6. (a) (i) Let $T: R^2 \rightarrow R^2$ be a linear transformation, $T(e_1) = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ and $T(e_2) = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$. Find the images of $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, where $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

(ii) Show that the transformation T defined by $T(x_1, x_2) = (2x_1 - 3x_2, x_1 + 4, 5x_2)$ is not linear. (4½,3)

(b) (i) Let $T: R^2 \to R^2$ be a linear transformation which rotates points (about the origin) through $-\pi/4$ radians clockwise. Find the standard matrix of T.

(ii) The vector x is in a subspace H with a basis $B = \{b_1, b_2\}$. Find $[x]_B$, the B-coordinate vector of x where

$$b_1 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}, \; b_2 = \begin{bmatrix} -2 \\ 7 \end{bmatrix}, \; x = \begin{bmatrix} -3 \\ 7 \end{bmatrix}$$ (4½,3)

(c) Find bases for Col A and Nul A. Hence find rank of A.

$$A = \begin{bmatrix} 1 & -3 & 2 & -4 \\ -3 & 9 & -1 & 5 \\ 2 & -6 & 4 & -3 \\ -4 & 12 & 2 & 7 \end{bmatrix}$$ (7½)

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1788 **GC-3** **Your Roll No...............**

Unique Paper Code : 32351101

Name of the Paper : C 1 – Calculus

Name of the Course : **B.Sc. (Hons.) / Maths – I (CBCS)**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** the sections are compulsory.
3. **All** questions carry equal marks.
4. Use of non-programmable scientific calculator is allowed.

SECTION – I

Attempt any **four** *questions from Section I.*

1. If $\cos^{-1}\frac{y}{b} = \log\left(\frac{x}{n}\right)^n$ then show that

$$x^2y_{n+2} + (2n+1)xy_{n+1} + 2n^2y_n = 0$$

2. Sketch the graph of

$f(x) = \frac{1}{3}x^3 - 9x + 2$ by finding intervals of increase and decrease, critical points, relative extrema and concavity for the given function.

P.T.O.

3. Find the horizontal asymptote to the graph of the function

$$f(x) = x^5\left[\sin\frac{1}{x} - \frac{1}{x} + \frac{1}{6x^3}\right]$$

4. It is projected that t years from now, the population of a certain country will be

$$P(t) = 50\,e^{0.02t} \text{ million}$$

(a) At what rate will the population be changing with respect to time 10 years from now.

(b) At what percentage rate will the population be changing with respect to time t years from now.

5. Sketch the graph of the curve in polar coordinates

$r^2 = 9\cos 2\theta$.

SECTION – II

Attempt any **four** *questions from Section – II.*

6. Find the reduction formula for $\int \sin^n x dx$ where n being positive integer and hence evaluate $\int_0^{\frac{\pi}{2}} \sin^n x dx$.

Further show that $\int_0^{\frac{\pi}{2}} \sin^n x dx = \int_0^{\frac{\pi}{2}} \cos^n x dx$.

7. Find the volume of the solid generated when the region enclosed by the curve $y = \sqrt{x}$, $y = 6 - x$ and $y = 0$ is revolved about x-axis.

8. Find the volume of the solid generated when the region enclosed by the curve $x = 2y - 2y^2$ and $x = 0$ is revolved about x-axis.

9. Find the arc length of the parametric curve $x = e^t \sin t$, $y = e^t \cos t$ for $0 \le t \le \frac{\pi}{2}$.

10. Find the area of the surface generated by revolving the curve $x = \sqrt{9 - y^2}$, $-2 \le y \le 2$, about y-axis.

SECTION – III

Attempt any **three** *questions from Section – III.*

11. Find the equation for a hyperbola passing through the origin with asymptotes

$y = 2x + 1$ and $y = -2x + 3$.

12. Find the equation of the ellipse whose foci are (1, 2) and (1, 4) and whose minor axis is of length 2.

13. Describe the graph of the equation $x^2 - 4y^2 + 2x + 8y - 7 = 0$.

14. Trace the conic $x^2 + 2\sqrt{3}xy + 3y^2 + 2\sqrt{3}x - 2y = 0$ by rotating the coordinate axes to remove the xy term.

SECTION – IV

Attempt any **four** *questions from Section – IV.*

15. Find tangent vector and parametric equation of tangent line to the graph of the vector function

$$\vec{F}(t) = t^2\hat{i} + (\cos t)\hat{j} + (t^2 \cos t)\hat{k} \quad \text{at} \quad t = \frac{\pi}{2}.$$

16. A shell is fired with muzzle speed 150 m/s and angle of elevation 45° from a position 10 m above ground level. Where does the projectile hit the ground and with what speed ?

17. Find the tangential and normal components of acceleration of an object that moves along the parabolic path $y = 4x^2$ at the instant the speed is $\frac{ds}{dt} = 20$.

18. An object moves along the curve

$$r = \frac{1}{1-\cos\theta} \text{ and } \theta = t$$

Find its velocity and acceleration in terms of unit polar vectors u_r and u_θ.

19. Find the curvature and radius of curvature for a curve

$x = 3\cos t,\ y = 4\sin t,\ z = t$ at $t = \frac{\pi}{2}$.

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1789 **FC-3** **Your Roll No...............**

Unique Paper Code : 32351102

Name of the Paper : C2 – Algebra

Name of the Course : **B.Sc. (H) Mathematics – I (CBCS)**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All **six** questions are compulsory.
3. Do any **two** parts from each question.

1. (a) Find the polar representation of the complex number (6)

$z = 1 + \cos\alpha + \sin\alpha, \alpha \in (0, 2\pi)$

(b) Compute (6)

$$z = \frac{\left((1-i)^{10}\left(\sqrt{3}+i\right)^{5}\right)}{\left(-1-i\sqrt{3}\right)^{10}}$$

(c) Find the three roots of unity of the complex number z = 1 + i and represent them in the complex plane. (6)

2. (a) For a, b ∈ Z/{0} define a ~ b if and only if ab > 0. (6)

(i) Prove that ~ defines an equivalence relation on Z.

(ii) What is the equivalence class of 5 ? What is the equivalence class of -5 ?

(b) Find the gcd (1800, 756). (6)

(c) Define $S : \mathbb{IR} \dashrightarrow \mathbb{IR}$ by $S(x) = x - \lfloor x \rfloor$. Is S is one to one ? Is it onto ? Explain. (6)

3. (a) Given natural numbers a and b, show that there are unique non – negative integers q and r with $0 \le r < b$ such that $a = bq + r$. (6)

(b) Show that the open intervals $(1,3)$ and $(0,\infty)$ have the same cardinality. (6)

(c) If $ac \equiv bc \pmod m$ and $(c, m) = 1$ then $a \equiv b \pmod m$. (6)

4. (a) Determine the values of h and k such that the system

$$x_1 + hx_2 = 2$$

$$4x_1 + 8x_2 = k$$

has (i) no solution (ii) a unique solution (iii) many solutions (6½)

(b) Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$, $v_2 = \begin{bmatrix} -3 \\ 1 \\ 8 \end{bmatrix}$ and $y = \begin{bmatrix} h \\ -5 \\ -3 \end{bmatrix}$.

For what values(s) of h is y in the plane generated by v_1 and v_2. (6½)

(c) Balance the given chemical equation where Boron Sulphide reacts violently with water to form boric acid and hydrogen sulphide gas. The unbalanced equation

$$B_2S_3 + H_2O \rightarrow H_3BO_3 + H_2S$$

Here, for each compound, construct a vector that lists the number of atoms of boron sulphur, hydrogen and oxygen. (6½)

5. (a) Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^4$ be defined as

$$T(x_1, x_2) = (2x_2 - 3x_1, x_1 - 4x_2, 0, x_2).$$

(i) Prove that T is a linear transformation.

(ii) Find the standard matrix of T. (6½)

(b) Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation and let A be the standard matrix for T. Then prove that

(i) T maps $\mathbb{R}^n$ onto $\mathbb{R}^m$ if and only if columns of A spans $\mathbb{R}^m$.

(ii) T is one to one if and only if columns of A are linearly independent. (6½)

(c) Find the basis for the column space and null space of the matrix

$$A = \begin{bmatrix} 4 & 5 & 9 & -2 \\ 6 & 5 & 1 & 12 \\ 3 & 4 & 8 & -3 \end{bmatrix}$$ (6½)

6. (a) (i) Define a subspace H of $\mathbb{R}^n$ and its dimension too.

Is $H = \{(a, b, c, d) \mid c = a + 2b + 3d\}$ a subspace of $\mathbb{R}^4$. Justify your answer. (6½)

(b) Determine the dimension of the subspace H of $\mathbb{R}^3$ spanned by the vectors

$$v_1 = \begin{bmatrix} 2 \\ -8 \\ 6 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 3 \\ -7 \\ -1 \end{bmatrix} \text{ and } v_3 = \begin{bmatrix} -1 \\ 6 \\ -7 \end{bmatrix}$$ (6½)

(c) Is $\lambda = 3$ an eigen value of the matrix $\begin{bmatrix} 1 & 2 & 2 \\ 3 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$? If so, find one corresponding eigen vector. (6½)

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1837 GC-3 **Your Roll No...............**

Unique Paper Code : 42351101

Name of the Paper : Mathematics – I : Calculus and Matrices (Course Code-235)

Name of the Course : **B.Sc. (Mathematical Sciences) / B.Sc. (Physical Sciences)**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** questions from each Section.

SECTION I

1. (a) Define basis of a vector space.

Is the set S,

$$S = \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

of vectors constitute a basis for R^3? (6)

(b) Define a linear Transformation. Let T: $R^2 \rightarrow R^2$ be the transformation denoting reflection about the line $y = -x$. Show that T is a linear transformation. Also find the standard matrix representing T. (6)

2. (a) Define rank of a matrix. Find the rank of the following matrix by using elementary row operations.

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 3 & 0 & 3 \\ 1 & -2 & -3 & 0 \\ 1 & 1 & 2 & 3 \end{bmatrix}$$ (6)

(b) Let

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{pmatrix}$$

Find eigen values of the matrix A and eigen vector corresponding to one of them. (6)

3. (a) Define a subspace of a vector space. Let W be the set of all points inside and on the unit circle in the xy-plane. Is W a subspace of xy-plane ? Justify. (6)

(b) Solve the system of equations :

$$x + y + 3z = 1$$

$$2x + 3y - z = 3$$

$$5x + 7y + z = 7$$ (6)

SECTION II

4. (a) Sketch the graph of $y = \frac{1}{2}x^2 - 3x + \frac{11}{2}$. Mention the transformation used at each step. (6)

(b) A certain culture of bacteria grows at a rate that is proportional to the number present. It is found that the number doubles in 4 hours. How may be expected at the end of 24 hours ? (6)

(c) Find $\frac{d^n y}{dx^n}$, where

$y = \sin(ax + b)$. (6)

5. (a) Discuss the convergence of the sequences : (6)

(i) $\left\langle \frac{\sin n}{n} \cdot \frac{n}{3n+1} \right\rangle$ (ii) $\left\langle 1 + \left(-\frac{1}{2}\right)^n \right\rangle$ (6)

(b) Show that

$$u(x,t) = 4\cos(2x + 2ct) + e^{x+ct},$$

is a solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$. (6)

(c) If $u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}, x^2 + y^2 + z^2 \neq 0$

Show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} = 0$. (6)

6. (a) If $y = \frac{\sin^{-1} x}{\sqrt{1 - x^2}}$ show that

$\left(1 - x^2\right) y_{n+2} - \left(2n + 3\right) x y_{n+1} - \left(n + 1\right)^2 y_n = 0$. (6)

(b) Find the n^{th} Maclaurins polynomial for $\frac{1}{1-x}$. (6)

(c) Draw the level curve of $f(x, y) = y^2 - x^2$ of height $k = 1$. (6)

SECTION III

7. (a) Give the geometrical representation of difference of two complex numbers. (3½)

(b) State Fundamental Theorem of Algebra. Also form an equation in lowest degree with real coefficients having $2+\sqrt{(-3)}$ and $3+\sqrt{(-5)}$ as two of its roots. (4)

8. (a) Solve the equation

$$z^4 + z^3 + z^2 + z + 1 = 0.$$ (4)

(b) Show that

$$(1 + \cos\theta + i \sin\theta)^n + (1 + \cos\theta - i \sin\theta)^n = 2^{n+1} \cos^n\frac{\theta}{2} \cos\frac{n\theta}{2}.$$ (3½)

9. (a) Find the equation of the circle described on the line joining the points $(-1 -3i)$ and $(5+7i)$ as extremities of one of its diameters. (4)

(b) Find the equation of the right bisector of the line joining the points z_1 and z_2. (3½)

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **618**

Unique Paper Code : **235354** **G**

Name of the Paper : **Mathematical Awareness**

Name of the Course : **B.A. (Hons.) Interdisciplinary Course**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *All* questions as per directed questionwise.

1. Do any *three* parts :

(*a*) (*i*) Who proved that π is a transcendental number ?

(*ii*) Which book helped Ramanujan to teach himself mathematics in high school.

(*iii*) What was the topic of Emmy Noether's dissertation ?

(*iv*) Name the mathematician who wrote the mathematical series named 'Elements'. 4

(*b*) (*i*) What was Ramanujan's area of research ?

(*ii*) Newton had a dispute with a French mathematician over the invention of Calculus. Name the French Mathematician.

(*iii*) Other than Hardy name another English mathematician who came in contact with Ramanujan ?

(*iv*) Give the name of *three* important books that Newton wrote. 4

P.T.O.

(*c*) State whether the following statements are True *or* False. If false, then give the *correct* answer :

(*i*) Emmy Noether showed a great relationship with her students.

(*ii*) Euclid is regarded as the pioneer in the invention of Complex Analysis.

(*iii*) The doctoral thesis of Riemann deal with the Number Theory.

(*iv*) Srinivas Ramanujan died in London. 4

(*d*) (*i*) Which property given by Riemann remains an open question till this day.

(*ii*) In which book did Newton develop the idea of gravitation based on the inverse square law ?

(*iii*) One professor of mathematics at the Presidency College at Madras provided financial support to Ramanujan for a while around 1910. Who was he ?

(*iv*) In which College in America did Emmy Noether find a temporary position in 1933 ? 4

2. Do any *three* parts :

(*a*) (*i*) Show that every square integer is of the form $4k$ or $4k + 1$ where k is an integer. 4

(*ii*) Using the above result show that no number in the sequence :

11, 111, 1111, 11111, ...

is a square. 3

(*b*) (*i*) Express greatest common divisor of 6237 and 2520 as a linear combination of 6237 and 2520. 4

(*ii*) Write $\frac{2520}{6237}$ as a continued fraction. 3

(*c*) (*i*) Verify that 2620 and 2924 form an amicable pair. 4

(*ii*) In how many ways the word "DAUGHTER" be arranged so that : 3

(1) All the vowels come together.

(2) Vowels occupy odd places.

(*d*) (*i*) Construct a magic square of order 7. What is its magic sum ? 4

(*ii*) What is a Fibonacci series ? Give *two* examples which show the existence of Fibonacci numbers in nature. 3

3. Do any *three* parts :

(*a*) Write short notes on :

(*i*) Perspective and Projection 3

(*ii*) Fermat and Mersenne numbers 2

(*iii*) Regular Polyhedra. 3

(*b*) (*i*) State the Four-Color map theorem. What is a Chromatic number ? Give the Chromatic numbers for a plane and a tours. 3

(*ii*) Give any *two* basic difference between the Mobius strip and the Klein Bottle. 2

(*iii*) Explain how the Konigsberg Bridge problem led to the discovery of Euler's formula for networks ? 3

(*c*) (*i*) Draw the graphs of the following functions and indicate where the function is increasing and decreasing : 3

(1) $f(x) = |x|$ in $[-1, 1]$

(2) $f(x) = \sqrt{1-x^2}$.

Also, find their domain and range.

(*ii*) Discuss Golden triangle and Golden spiral with respect Golden ratio. What is the significance of Golden ratio in nature ? 3

(*iii*) Verify which of the following functions are even or odd via graphs : 2

(1) $f(x) = \tan x$

(2) $f(x) = \sec x$

(*d*) (*i*) Explain how the Snow flake curve is formed. What can be said about its perimeter and area ? 3

(*ii*) Explain "Fractals in nature" with examples. 3

(*iii*) What is Basic Tilings ? Discuss. 2

4. Do any *two* parts :

(*a*) (*i*) A die is thrown twice. What is the probability of getting a sum greater than or equal to 9 ? 3

(*ii*) Find two numbers whose A.M. is 10 and G.M. is 8. 2

(*iii*) Use the graphical method to solve the following Linear Programming Problem : 4

Max $Z = 3x + 4y$

Subject to the constraints

$$x + 4y \leq 24$$

$$7x + y \leq 21$$

$$x + y \leq 9$$

$$x, y \geq 0$$

(*b*) (*i*) An urn contains 7 red and 4 blue balls. Two balls are drawn at random, without replacement. What is the probability that both the balls are red ? 3

(*ii*) How are standard deviation and the variance same and how are they different ? 2

(*iii*) What is Optimal Feasible solution ? Use the graphical method to solve the following Linear Programming Problem : 4

Max $Z = 2x + 3y$

Subject to the constraints

$$3x + y \leq 21$$

$$x + 4y \leq 24$$

$$x + y \leq 9$$

$$x, y \geq 0$$

(c) (i) Three envelopes are addressed for three secret letters written in invisible in[k]. [A] secretary randomly places each of the letters in an envelope and mails them. W[hat] is the probability that at least one person receives the correct letter ?

(ii) Explain the meaning of skewness. What are the objectives of measuring it ?

(iii) Define Basic Feasible Solution. Use the graphical method to solve the follow[ing] Linear Programming Problem :

Max $Z = x - 7y$

Subject to the constraints

$$x + y \leq 8$$

$$x \leq 5$$

$$y \leq 5$$

$$x + y \geq 4$$

$$x, y \geq 0$$

This question paper contains **4** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **849**

Unique Paper Code : **235362** **G**

Name of the Paper : **Mathematics–I [PHHT-310]**

Name of the Course : **B.Sc. (H) Physics**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

1. (*a*) Define the convergence of a sequence. Prove that : 7½

$$\lim_{n \to \infty} (a)^{1/n} = 1, a > 0.$$

(*b*) Prove that every convergent sequence is a Cauchy sequence. Apply it to prove that the sequence $<a_n>$ defined by : 7½

$$a_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}$$

does not converge.

(*c*) Show that the sequence $\langle a_n \rangle$ defined by : 7½

$$a_{n+1} = \sqrt{7 + a_n}, \quad a_1 = \sqrt{7}$$

converges to a positive root of the equation $x^2 - x - 7 = 0$.

P.T.O.

2. (*a*) Show that the series : 7½

$$1 + r + r^2 + r^3 + \ldots + r^n \ \ldots\ldots, (r > 0)$$

converges if $r < 1$ and diverges if $r \geq 1$.

(*b*) State Raabe's test. Show that the series : 7½

$$\sum \frac{3.6.9 \ldots\ldots 3n}{7.10.13 \ldots.(3n+4)} x^n, x > 0$$

converges for $x \leq 1$ and diverges for $x > 1$.

(*c*) State Leibnitz test for alternating series. Show that the series : 7½

$$\sum \frac{(-1)^{n+1}}{n^p}$$

is absolutely convergent for $p > 1$, but conditionally convergent for $0 < p \leq 1$

3. (*a*) Show that the function :

$$f(x) = \frac{1}{x^2}$$

is uniformly continuous on $A = [1, \infty)$, but it is not uniformly continuous on $B = (0, \infty)$ 7½

(*b*) Let R be the set of real numbers. State intermediate value theorem for a continuous function. Let the function $f : [-1, 1] \rightarrow R$ be defined as follows :

$$f(x) = \begin{cases} -1 - x, & -1 \leq x < 0 \\ 1 - x, & 0 \leq x \leq 1 \end{cases}$$

Show that f is not continuous at $x = 0$ and conclusion of intermediate value theorem does not hold for this function. 7½

(c) Obtain and plot the Taylor's polynomial of order 0, 1 and 2 generated by $f(x) = e^x$ at $x = 0$. Also obtain the Maclaurin series for this function. 7½

4. (a) Investigate the maxima and minima of the function : 7½

$$f(x, y) = x^3 + y^3 - 3x - 12y + 20$$

(b) Show that the function :

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$$

is continuous but not differentiable at origin. 7½

(c) State Young's theorem. For the following function :

$$f(x,y) = \begin{cases} \dfrac{(x^2 y + xy^2) \sin (x - y)}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$$

prove or disprove : 7½

$$f_{xy}(0, 0) = f_{yx}(0, 0).$$

5. (a) If :

$$f(x) = \begin{cases} 1, & \text{when } x \text{ is rational} \\ -1, & \text{when } x \text{ is irrational} \end{cases}$$

then show that f is not Riemann integrable on any interval $[a, b]$. 7½ (*b*) If M, m are the bounds of an integrable function f on $[a, b]$, then show that : 7½

$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a).$$

(*c*) If a function f is monotonic on $[a, b]$, then prove that it is Riemann integrable on $[a, b]$. 7½

[This question paper contains 2 printed pages.]

Sr. No. of Question Paper : 1116 **G** **Your Roll No...............**

Unique Paper Code : 235304

Name of the Paper : III.3 – Algebra II

Name of the Course : **B.Sc. (Hons.) Mathematics, Part II**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory.

1. (a) Let $G = Q\setminus\{-1\}$. Define * on G by $a*b = a + b + ab$ for all $a,b \in G$. Prove that $(G,*)$ is an abelian group. (6)

   (b) (i) Let G be a group and H be a non-empty subset of G. Prove that H is a subgroup of G iff $a.b^{-1} \in H$ for all $a,b \in H$. (3)

   (ii) Prove that the group of positive rational numbers under multiplication is not cyclic. (3)

   (c) Define centralizer of $a \in G$, where G is a group.

   (i) Prove that C(a) is an subgroup of G.

   (ii) Let $G = GL(2, R)$. Find $C\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$. (6)

2. (a) For every non-negative integer n, prove that nZ is a subgroup of (Z,+). Moreover, prove that every subgroup of Z is of the form mZ for some non-negative integer m. (6)

   (b) Let G be a group. Let $a,b \in G$ such that $ab = ba$ and $|a| = m$ and $|b| = n$. If $\langle a\rangle \cap \langle b\rangle = \{e\}$. Prove that G has an element of order l.c.m.(m, n). (6)

   (c) Prove that the order of a cyclic group is equal to the order of its generator. (6)

3. (a) Let H and K be two subgroup of G. Prove that HK is a subgroup of G iff HK = KH. (6.5)

(b) (i) Show that for $n \leq 3$, $Z(S_n) = \{e\}$. (3.5)

(ii) Suppose G is an abelian group with odd number of elements. Show product of all elements of G is identity. (3)

(c) State Lagrange's Theorem and prove with the help of example that converse of Lagrange's Theorem does not hold in general. (6.5)

4. (a) Prove that every permutation of a finite set can be written as product of disjoint cycles. (6.5)

(b) (i) Let H be a subgroup of G and let $a,b \in G$. Prove that either $aH = bH$ or $aH \cap bH = \phi$. (3)

(ii) Let N be a normal subgroup of a group G. If N is cyclic, prove that every subgroup of N is also normal in G. (3.5)

(c) (i) Given that G is a cyclic group, prove that G/N is also cyclic where N is a subgroup of G. Also give an example to show that converse is not true. (3.5)

(ii) If N is a normal subgroup of G and $|G/N| = m$, prove that $x^m \in N$ for all x in G. (3)

5. (a) Define automorphism and inner automorphism induced by an element 'a' of a group G. Prove that Inn (G) is a normal subgroup of Aut G. (6)

(b) Let ϕ be a homomorphism from a group G to a group $\bar{G}$. If $\bar{K}$ is a normal subgroup of $\bar{G}$, then prove that $\phi^{-1}(\bar{K})$ is a normal subgroup of G. (6)

(c) Let G = U(10). Find the left regular representation $\bar{G}$ of G and verify that $G \approx \bar{G}$. (6)

6. (a) If M and N are normal subgroups of G and $N \leq M$, prove that $(G/N)/(M/N) \approx G/M$. (6.5)

(b) (i) Let $\phi: G \to \bar{G}$ be a homomorphism. Define Ker ϕ and prove that Ker ϕ is a normal subgroup of G. (3)

(ii) Show $\phi: Z_{12} \to Z_{12}$ by $\phi(x) = 3x$ is a homomorphism and find Ker ϕ. Also find $\phi^{-1}(6)$. (3.5)

(c) Let k be a divisor of n. Prove that $Z_n/\langle k\rangle \approx Z_k$. (6.5)

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1456 **F-7** **Your Roll No...............**

Unique Paper Code : 2351301

Name of the Paper : Algebra II (Group Theory – I)

Name of the Course : **B.Sc. (Hons) Mathematics (FYUP)**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. Attempt **any two parts** from each question.
3. **All** questions are compulsory.

1. (a) (i) Prove that a group G is abelian if and only if $(ab)^{-1} = a^{-1} b^{-1}$.

(ii) Define a cyclic group. Find the generators of U(10).

(b) Let G be a group and H a non empty subset of G. Then prove that H is a subgroup of G if ab^{-1} is in H whenever a and b are in H.

(c) Let G be a group and let $a \in G$. Prove that if a has infinite order then all distinct powers of a are distinct group elements. Also prove that if a has finite order say n then $\langle a \rangle = \{e, a, \ldots, a^{n-1}\}$ and $a^i = a^j$ if and only if n divides $i - j$. (6×2=12)

2. (a) (i) Prove that in a group G there is only one identity element.

(ii) Show that in a group G the right and left cancellation law hold. Is the converse true ? Justify your answer.

(b) Define subgroup of a group G. Let G be an Abelian group with identity e. Then show that $H = [x \in G \mid x^2 = e\}$ is a subgroup of G. Is H a normal subgroup of G ? Justify.

(c) Prove that every subgroup of a cyclic group is cyclic. (6×2=12)

3. (a) Prove that if the identity permutation $\varepsilon = \beta_1 \ldots \beta_r$ where the β's are 2-cycles, then r is even.

(b) Define (i) Orbit of a point (ii) Stabilizer of a point. State and prove orbit stabilizer theorem.

(c) (i) Prove that A_n is a normal subgroup of S_n.

(ii) Prove that if a subgroup H of a group G has index 2 then H is normal in G. (6×2=12)

4. (a) Prove that the disjoint cycles commute.

(b) If G is a group with more than one element and G has no proper, nontrivial subgroup. Then prove that G is a finite group of prime order.

(c) (i) Prove that the center Z(G) of a group G is a normal subgroup of G.

(ii) H is a subgroup of G such that H is contained in the Z(G). Then prove that H is a normal subgroup of G. Is the converse true ? Justify your answer. (6.5×2=13)

5. (a) Let ϕ be a group homomorphism from a group G to a group G* then prove that

(i) $|\phi(x)|$ divides $|x|$, for all x in G.

(ii) ϕ is one-one if and only if $|\phi(x)| = |x|$, for all x in G.

(b) Let C be the set of complex numbers. Let $M = \left\{ \begin{pmatrix} a & -b \\ b & a \end{pmatrix} : a, \in \mathbb{R} \right\}$

Prove that

(i) $\langle C,+\rangle \approx \langle M,+\rangle$.

(ii) $\langle C^{*},\times\rangle \approx \langle M^{*},\times\rangle$ where C^{*} is the set of all non-zero complex number D and M^{*} is the set of all non-zero matrices from M.

(c) Prove that a finite cyclic group of order n is isomorphic to Z_n, the group of integers under addition modulo n. (6.5×2=13)

6. (a) State and prove Cayley's theorem.

(b) Prove that the external direct product of a finite no. of groups is a group under component-wise product.

(c) Let H be a subgroup of G and K be a normal subgroup of G. Prove that

$$HK/K \approx H/H\cap K.$$ (6.5×2=13)

[This question paper contains 2 printed pages.]

Sr. No. of Question Paper : 1457 **F-7** **Your Roll No................**

Unique Paper Code : 2351302

Name of the Paper : Analysis – II (Real Functions)

Name of the Course : **B.Sc. (H) Maths – II (Erstwhile FYUP)**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **three** parts from each question.

1\. (a) Let $A \subseteq R$ and $c \in R$ be a cluster point of A and $f : A \to R$, then define limit of function f at 'c'. Also show that, f can have only one limit at 'c'. (5)

(b) Let c E R. Use $\varepsilon - \delta$ definition to show that $\lim_{x \to c} x^2 = c^2$. (5)

(c) State and prove Sequential Criterion of Limits. (5)

(d) Show that $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ does not exist, where as $\lim_{x \to 0} x \sin\left(\frac{1}{x}\right) = 0$. (5)

2\. (a) Let $A \subseteq R$, let f and g be functions on A to R and let $c \in R$ be a cluster point of A.

Show that if $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$ then $\lim_{x \to c} (fg)(x) = LM$. (5)

(b) Let $f(x) = e^{1/x}$ for $x \neq 0$, then find $\lim_{x \to 0-} f(x)$ and $\lim_{x \to 0+} f(x)$. (5)

(c) Show that $\varphi(x) = \frac{1}{x}$ is continuous on $A = \{x \in R: x > 0\}$. (5)

(d) Let A = R and let f be the Dirichlet's function defined by

$$f(x) = \begin{cases} 1, & \text{for } x \text{ rational} \\ -1, & \text{for } x \text{ irrational} \end{cases}$$

Show that f is discontinuous at every point of R. (5)

3. (a) Let $A \subseteq R$ and $f : A \to R$. Show that if f is continuous at $c \in A$ then $|f|$ is continuous at c. Is the converse true ? Justify your answer. (5)

(b) Let f, g be continuous from R to R and suppose that $f(r) = g(r)$ for all rational numbers r. Show that $f(x) = g(x)$ for all $x \in R$. (5)

(c) Let f be a continuous real valued function defined on [a, b]. Show that f is a bounded on [a,b]. (5)

(d) Suppose that f is a real valued continuous function on R and that $f(a)f(b) < 0$ for some a, $b \in R$. Prove that there exists x between a and b such that $f(x) = 0$. Prove that $x2^x = 1$ for some x in (0,1). (5)

4. (a) Let I be a closed and bounded interval and let $f : I \to R$ be continuous on I. Then prove that f is uniformly continuous on I. (5)

(b) Show that the function $f(x) = x^2$ is not uniformly continuous on R. (5)

(c) If $f : I \to R$ has a derivative at $c \in I$, then show that f is continuous at c. Is the converse true? Justify your answer. (5)

(d) Let $f : I \to R$ be differentiable on the interval I. Prove that f is decreasing on I if and only if $f'(x) \leq 0$ for all $x \in I$. (5)

5. (a) State and prove Mean Value Theorem. (5)

(b) Show that $|\sin x - \sin y| < |x - y| \cdot \forall x,y \in R$. (5)

(c) For the function $f : R \to R$ defined as $f(x) = x^2 - 3x + 5$, find the points of relative extrema. Also, find the intervals on which the function is increasing, and those on which it is decreasing. (5)

(d) Obtain Maclaurin's series expansion for the function $f(x) = \sin x$, $x \in R$. (5)

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1458**

Unique Paper Code : **2351303** **F-7**

Name of the Paper : **Numerical Methods**

Name of the Course : **B.Sc. (H) Mathematics Under Erstwhile FYUP**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the *six* questions are compulsory.

Attempt any *two* parts from each question.

Marks are indicated against each question.

Choice is given within the question.

Use of Scientific Calculator is allowed.

1. (*a*) Perform three iterations of Newton's method to find root of the equation $x^4 - x - 10 = 0$ and starting approximation as 1.5.

(*b*) Explain rate of convergence of an iterative method for finding an approximation to the location of a root of $f(x) = 0$. Find rate of convergence of the false position method.

(c) Verify that the function $f(x) = x^3 + 2x^2 - 3x - 1$ has a zero on the interval (1, 2). Perform three iterations of bisection method. 13

2. (a) Verify that the equation $x^5 + 2x - 1 = 0$ has a root in the interval (0, 1). Perform three iterations of secant method to approximate the root.

(b) Perform three iterations of the Regula Falsi method to approximate the root of the function $f(x) = \cos x - xe^{-x}$ in the interval (0, 1).

(c) Define asymptotic error constant. Find out the error equation in rate of convergence of Secant method. 13

3. (a) Find an LU decomposition of the matrix :

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$$

and use it to solve the system $Ax = [1 \quad 6 \quad 4]^T$.

(b) Perform three iterations of Seidal method to solve the system of equations AX = B, for the given coefficient matrix and right hand side vector, starting with the initial vector :

$$x^{(0)} = (0.5, -0.5, -0.5)$$

$$A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 5 & 2 \\ 1 & 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ -6 \\ -4 \end{bmatrix}.$$

(*c*) Perform three iterations of Jacobi method to solve the system of equations AX = B, for the given coefficient matrix and right hand side vector, starting with the initial vector $x^{(0)} = (0, 0, 0)$:

$$A = \begin{bmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{bmatrix}, \; B = \begin{bmatrix} 10 \\ -14 \\ 33 \end{bmatrix}.$$ 13

4. (*a*) Use Newton Divided difference Method to estimate sin(0.15) from the following data set :

X	0.1	0.2
$F(x) = \sin X$	0.09983	0.19867

(*b*) Find the Lagrange interpolation polynomial for the given data set (0, 1), (1, 3) and (3, 55).

(*c*) Find the maximum value of the step size *h* that can be used in the interpolation $f(x) = e^x$ in [0, 1] so that the error in linear interpolation of $f(x)$ is less then 5×10^{-4}. 12

5. (*a*) Define the forward difference operator (Δ) and backward difference operator (∇). Prove that :

(*i*) $\mu = \sqrt{1 + \frac{\delta^2}{4}}$

(*ii*) $\nabla = -\frac{\delta^2}{2} + \delta\sqrt{1 + \frac{\delta^2}{4}}$.

P.T.O.

(*b*) If $f(x) = \frac{1}{x}$ then evaluate the *n*th Divided difference $f[x_0, x_1, x_2, \ldots\ldots, x_n]$.

(*c*) Derive the following backward difference approximation formula for the first order derivative where *h* is the spacing between the points :

$$f'(x_0) = \frac{1}{2h}\left(3f(x_0) - 4f(x_0 - h) + f(x_0 - 2h)\right).$$ 12

6. (*a*) Evaluate $\int_1^2 \frac{dx}{d}$ by Trapezoidal Rule and obtain the theoretical error bound.

(*b*) Apply Euler's method to approximate the solution of the initial value problem

$$\frac{dx}{dt} = tx^3 - x, \qquad 0 \le t \le 1, \qquad x(0) = 1.$$

over the interval [0, 1] using four steps.

(*c*) Verify that the forward difference approximation :

$$f'(x_0) = \frac{1}{2h}\left(-3f(x_0) + 4f(x_0 + h) - f(x_0 + 2h)\right).$$

For the first order derivative provide the exact value of the derivative, regardless of the value of *h*, for the function $f(x) = 1$, $f(x) = x$, $f(x) = x^2$ but not for the function $f(x) = x^3$. 12

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1517 **F-7** **Your Roll No...............**

Unique Paper Code : 2362401

Name of the Paper : Inventory and Production Management

Name of the Course : **Mathematics (Hons.) FYUP – Allied Course**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. Attempt any **Five** questions.
3. **All** questions carry equal marks.
4. Use of Simple Calculator is allowed.

1. (a) Explain the term 'inventory' in the organization. Describe different types of cost incurred in an inventory system.

   (b) An electric motor company works for 50 weeks in a year with the following characteristics :

   Demand = 20 units a week, unit cost = Rs. 2,500 per unit, reordering cost = Rs. 50 and holding cost = Rs. 660 per unit per year. What is the optimal order quantity ? Would it make much difference if this number were rounded up or down to the nearest integer ?

   (8,7)

2. (a) The annual demand for an item is 2,000 units, each order costs Rs. 10 and annual holding cost is 40% of unit cost. The unit cost depends on the quantity ordered as follows :

   (i) Rs. 1 for order quantities less than 500

   (ii) Rs. 0.80 for quantities between 500 and 999

   (iii) Rs. 0.60 for quantities of 1,000 and more

   What is the optimal order size ?

   (b) Obtain an expression for the EOQ for the deterministic demand inventory system in which supply is instantaneous, demand rate is uniform, backorders are allowed and are fully backlogged. (7,8)

3. (a) A retailer realizes that demand for an item is normally distributed with a mean of 2,000 units per year and standard deviation of 400 units. Unit cost is Rs. 100, reorder cost is Rs. 200, holding cost is 20% of value a year and lead time is fixed and equal to 3 weeks. Describe an ordering policy that gives him a 95% of service level. What will be the cost of the safety stock ?

   (b) Discuss the Newsboy problem. Formulate and derive the single period, discrete and stochastic demand model. (7,8)

4. (a) A store wants to improve the control of its stock and is looking at the possibility of using ABC analysis. Records from eight types of item show the current sales and costs as follows :

Item	No. of Sales	Cost (Rs.)
1	25	1400
2	150	14
3	30	680
4	80	20
5	10	1020
6	40	150
7	1000	20
8	100	30

Perform the ABC analysis.

(b) What is the main feature of JIT and how JIT's approach to inventory management differs from other methods? What is the difference between "Push" and "Pull" systems ? (7,8)

5. (a) Explain any three of the following :

(i) Reorder level

(ii) Safety stock

(iii) Lead time

(iv) Shortages cost (3×3)

(b) Demand for an item is steady at 1200 units per year with an ordering cost of Rs. 16 and holding cost of Rs. 0.24 per unit per year. Describe an appropriate ordering policy if the lead time is constant at (i) 3 months (ii) 9 months (iii) 18 months. (6)

6. (a) Demand for an item is constant at 1800 units per year. The item can be made at a constant rate of 3500 units a year. Unit cost is Rs. 50, batch setup cost is Rs. 650 and holding cost is 30% of unit cost per year. What is the optimal batch size for the item? If production setup time is 2 weeks, when should this is started ?

(b) What is material requirement planning (MRP) ? What are its advantages and disadvantages ? (7,8)

7. Derive the optimal inventory policy in a multi-item inventory system when there is

(i) Constraint on storage space

(ii) Constraint on average investment in stock (15)

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1520 **F-7** **Your Roll No................**

Unique Paper Code : 2352701

Name of the Paper : Real Analysis

Name of the Course : **Allied Courses [Erstwhile FYUP]**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. Attempt any **two** parts from each question.

1. (a) For any two real numbers x and y, prove that

   (i) $|x + y| \leq |x| + |y|$. (4½)

   (ii) Solve for x, if $|x - 8| = 3|x - 2|$. (3)

   (b) Let A be a non empty bounded set and define $B = \{x + k : x \in A\}$ where k is a fixed real number. Show that

   $$\text{lub } B = \text{lub } A + k$$ (7½)

   (c) Prove that for every pair of real numbers a and b with $a < b$, there is a rational number r such that $a < r < b$. (7½)

2. (a) Define a convergent sequence of real numbers. Determine whether the following sequences are convergent or not.

(i) $\{x_n\} = \left\{\frac{3n+4}{2n}\right\}$

(ii) $\{x_n\} = \left\{\frac{2+3n^4}{n^2}\right\}$

(iii) $\{x_n\} = \left\{\frac{(1+2n)^2}{5+3n+3n^2}\right\}$ (7½)

(b) Prove that, if $\{x_n\}$, $\{y_n\}$ and $\{z_n\}$ are sequences of real numbers such that $\{x_n\} \to a$, $\{z_n\} \to a$ and $x_n \leq y_n \leq z_n$ for all n, then $\{y_n\} \to a$. (7½)

(c) Define cluster point of sequence $\{x_n\}$ of real numbers. Determine the cluster points of the following sequences :

(i) $\{x_n\} = \{(-1)^n\}$

(ii) $\{x_n\} = \{\sin(n\pi/4)\}$

(iii) $\{x_n\} = \{(-1)^n n\}$ (7½)

3. (a) Prove that the sequence $\{x_n\}$, where

$$x_n = \sum_{k=1}^{n} \frac{1}{k^2} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2}$$

is a cauchy sequence. (7½)

(b) If $a \neq 0$, then show that the geometric series $\sum_{k=1}^{\infty} ar^{k-1}$ converges if and only if $|r| < 1$, and when convergent, the sum is $a/(1-r)$. (7½)

(c) Test for convergence or divergence of the series given below :

(i) $\sum_{n=1}^{\infty} \frac{1}{2n+1}$

(ii) $\sum_{n=1}^{\infty} \frac{(n+1)}{n2^n}$

(iii) $\sum_{n=1}^{\infty} (-1)^n$ (7½)

4. (a) State Taylor's theorem and hence find the Taylor's series expansion for the function $f(x) = \cos x$ about $x = \pi/2$. (7½)

(b) Examine the continuity of the function

$$f(x) = \begin{cases} -x^2, & \text{if} \quad x \leq 0 \\ 5x-4, & \text{if} \quad 0 < x \leq 1 \\ 4x^2-3x, & \text{if} \quad 1 < x \end{cases}$$

at $x = 0,1$. (7½)

(c) (i) State Intermediate Value Theorem. (2)

(ii) Prove that the function $f(x) = \frac{x-1}{x+2}$ is uniformly continuous on $[0, \infty)$. (5½)

5. (a) Define local maxima and local minima. Determine the local maxima and local minima of the function $f(x) = x^4 - 4x^3 + 1$ on $[-1, 4]$. (7½)

(b) State Rolle's theorem and verify it for $f(x) = \sqrt{1-x^2}$ in $[-1, 1]$. (7½)

(c) Show that the sequence of functions $\{f_n\}$ defined by $f_n(x) = \dfrac{x}{1+nx^2}$ converges uniformly on R. (7½)

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1846 **GC-3** **Your Roll No...............**

Unique Paper Code : 42354302

Name of the Paper : Paper III - Algebra

Name of the Course : **B.Sc. Physical Sciences / Mathematical Sciences (Part – II)**

Semester : III (Under CBCS)

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory.
4. Marks are indicated.

UNIT – I

1. (a) Define Group. Give examples of each of following :

   (i) Finite abelian group.

   (ii) Finite non-abelian group.

   (iii) Infinite abelian group.

   (iv) Infinite non-abelian group.

   (v) Cyclic group.

   (vi) Abelian group which is not cyclic. (6)

(b) Let G be a group. Show that $Z(G) = \bigcap_{a\in G} C(a)$. Where Z(G) is the center of the group G and C(a) is the centralizer of a in G. (6)

(c) Find the inverse of the element $\begin{bmatrix} 2 & 6 \\ 3 & 5 \end{bmatrix}$ in $GL(2, Z_{11})$. (6)

2. (a) Let H be a nonempty finite subset of a group G. Then show that if is a subgroup of G if H is closed under the operation of G. (6)

(b) Define cyclic group. Give an example of a noncyclic group, all of whose proper subgroups are cyclic. (6)

(c) Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{bmatrix}$

Compute each of the following :

(a) α^{-1}

(b) $\beta\alpha$

(c) $\alpha\beta$ (6)

3. (a) State and prove Lagrange's theorem for finite group. (6)

(b) Find all left cosets of {1,11} in U(30). (6)

(c) Show that the order of a permutation on a finite set written in disjoint cycle form is the least common multiple of the lengths of the cycles. (6)

UNIT – II

4. (a) Prove that a nonempty subset S of a ring R is a subring of R if and only if

(i) $a - b \in S$ and (ii) $ab \in S$ for all $a,b \in S$.

Hence show that if a is a fixed element of a ring R then $I_a = \{x \in R : ax = 0\}$ is a subring of R. (6½)

(b) Let R be a commutative ring. Then show that R is an integral domain if and only if $ab = ac \Rightarrow b = c$, where $a,b,c \in R$ and $a \neq 0$. (6½)

(c) Define an ideal of a ring R and prove that intersection of two ideals of a ring is an ideal but union is not so. (6½)

UNIT – III

5. (a) Prove that a nonempty subset W of a vector space V(F) is a subspace of V if and only if $\alpha w_1 + \beta w_2 \in W \quad \forall \alpha, \beta \in F$ and $w_1, w_2 \in W$.

Give an example of a nonempty subset W of a vector space V(F) which is not a subspace of V. (6½)

(b) Show that the vectors (1,2,3,4),(0,1,–1,2),(1,5,1,8),(3,7,8,14) in $\mathbb{R}^4$ are linearly dependent over $\mathbb{R}$. (6½)

(c) Determine whether or not the vectors (1,–3,2), (2,4,1) and (1,1,1) form a basis of $\mathbb{R}^3$. (6½)

6. (a) Define a basis of a vector space over a field F and prove that every element of a vector space is uniquely expressible as a linear combination f elements of the basis. (6½)

(b) (i) Show that the mapping $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by

$T(x_1,x_2) = (x_1 + x_2, x_1 - x_2, x_2)$ is a linear transformation.

(ii) Let $T : V \to W$ be a linear transformation. Prove that the vectors $v_1, v_2, v_3 \in V$ are linearly independent, if $T(v_1), T(v_2), T(v_3)$ are linearly independent. (6½)

(c) Let $T : V \to U$ be a linear transformation. Define null space $N(T)$ and range space $R(T)$ of T. Show that $N(T)$ is a subspace of V and $R(T)$ is a subspace of U. (6½)

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 2064 **GC-3** **Your Roll No................**

Unique Paper Code : 32351301

Name of the Paper : C5 Theory of Real Functions

Name of the Course : **B.Sc. (Hons.) Mathematics – C.B.C.S.**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **three** parts from each question.

1. (a) Use the $\epsilon - \delta$ definition of limit of a function to find $\lim_{x \to 1} \frac{x^2 - x + 1}{x + 1}$. (5)

   (b) State Sequential Criterion for Limits. Using Sequential Criterion for limits, prove that $\lim_{x \to 0} \sin \frac{1}{x^2}$ does not exist. (5)

   (c) State Squeeze Theorem. Use the theorem to show that $\lim_{x \to 0} \frac{\sin x}{x} = 1$. (5)

   (d) Let f be defined on $A \subseteq R$ to R and let c be a cluster point of A. If $\lim_{x \to c} f > 0$, then show that there exists a neighbourhood $V_\delta(c)$ of c such that $f(x) > 0$ for all $x \in A \cap V_\delta(c)$, $x \neq c$. (5)

P.T.O.

2\. (a) Let f be defined on $A \subseteq R$ to R and let c be a cluster point of $A \cap (c,\infty)$ and $A \cap (-\infty,c)$. Then show that $\lim_{x\to c} f(x) = L$ exists if and only if $\lim_{x\to c+} f(x) = L = \lim_{x\to c-} f(x)$. (5)

(b) Prove that

(i) $\lim_{x\to\infty} \frac{1}{x^2} = 0$

(ii) $\lim_{x\to 0} \frac{1}{x^2} = \infty$ (5)

(c) Let g: $R \to R$ be defined by

$$g(x) = \begin{cases} x, & \text{for } x \text{ rational} \\ 0, & \text{for } x \text{ irrational} \end{cases}$$

Find all the points at which g is continuous. (5)

(d) Determine the points of continuity of the function $f(x) = [\![x]\!]$, $x \in R$, where $[\![x]\!]$ denotes the greatest integer $n \in \mathbb{Z}$ such that $n \leq x$. (5)

3\. (a) Let $A, B \subseteq R$. Let $f : A \to R$ and $g : B \to R$ be functions such that $f(A) \subseteq B$. If f is continuous at $c \in A$ and g is continuous at $b = f(c) \in B$ then show that the composite function $gof : A \to R$ is continuous at c. (5)

(b) Let f, g be continuous from R to R and suppose that $f(r) = g(r)$ for all rational numbers r. Show that $f(x) = g(x)$ for all $x \in R$. (5)

(c) Let f be a continuous real valued function defined on [a, b]. By assuming that f is a bounded function, show that f attains its maximum value on [a, b]. (5)

(d) Suppose that f is continuous on [0,2] and that f(0) = f(2). Prove that there exist x, y in [0,2] such that $|y - x| = 1$ and f(x) = f(y). (5)

4. (a) Let I be a closed and bounded interval and let $f : I \to R$ be continuous on I. Then prove that f is uniformly continuous on I. (5)

(b) Show that the function $f(x) = x^2$ is not uniformly continuous on R. (5)

(c) State and prove Caratheodory's Theorem. (5)

(d) Let $f: R \to R$ be defined by

$$f(x) = \begin{cases} x^2, & \text{for x rational} \\ 0, & \text{for x irrational} \end{cases}$$

Show that, f is differentiate at x = 0 and find $f'(0)$. (5)

5. (a) Let f be continuous on [a,b] and differentiable on (a,b). Prove that if $f'(x) < 0\ \forall x \in [a,b]$, then f is strictly decreasing on [a,b]. Is the converse true ? Justify. (5)

(b) Let $g: [-1,1] \to R$ be defined by

$$g(x) = \begin{cases} 2, & 0 < x \le 1 \\ 0, & x = 0 \\ 1, & -1 \le x < 0 \end{cases}$$

Show that g is not the derivative on [−1,1] of any function. (5)

(c) Find the Taylor series for sin x and indicate why it converges to $\sin x\ \forall x \in R$. (5)

(d) Define a convex function on [a,b]. Show that the function $f(x) = |x|$, $x \in [-1,1]$ is convex but not differentiable on $[-1,1]$. (5)

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 2065 **GC-3** **Your Roll No...............**

Unique Paper Code : 32351302

Name of the Paper : C6 Group Theory 1

Name of the Course : **B.Sc. (Hons) Mathematics – CBCS**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory.

1. (a) Describe the elements of the dihedral group D_3 and show that they form a group under composition of mappings. (6)

   (b) Prove that if H and K are subgroups of G, then so is $H \cap K$. Is $H \cup K$ a subgroup of G ? Justify your answer. (6)

   (c) Define Centralizer C(G) of a group G. Is C(G) Abelian ? Justify your answer. (6)

2. (a) Let $G = \langle a \rangle$ be a cyclic group of order n. Show that $G = \langle a^k \rangle$ if and only if $\gcd(k,n) = 1$. (6)

(b) List the subgroups of Z_{30} and their generators. (6)

(c) (i) Let G be an Abelian group with identity e. Prove that

$H = \{x \in G | x^2 = e\}$ is a subgroup of G. (3)

(ii) Let a and b be elements of a group. If $|a| = 10$ and $|b| = 21$, show

that $\langle a \rangle \cap \langle b \rangle = \{e\}$. (3)

3. (a) Prove that every permutation on a finite set can be written as a cycle or product of disjoint cycles. (6)

(b) Prove that the set of even permutations in S_n forms a subgroup of S_n. (6)

(c) Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 1 & 3 & 5 & 4 & 7 & 6 & 8 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{bmatrix}$

Write α and β as

(i) products of disjoint cycles.

(ii) products of 2 – cycles. (6)

4. (a) Prove that any finite cyclic group of order n is isomorphic to $\mathbb{Z}_n$ and any infinite cyclic group is isomorphic to $\mathbb{Z}$. (6½)

(b) Let $\overline{G}$ denote the left regular representation of the group G (as defined in Cayley's Theorem). Calculate $\overline{U(12)}$. (6½)

(c) Give statements only, of Lagrange's Theorem and its converse. Is the converse true ? Justify your answer. (6½)

5. (a) Let H and K be subgroups of a finite group G. Prove that

$$|HK| = \frac{|H||K|}{|H \cap K|}$$ (6½)

(b) Let G be a finite Abelian group and let p be a prime number that divides the order of G. Prove that G has an element of order p. (6½)

(c) Let φ be a homomorphism from a group G to a group $\bar{G}$ and let H be a subgroup of G. Prove that

(i) if H is cyclic, then φ(H) is cyclic. (2)

(ii) if H is Abelian, then φ(H) is Abelian. (2)

(iii) if H is normal in G, then φ(H) is normal in φ(G). (2½)

6. (a) State and prove The Second Isomorphism Theorem. (6½)

(b) Let G be a subgroup of some dihedral group. For each x in G, define

$$\varphi(x) = \begin{cases} +1 & \text{if x is a rotation} \\ -1 & \text{if x is a reflection} \end{cases}$$

Prove that φ is a homomorphism from G to the multiplicative group $\{+1, -1\}$. What is the kernel of φ ? (6½)

(c) Determine all homomorphisms from $\mathbb{Z}_{12}$ to $\mathbb{Z}_{30}$. (6½)

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 2066 **GC-3** **Your Roll No................**

Unique Paper Code : 32351303

Name of the Paper : C-7 Multivariate Calculus (Including Practicals)

Name of the Course : **B.Sc. (Hons.) Mathematics – II – C.B.C.S.**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. **All** sections are compulsory.
3. Attempt any **five** questions from each section.
4. **All** questions carry equal marks.

SECTION I

1. Show that f is continuous at (0, 0) where

$$f(y,) = \begin{cases} y \sin 1/x & x \neq 0 \\ 0 & x = 0 \end{cases}$$

2. A closed box is found to have length 2 ft, width 4 ft and height 3 ft, where the measurement of each dimension is made with a maximum possible error of ± 0.02ft. The top of the box is made from material that costs \$ 2/ft^2; and the material for the sides and bottom costs only \$ 1.50/ft^2. What is the maximum error involved in the computation of the cost of the box ?

3. If $z = f(u - v, v - u)$, show that

$$\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v} = 0$$

4. Suppose at point $P_0(-1, 2)$, a certain function $f(x, y)$ has directional derivative 8 in the direction of $v_1 = 3i - 4j$ and 1 in the direction of $v_2 = 12i + 5j$. What is the directional derivative of f at P_0 in the direction of $v = 3i - 5j$?

5. Find the absolute extrema of $f(x, y) = xy - 2x - 5y$ on the closed bounded set S where S is the triangular region with vertices (0, 0), (7, 0) and (7, 7).

6. Find the point of intersection of the plane $x + 2y + z = 10$ and the paraboloid $z = x^2 + y^2$ that is closest to the origin. You may assume that such a point exists.

SECTION II

7. Compute $\iint_R \frac{xy}{x^2 + y^2} dA$, where R is the rectangle $1 \le x \le 3$, $1 \le y \le 2$.

8. Evaluate the integral $\int_0^1 \int_{\tan^{-1} y}^{\pi/4} \sec x \, dx \, dy$.

9. Evaluate $\iint_D \frac{1}{x} dA$, where D is the region that lies inside the circle $r = 3\cos\theta$ and outside the cardioid $r = 1 + \cos\theta$.

10. Find the volume V of the solid, using spherical coordinates, bounded by the sphere $(x-1)^2 + (y-2)^2 + (z-3)^2 = 1$.

11. Find the volume of the solid D where D is the intersection of the solid sphere $x^2 + y^2 + z^2 \le 9$ and the solid cylinder $x^2 + y^2 \le 1$.

12. Evaluate $\iint_D e^{-(4x^2+5y^2)} dA$, using change of variable, where D is the elliptical disk $\frac{x^2}{5} + \frac{y^2}{4} \le 1$.

SECTION III

13. Let $F(x, y) = e^y i + xe^y j$. Verify that the vector field F is conservative on the entire xy-plane. Use a potential function of F to find the work done by the field F on a particle that moves in anticlockwise direction from (1, 0) to (–1, 0) along the semi-circular path

$$y = \sqrt{1-x^2}; \quad -1 \le x \le 1.$$

14. State Green's theorem for simply connected regions. Verify Green's theorem for the field $F(x, y) = (x - y)i + xj$ and the region R bounded by the unit circle $C : r(t) = \cos t\, i + \sin t\, j;\ 0 \le t \le 2\pi$.

15. Let E be the solid bounded by the xy-plane and the paraboloid $z = 4 - x^2 - y^2$. Let S be the boundary of E (that is, piece of the paraboloid and a disk in the xy-plane), oriented with the outward-pointing normal n.

    If $F(x,y,z) = (xz \sin(yz) + x^3)i + \cos(yz)j + (3zy^2 - e^{x^2+y^2})k$.

    Find $\iint F.n\ ds$ over the surface S using divergence theorem.

16. Evaluate $\iint (x + y + z)\ ds$ over the surface S defined parametrically by

    $R(u, v) = (2u + v)i + (u - 2v)j + (u + 3v)k$ for $0 \le u \le 1;\ 0 \le v \le 2$.

17. State and prove the Stokes' theorem.

18. State fundamental theorem of line integrals. If a semi-circular wire has the equation $y = \sqrt{25 - x^2}$ and its mass density function is $\psi(x, y) = 15 - y$. Find the mass of the wire.

This question paper contains 3 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **32**

Unique Paper Code : **235566** **G**

Name of the Paper : **MAPT-505, Real Analysis**

Name of the Course : **B.Sc. Mathematical Sciences/B.Sc. Physical Sciences**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

1. (*a*) State order completeness property. Show that the set of irrational numbers is not order complete.

   (*b*) State and prove Archimedean property of real numbers.

   (*c*) Show that the set of positive rational numbers is countable. 6,6

2. (*a*) Prove that the set $S = \{1/n | n \in N\}$ has no limit point other than zero in the set of real numbers.

   (*b*) Show that every subset of a countable set is countable. Is every superset of a countable set countable ? Justify.

   (*c*) If $\lim a_n = a$ and $a_n \geq 0$ for all n, then prove that $a \geq 0$. Deduce that if $\{a_n\}$ and $\{b_n\}$ are two sequences such that $a_n \geq b_n$ for all n, then $\lim a_n \geq \lim b_n$. 6,6

P.T.O.

3. (*a*) State and prove Bolzano-Weierstrass theorem for sequences.

(*b*) Define a Cauchy sequence. Show that the sequence $\{a_n\}$ is not convergent, where :

$$a_n = 1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \dots .$$

(*c*) Let $\{x_n\}$ be a sequence defined by :

$$x_1 = 1,\ x_{n+1} = \frac{3+2a_n}{2+a_n},\ n \geq 1.$$

Show that the sequence $\{x_n\}$ is convergent. Also find its limit. 6½,6½

4. (*a*) Define the convergence of an infinite series. Show that the series :

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots..$$

is convergent.

(*b*) Test the convergence of the following series :

(*i*) $\sum \frac{\sqrt{n+1} - \sqrt{n-1}}{n}$

(*ii*) $1 + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \dots\dots$ for all $x > 0$.

(*c*) Let Σu_n be a positive term series such that

$$\lim_{n\to\infty} (u_n)^{\frac{1}{n}} = L.$$

Show that Σu_n converges for $L < 1$. What happens for $L = 1$? Justify. 6,6

5. (*a*) State Leibnitz test for convergence of an alternating series. Test for convergence and absolute convergence, the series :

$$1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + \dots .$$

Is the series conditionally convergent ?

(*b*) Define radius of convergence and interval of convergence of a power series. Find the radius of convergence and exact interval of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{(x-1)^n}{2^n}.$$

(*c*) Define sine function S(x) and cosine function C(x) in terms of power series. Specify the domains of convergence of the respective power series. 6½, 6½

6. (*a*) If $\{f_n\}$ is a sequence of continuous functions converging uniformly to a function f on $[a, b]$, then show that f is continuous on $[a, b]$.

(*b*) Test the sequence $\{f_n\}$ where

$$f_n(x) = \frac{nx}{1 + n^2 x^2}$$

for uniform convergence on [0, 1]. Verify :

$$\lim_{n\to\infty} \int_0^1 f_n(x)\,dx = \int_0^1 \left(\lim_{n\to\infty} f_n(x)\right) dx.$$

(*c*) (*i*) Show that the series :

$$\sum \frac{1}{n^4 + n^2 x^2}$$

converges uniformly for all real values of x.

(*ii*) Show that the series :

$$x^4 + \frac{x^4}{1+x^4} + \frac{x^4}{\left(1+x^4\right)^2} + \frac{x^4}{\left(1+x^4\right)^3} + \ldots$$

is not uniformly convergent on [0, 1]. 6½,6½

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1118 **G** **Your Roll No...............**

Unique Paper Code : 235503

Name of the Paper : Analysis – IV (MAHT-502)

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.

1. (a) Let d_1 and d_2 be two metrics on a non-empty set X. Let

$$d(x, y) = \sqrt{d_1^2(x, y) + d_2^2(x, y)}$$

$$d^*(x, y) = \max(d_1(x, y), d_2(x, y)) \text{ for all } x, y \in X.$$

What is the relation between these two metrics ?

(b) Suppose (x, d) is a metric space, Z is a set and $f : Z \to X$ is an injection function.

Show that $(a,b) \to d(f(a), f(b))$ is a metric on Z.

(c) Suppose X is a metric space, $z \in X$ and S is a subset of X. Show that $z \in acc(S)$ if, and only if, $z \notin iso(S)$ and $dist(z, S) = 0$.

2. (a) Suppose X is a metric space, $x \in X$ and S is a subset of X. Show that $x \in$ boundary (S) if and only if, every open ball of X centred at x has non-empty intersection with both S and S^c.

(b) What is an isometry ? Show that an isometry from a metric space X onto a metric space (Y, d_y) is a homeomorphism.

(c) Suppose (X, d) is a metric space, $w \in X$ and A is a subset of X. Then, show that

$$dist(w, cl(A)) = dist(w, A),$$

where cl(A) is the closure of A.

3. (a) Suppose X is a metric space and $S \subseteq X$. Prove that

(i) $(S^0)^c = Cl(S^c)$

(ii) $S^0 = \{x \in X: dist(x, S^c) > 0\}$

(b) Suppose X is a metric space and S is a non-empty subset of X. Then prove that

$$diam(cl(S)) = diam(S).$$

Do we also have $diam(int(S)) = diam(S)$? Justify your answer.

(c) Suppose (X, d) is a metric space and Z is a metric subspace of X. Show that the collection of open subsets of Z is $\{U \cap Z \mid U \text{ is open in } X\}$.

4. (a) Suppose X is a metric space and $S \subseteq X$. Prove that

(i) $\partial(\partial S) \subseteq \partial S$.

(ii) $\overline{S}$ is the smallest superset of S that is closed in X.

(b) Let A be a nonempty subset of a metric space X. Show that $z \in cl(A)$ if and only if, there is a sequence in A that converges to z in X.

(c) If $x = \langle x_n \rangle$ has a subsequence which converges to z. Show that

$$dist(z, \{x_n : n \in N\}) = 0.$$

Give an example to show that converse to above statement is not necessarily true.

5. (a) Suppose X is a metric space and S is a bounded subset of X,

(i) Show that closure of S is bounded in X.

(ii) Is S also a totally bounded ? Justify your answer.

(b) Let $f : X \to Y$ be a function from the metric space X into other metric space. Show that f is a continuous at every isolated point of X.

(c) Let $f : (X, d) \to (Y, e)$ be uniformly continuous. Prove that the sequence $\langle x_n \rangle$ is Cauchy in X if and only if the sequence $\langle f(x_n) \rangle$ is Cauchy in Y.

6. (a) State Banach contraction Property. Give an example of a complete metric space X and a function $f : X \to X$ such that $d(f(x), f(y)) <= d(x,y)$ for all $x, y \in X$, but f has no fixed point in X.

(b) Define a connected space. Prove that a metric space X is connected if and only if, every continuous function $f : X \to \{0, 1\}$ (a discrete space) is a constant function.

(c) Define a compact metric space. Prove that the continuous image of a compact metric space is compact.

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1119 **G** **Your Roll No................**

Unique Paper Code : 235504

Name of the Paper : Algebra IV (MAHT-503)

Name of the Course : **B.Sc. (H) MATHEMATICS – III**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **five** parts from Question **1.** Each part carries **three** marks.
3. Attempt any **two** parts from each of the Questions **2** to **6.** Each part carries **six** marks.

1. (i) Show that $\mathbb{Q}\left(\sqrt{3},\sqrt{5}\right)=\mathbb{Q}\left(\sqrt{3}+\sqrt{5}\right)$.

   (ii) Find a basis for $\mathbb{Q}\left(\sqrt{2i}\right)$ over $\mathbb{Q}$.

   (iii) Prove that an angle θ is constructible if and only if $\sin\theta$ is constructible.

   (iv) Find the dual basis β^* of an ordered basis $\beta = \{(2, -1), (1, -1)\}$ for $\mathbb{R}^2$.

   (v) Find all the eigen vectors of the matrix $A=\begin{pmatrix}2 & 1\\4 & 2\end{pmatrix}$.

   (vi) Let β be a basis for a finite-dimensional inner product space. Prove that if $\langle x,z\rangle=\langle y,z\rangle$ for all $z \in \beta$, then $x = y$.

(vii) Let $S = \{(1, i, 0), (1,1,2)\}$ in $\mathbb{C}^3$. Compute $S^{\perp}$.

(viii) For inner product space V and linear operator T on V, evaluate T* at a vector x in V where $V = \mathbb{R}^2$, $T(a, b) = (a + 2b, -3a + b)$, $x = (5,3)$.

2. (a) Let F be a field and f(x) be a non-constant polynomial in F[x]. Show that there is an extension field E of F in which f(x) has a zero.

(b) Prove or disprove that $\mathbb{Q}\left(\sqrt{5}\right)$ and $\mathbb{Q}\left(\sqrt{-5}\right)$ are field-isomorphic.

(c) Suppose that E, is an extension of F of prime degree. Show that, for every a in E, $F(a) = F$ or $F(a) = E$.

3. (a) Prove that cos 2θ is constructible if and only if sin θ is constructible.

(b) Let a and b belong to some extension of F and let b be algebraic over F. Prove that $[F(a,b) : F(a)] \le [F(a,b): F]$.

(c) Show that no finite field is algebraically closed.

4. (a) Let W_1 and W_2 be two subspaces of a vector space. Prove that

$$(W_1 + W_2)^0 = W_1^0 \cap W_2^0$$

(b) Let T be the linear operator on $\mathbb{R}^4$ defined by

$$T(a,b,c,d\} = (a + b + 2c - d,\ b + d,\ 2c - d,\ c + d)$$

and let $W = \{(t, s, 0,0) : t, s \in \mathbb{R}\}$.

Show that the characteristic polynomial of T_W, the restriction of T to W, divides the characteristic polynomial of T.

(c) Let T be the linear operator on $\mathbb{R}^3$ defined by

$$T\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} a_1 & & + 4a_3 \\ 2a_1 - 3a_2 & & + 2a_3 \\ 4a_1 & & + a_3 \end{pmatrix}$$

Find all the eigen vectors of T.

5. (a) Let $\{v_1, v_2, ..., v_k\}$ be an orthonormal set in an inner product space V, and let $a_1, a_2, ..., a_k$ be scalars. Prove that

$$\left\| \sum_{i=1}^{k} a_i v_i \right\|^2 = \sum_{i=1}^{k} |a_i|^2$$

(b) Let V be a finite-dimensional inner product space, and let T be a linear operator on V. Then prove that there exists a unique function $T^*: V \to V$ such that

$$\langle T(x), y \rangle = \langle x, T^*(y) \rangle \text{ for all } x, y \in V.$$

Further, prove that T^* is linear.

(c) For inner product space V and linear operator T on V, evaluate T^* at a vector f in V where,

$$V = P_1(\mathbb{R}) \text{ with } \langle f, g \rangle = \int_{-1}^{1} f(t)g(t)\,dt,\ T(f) = f' + 3f,\ f(t) = 4 - 2t$$

6. (a) Let T be a linear operator on a finite-dimensional vector space, and let p(t) be the minimal polynomial of T. Prove that T is not invertible if and only if $p(0) = 0$.

(b) Let T be a linear operator on an inner product space V. If $\langle T(x), y\rangle = 0$ for all $x, y \in V$, prove that $T = T_0$ (the zero operator on V). Prove that the same result is true if the equality holds for all x and y in some basis of V.

(c) Let K be a finite extension field of a finite field F. Show that there is an element a in K such that = F(a).

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1120 G **Your Roll No...............**

Unique Paper Code : 235505

Name of the Paper : Linear Programming and Theory of Games (Paper V.4)

Name of the Course : **B.Sc. (Hons.) MATHEMATICS**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts of each question.
3. **All** questions carry equal marks.

1. (a) Given a basic feasible solution $x_B = B^{-1}b$ with $z_0 = c_B x_B$ as the value of the objective function to the linear programming problem :

Minimize $z = c\ x$

Subject to $Ax = b$

$x \geq 0$

such that $z_j - c_j \leq 0$ for every column a_j in A. Show that z_0 is the minimum value of z subject to the constraints and that the basic feasible solution is optimal feasible solution.

(b) Prove that if there is a feasible solution to the linear programming problem :

Minimize $z = c\ x$

Subject to $Ax = b$

$x \geq 0$

then, there is a basic feasible solution to it.

(c) $x_1 = 2, x_2 = 3, x_3 = 1$ is a feasible solution to the system of equations :

$$2x_1 + x_2 + 4x_3 = 11$$
$$3x_1 + x_2 + 5x_3 = 14$$

Is this a basic feasible solution ? If not, reduce it to a basic feasible solution.

2. (a) Using Two Phase method, solve the linear programming problem :

Minimize $z = x_1 - 2x_2$

Subject to $x_1 + x_2 \geq 2$

$-x_1 + x_2 \geq 1$

$x_2 \leq 3$

$x_1, x_2 \geq 0$

(b) Using simplex method, solve the system of solution :

$$x_1 + x_2 = 1$$
$$2x_1 + x_2 = 3$$

and find the inverse of the coefficient matrix $\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$.

(c) Use the big- M method to solve the following linear programming problem :

Maximize $z = 3x_1 + 2x_2 + 3x_3$

Subject to $2x_1 + x_2 + x_3 \leq 2$

$3x_1 + 4x_2 + 2x_3 \geq 8$

$x_1, x_2, x_3 \geq 0$

3. (a) (i) State and prove the relationship between the objective function values of the primal and dual linear programming problems.

(ii) Write the dual of the following linear programming problem :

Maximize $z = 8x_1 + 3x_2 - 2x_3$

Subject to $x_1 - 6x_2 + x_3 \geq 2$

$5x_1 + 7x_2 - 2x_3 = -4$

$x_1 \leq 0, x_2 \geq 0, x_3$ unrestricted,

and verily that dual of the dual is primal problem. (2.5,5)

(b) Verify that $(w_1, w_2) = \left(\frac{8}{5}, \frac{1}{5}\right)$

is a feasible solution of the dual of the linear programming problem :

Minimize $2x_1 + 15x_2 + 5x_3 + 6x_4$

Subject to $x_1 + 6x_2 + 3x_3 + x_4 \geq 2$

$-2x_1 + 5x_2 - x_3 + 3x_4 \leq -3$

$x_1, x_2, x_3, x_4 \geq 0$

and use this information to find the optimal solution of the primal and dual problems.

(c) Apply the principle of duality to solve the linear programming problem :

Maximize $x = 3x_1 + 2x_2$

Subject to $x_1 + x_2 \geq 1$

$x_1 + x_2 \leq 7$

$x_1 + 2x_2 \leq 10$

$x_2 \leq 3$

$x_1, x_2 \geq 0$

4. (a) Solve the following cost-minimizing transportation problem :

	D_1	D_2	D_3	Supply
O_1	10	9	8	8
O_2	10	7	10	7
O_3	11	9	7	9
O_4	12	14	10	4
Demand	10	10	8	

(b) Solve the following cost-minimizing assignment problem :

	A	B	C	D
I	18	26	17	11
II	13	28	14	26
III	38	19	18	15
IV	19	26	24	10

(c) Define saddle point of a two person zero sum game. Use the minimax criteria to find the best strategy for each player for the game having the following pay off matrix :

Player II

$$\text{Player I} \begin{bmatrix} 1 & -1 \\ -2 & 0 \\ 3 & 1 \end{bmatrix}$$

Is it a stable game ? (3.5,4)

5. (a) Solve graphically the game whose payoff matrix is

$$\begin{bmatrix} 2 & 4 & 11 \\ 7 & 4 & 2 \end{bmatrix}$$

(b) Use the relation of dominance to solve the game whose payoff matrix is given by

$$\begin{bmatrix} 1 & 7 & 3 & 4 \\ 5 & 6 & 4 & 5 \\ 7 & 2 & 0 & 3 \end{bmatrix}$$

(c) Reduce the following game to a Linear Programming Problem and then solve by simplex method.

$$\begin{bmatrix} 1 & -3 & 2 \\ -4 & 4 & -2 \end{bmatrix}$$

This question paper contains 2 printed pages. Your Roll No. ....

Sl. No. of Ques. Paper	: **1355**	**F–7**
Unique Paper Code	: **2351502**	
Name of Paper	: **Analysis – V (Complex Analysis)**	
Name of Course	: **B.Sc. (Hons.) Mathematics (Erstwhile FYUP)**	
Semester	: **V**	
Duration :	: **3 hours**	
Maximum Marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any five parts from Q. No. 1 and any two parts from each of Q. Nos. 2 to 6.
All questions are compulsory.

1. Attempt any five parts. Each part carries 3 marks.

(a) Find all accumulation points of the set $S = \{(-1)^n + \frac{i}{n} : n \in \mathbb{N}\}$.

(b) Sketch the image of the set $r \leq 1, 0 < \theta \leq \pi/4$ under the map $f(z) = z^2$ with justification.

(c) Let f be analytic in a domain D such that $f(z)$ is purely imaginary for all $z \in D$. Show that f must be constant.

(d) Determine the set of points at which the function $f(z) = \bar{z}$ is differentiable.

(e) Show that $\overline{\sin(i z)} = \sin(i \bar{z})$ if and only if $z = n\pi i$ $(n = 0, \pm 1, \pm 2, \dots)$.

(f) Let $f: \mathbb{C} \to \mathbb{C}$ be any function. Define the following notation:

$$\lim_{z \to \infty} f(z) = \infty.$$

(g) Find the order of the pole of the function

$$f(z) = \frac{1 - \exp(2z)}{z^4}$$

and find its residue.

(h) Write the principal part of the function $f(z) = z \exp(1/z)$ at its isolated singular point and determine whether that point is a pole, a removable singularity or an essential singular point.

P.T.O.

2. (a) Let the function f on $\mathbb{C}$ be given by $f(z) = \bar{z}^2/z$ if $z \neq 0$ and $f(0) = 0$. Show that $f'(0)$ does not exist. 6

(b) If $f(z) = u(x,y) + i\,v(x,y)$ is differentiable at $z_0 = x_0 + i\,y_0$, prove that the first order partial derivatives of u and v must exist at (x_0, y_0) and they must satisfy Cauchy-Riemann equations $u_x = v_y$ and $u_y = -v_x$ there. 6

(c) If $f(z) = z\,Im\,z$, determine the set of points where $f'(z)$ exists, and find the value of $f'(z)$ at all such points. 6

3. (a) Show that the function $f(z) = \exp \bar{z}$ is not analytic anywhere. 6

(b) If $z = x + i\,y$, prove that $|\sin z|^2 = \sin^2 x + \sinh^2 y$ and $|\cos z|^2 = \cos^2 x + \sinh^2 y$. Deduce that $\sin z$ and $\cos z$ are unbounded on $\mathbb{C}$. 6

(c) Find $\int_C f(z)dz$, where $f(z) = \pi\, e^{\pi \bar{z}}$ and C is the boundary of the square with vertices $0, 1, 1+i$ and i, the orientation of C being in the counter-clockwise direction. 6

4. (a) Let C denote the line segment from $z = i$ to $z = 1$. Without evaluating the integral, prove that $\left|\int_C \frac{dz}{z^4}\right| \leq 4\sqrt{2}$. 6

(b) State and prove the Cauchy Integral Formula. 6

(c) Show that if f is analytic within and on a simple closed contour C and z_0 is not on C, then $\int_C \frac{f'(z)}{z-z_0}\, dz = \int_C \frac{f(z)}{(z-z_0)^2}\, dz$. 6

5. (a) State Liouville's Theorem and use it to prove the Fundamental Theorem of Algebra. 6

(b) Obtain the Maclaurin's series for the function $f(z) = \sin z$, stating the region in which it is valid. 6

(c) Give two Laurent series expansion in powers of z for the function

$$f(z) = \frac{1}{z^2(1-z)}$$

and specify the regions in which these expansions are valid. 6

6. (a) If a power series $\sum_{n=0}^{\infty} a_n (z - z_0)^n$ converges when $z = z_1 (z_1 \neq z_0)$, then prove that it is absolutely convergent at each point of the open disk $|z - z_0| < R_1$, where $R_1 = |z_1 - z_0|$. 6

(b) State Cauchy's Residue Theorem. Use it to evaluate the integral

$$\int_C \frac{5z-2}{z(z-1)} dz.$$ 6

(c) Evaluate

$$\int_0^{2\pi} \frac{d\theta}{1 + a\cos\theta}, \quad |a| < 1.$$ 6

200

This question paper contains 4 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper	: 1356	F–7
Unique Paper Code	: 2351503	
Name of Paper	: Calculus – II (Multivariate Calculus)	
Name of Course	: B.Sc. (Hons.) Mathematics (Erstwhile FYUP)	
Semester	: V	
Duration :	: 3 hours	
Maximum Marks	: 75	

(Write your Roll No. on the top immediately on receipt of this question paper.)

All Sections are compulsory. Attempt any five questions from each Section.

Section-I

1. (a) Let f be the function defined by :

$$f(x,y)=\begin{cases}\dfrac{xy^3}{x^2+y^6}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

Is f continuous at (0, 0)? Justify your answer.

(b) Compute the slope of the tangent line to the graph of $f(x,y)=x\ln(x+y^2)$ at the point $P_0(e,0,e)$ in the direction parallel to the xz-plane.

$\left(2\frac{1}{2}+2\frac{1}{2}\right)$

2. A closed box is found to have length 2 ft, width 4 ft and height 3 ft where the measurement of each dimension is made with a maximum possible error of ± 0.02 ft. The top of the box is made from material that costs \$ 2/ft^2 and the material for sides and bottom costs only \$ 1.50/ft^2. What is the maximum error involved in the computation of cost of the box?

(5)

3. Find $\dfrac{\partial w}{\partial s}$,

if $w=4x+y^2+z^3$, where $x=e^{rs^2}$, $y=\ln\dfrac{r+s}{t}$, and $z=rst^2$.

(5)

4. (a) Find the gradient of the function $f(x,y,z)=\dfrac{xy-1}{z+x}$.

(b) Find the directional derivative of $f(x,y)=x^2+xy+y^2$ at the point $P_0(1,-1)$ in the direction towards the origin. $\left(2\frac{1}{2}+2\frac{1}{2}\right)$

5. Find the absolute extrema of the function $f(x,y)=2\sin x+5\cos y$ on the closed bounded rectangular region with vertices (0, 0), (2, 0), (2, 5) and (0, 5) **(5)**

6. Find the maximum and minimum distance from the origin to the ellipse $5x^2-6xy+5y^2=4$

(5)

Section-II

7. Sketch the region of integration and write an equivalent integral with the order of integration reversed:

$$\int_{-1}^{2}\int_{x^2-2}^{x} f(x,y)\,dy\,dx$$

(5)

8. Evaluate

$$\int_{0}^{2}\int_{0}^{\sqrt{2x-x^2}} y\sqrt{x^2+y^2}\,dy\,dx$$

by converting to polar coordinates.

(5)

9. Use a suitable change of variables to compute

$$\iint_D \exp\left(\frac{y-x}{y+x}\right)dy\,dx,$$

Where D is the triangular region with vertices (0, 0), (2, 0), (0, 2).

(5)

10. Find the volume of the tetrahedron T bounded by the plane $2x+y+3z=6$ and the coordinate planes $x=0$, $y=0$, and $z=0$ **(5)**

11. Evaluate the integral

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{\sqrt{2-x^2-y^2}} z\,dz\,dy\,dx$$

by transforming to cylindrical coordinates. **(5)**

12. Find the average value of the function $f(x,y,z) = x + y + z$ over the sphere

$x^2 + y^2 + z^2 = 4$. **(5)**

Section-III

13. Evaluate

$$\int_C [5xy\,dx + 10yz\,dy + z\,dz],$$

where C is the parabolic arc $x = y^2$ from (0, 0, 0) to (1, 1, 0) followed by the line segment given by $x = 1,\ y = 1,\ 0 \le z \le 1$.

(5)

14. Prove that the force field $\vec{\mathbf{F}} = \left[(2x - x^2 y)e^{-xy} + \tan^{-1} y\right]\hat{\mathbf{i}} + \left[\dfrac{x}{y^2+1} - x^3 e^{-xy}\right]\hat{\mathbf{j}}$ is conservative in $\mathbf{R}^2$ and using this evaluate the line integral $\int_C \vec{\mathbf{F}}.d\vec{R}$, where C is curve with parametric equations $x = t^2 \cos \pi t,\ y = e^{-t} \sin \pi t,\ 0 \le t \le 1$.

(5)

15. State Green's theorem for a simply connected region in $\mathbb{R}^2$. Use it to find the work done by the force field $\vec{\mathbf{F}}(x, y) = (3y - 4x)\hat{\mathbf{i}} + (4x - y)\hat{\mathbf{j}}$ when an object moves once counter-clockwise around the ellipse $4x^2 + y^2 = 4$. **(5)**

16. Evaluate

$$\iint_S (x + y + z)\, dS,$$

where S is the surface defined parametrically by

$\vec{\mathbf{R}}(u,v) = (2u + v)\ \hat{\mathbf{i}} + (u - 2v)\ \hat{\mathbf{j}} + (u + 3v)\ \hat{\mathbf{k}}$ for $0 \le u \le 1, 0 \le v \le 2$. **(5)**

17. Use Stokes' theorem to evaluate

$$\oint_C [(x+2z)dx+(y-x)dy+(z-y)dz],$$

where C is the boundary of the triangular region with vertices (3, 0, 0), (0, $3/2$, 0), (0, 0, 3) traversed counter clockwise as viewed from above.

(5)

18. Use the divergence theorem to evaluate the surface integral $\iint_S \vec{\mathbf{F}}\cdot\vec{\mathbf{N}}\,dS$, where

$\vec{\mathbf{F}}(x,y,z)=\cos yz\,\hat{\mathbf{i}}+e^{xz}\,\hat{\mathbf{j}}+3z^2\,\mathbf{k}$ and S is the closed hemispherical surface

$z=\sqrt{4-x^2-y^2}$ together with the disk $x^2+y^2\leq 4$ in the xy-plane and $\vec{\mathbf{N}}$ is the outward unit normal vector field.

(5)

19. State and prove the Gauss-divergence theorem.

(5)

This question paper contains 4 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper	**: 1357**	**F–7**
Unique Paper Code	**: 2351504**	
Name of Paper	**: Probability & Statistics**	
Name of Course	**: B.Sc. (Hons.) Mathematics (Erstwhile FYUP)**	
Semester	**: V**	
Duration :	**: 3 hours**	
Maximum Marks	**: 75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

In all there are six questions. Question No. 1 is compulsory and it contains seven parts of 3 marks each, out of which any five parts are to be attempted. In Question Nos. 2 to 6, attempt any two parts from three parts. Each part carries 6 marks.

Use of scientific calculator is allowed.

Q. 1 Attempt any five parts.

(i) There are 5 red chips and 3 blue chips in a bowl. The red chips are numbered 1, 2 ,3 4, 5, respectively, and the blue chips are numbered 1,2,3, respectively. If 2 chips are to be drawn at random and without replacement, find the probability that these chips have either the same number or the same color.

(ii) Verify that $b(x; n, \theta) = b(n\text{-}x; n, 1-\theta)$

(iii) Suppose X has the pdf

$$f_X(x)=\begin{cases} cx^3 & 0 < x < 2 \\ 0 & elsewhere \end{cases}$$

then find $P\left(\frac{1}{4} < X < 1\right)$.

(iv) Let X_1 and X_2 have the joint pmf $p(x_1, x_2)$ described as follows:

(x_1, x_2)	(0,0)	(0,1)	(1,0)	(1,1)	(2,0)	(2,1)
$p(x_1, x_2)$	1/18	3/18	4/18	3/18	6/18	1/18

and $p(x_1, x_2)$ is equal to zero elsewhere. Find $E(X_2 \mid x_1)$, when $x_1 = 1$.

P.T.O

(v) Let X be a random variable such that $E[(X-b)^2]$ exists for all real b. Show that $E[(X-b)^2]$ is a minimum when $b = E(X)$.

(vi) Find the median of the distribution

$$f(x)=\frac{1}{\pi(1+x^2)}, \quad -\infty < x < \infty$$

(vii) Suppose that the number of items produced in a factory during a week is a random variable X with mean 500. If the variance of a week's production is known to equal 100, then find the lower bound for $P\{|X-500|<100\}$.

Q.2 (i) Show that in the limit when $n \to \infty$, $\theta \to 0$ and $n\theta = \lambda$ remains constant, the number of successes in the binomial distribution $B(n,\theta)$ is a random variable having a Poisson distribution with the parameter λ.

(ii) Show that if X is a random variable having a binomial distribution with the parameters n and θ, then the moment generating function of

$$Z=\frac{X-n\theta}{\sqrt{n\theta(1-\theta)}}$$

approaches that of the standard normal distribution when $n \to \infty$.

(iii) Show that the moment-generating function of the geometric distribution is given by

$$M_X(t)=\frac{\theta e^t}{1-e^t(1-\theta)}$$

Use it to show that $\mu=\frac{1}{\theta}$ and $\sigma^2=\frac{1-\theta}{\theta^2}$.

Q.3 (i) Show that the rth moment about the origin of the gamma distribution is given by

$$\mu_r'=\frac{\beta^r\Gamma(\alpha+r)}{\Gamma(\alpha)}$$

and hence show that the mean and variance are given by $\mu=\alpha\beta$ and $\sigma^2=\alpha\beta^2$.

(ii) Let X_1 and X_2 have the pdf

$$f(x_1,x_2)=\begin{cases}8x_1x_2 & 0<x_1<x_2<1\\ 0 & elsewhere\end{cases}$$

Evaluate $E(7X_1X_2^2+5X_2)$.

(iii) Let $f_{1/2}(x_1/x_2) = c_1 x_1/x_2^2$, $0 < x_1 < x_2, 0 < x_2 < 1$, zero elsewhere, and

$f_2(x_2) = c_2 x_2^4, 0 < x_2 < 1$, zero elsewhere, denote respectively , the conditional pdf of X_1

, given $X_2 = x_2$, and the marginal pdf of X_2. Determine:

(a) The constants c_1 and c_2.
(b) The joint pdf of X_1 and X_2.
(c) $P\left(\frac{1}{4} < X_1 < \frac{1}{2} / X_2 = \frac{5}{8}\right)$.
(d) $P\left(\frac{1}{4} < X_1 < \frac{1}{2}\right)$.

Q. 4 (i) Let the random variables X and Y have the joint pdf

$$f(x,y) = \begin{cases} x+y & 0 < x < 1, 0 < y < 1 \\ 0 & elsewhere \end{cases}$$

Compute a) $P\left(X_1 \leq \frac{1}{2}\right)$

b) $P(X_1 + X_2 \leq 1)$

c) ρ_{XY}

(ii) Let $f(x_1, x_2) = 21 x_1^2 x_2^3$, $0 < x_1 < x_2 < 1$, zero elsewhere, be the joint pdf of X_1 and X_2.

(a) Find the conditional mean and variance of X_1, given $X_2 = x_2$, $0 < x_2 < 1$.
(b) Find the distribution of $Y = E(X_1/X_2)$

(iii) Let the continuous type random variables X and Y have the joint pdf

$$f(x,y) = \begin{cases} e^{-y} & 0 < x < y < \infty \\ 0 & elsewhere \end{cases}$$

Find

(a) mgf of joint distribution.
(b) $Cov(X,Y)$
(c) Are the random variables dependent?

Q.5 (i) If X and Y have a bivariate normal distribution, show that the conditional density of Y given $X = x$ is a normal distribution with the mean

$$\mu_{Y/x} = \mu_2 + \rho\frac{\sigma_2}{\sigma_1}(x-\mu_1)$$

and the variance

$$\sigma^2_{Y/x} = \sigma^2_2(1-\rho^2)$$

(ii) Given the joint density

$$f(x,y) = \begin{cases} xe^{-x(1+y)} & x>0 \; and \; y>0 \\ 0 & elsewhere \end{cases}$$

find $\mu_{Y/x}$.

(iii) Given the joint density

$$f(x,y) = \begin{cases} 24xy & x>0, y>0 \; and \; x+y<1 \\ 0 & elsewhere \end{cases}$$

Find the regression equation of Y on X using the formula

$$\mu_{Y/x} = \mu_2 + \rho\frac{\sigma_2}{\sigma_1}(x-\mu_1)$$

Q.6 (i) State and prove Chebyshev's Inequality.

(ii) Let X be the number of times that a fair coin, flipped 40 times, lands heads. Find the probability that $X = 20$. Use the normal approximation and then compare it to the exact solution.

(iii) Suppose that the chance of rain tomorrow depends on previous weather conditions only through whether or not it is raining today and not on past weather conditions. Suppose also that if it rains today, then it will rain tomorrow with probability α ; and if it does not rain today, then it will rain tomorrow with probability β .If $\alpha = 0.7$ and $\beta = 0.4$, then calculate the probability that it will rain four days from today given that it is raining today.

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1962 **GC-3** **Your Roll No...............**

Unique Paper Code : 62351101

Name of the Paper : Calculus

Name of the Course : **B.A. (Prog.) Mathematics (CBCS)**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two parts** from each question.

1. (a) Discuss the existence of the limit of the function $f(x) = |2x - 1|$ at $x = \frac{1}{2}$. (6)

(b) Examine the continuity of the function at x = 0 for

$$f(x) = \begin{cases} x\dfrac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

Also state the kind of discontinuity, if any. (6)

(c) Examine the following function for differentiability at x = 0 and x = 1 :

$$f(x) = \begin{cases} x^2 & x \leq 0 \\ 1 & 0 < x \leq 1 \\ 1/x & x > 1 \end{cases}$$ (6)

2. (a) If $V = r^m$, where $r^2 = x^2 + y^2 + z^2$, show that

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = m(m+1)r^{m-2}.$$ (6)

(b) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$, prove that

$$x^2 y_{n+2} + (2n+1)x\, y_{n+1} + 2n^2 y_n = 0.$$ (6)

(c) If $z = \sec^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$, prove that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2 \cot z.$$ (6)

3. (a) If the tangent to the curve $\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1$ cuts off intercept p and q from the axis of x & y respectively. Show that $\frac{p}{a} + \frac{q}{b} = 1$. (6)

(b) Find the point where the tangent to the curve $y = x^2 - 3x + 2$ is perpendicular to the line $y = x$. (6)

(c) Show that the radius of curvature for the curve $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ is $4a \cos(\theta/2)$. (6)

4. (a) Find the asymptotes of the curve

$$x^3 + x^2y - xy^2 - y^3 + 2xy + 2y^2 - 3x + y = 0.$$ (6½)

(b) Find the position and nature of the double points on the curve

$$x^4 + y^3 - 2x^3 + 3y^2 - a^4 = 0$$ (6½)

(c) Trace the curve

$$x^2(x^2 + y^2) = 4(x^2 - y^2).$$ (6½)

5. (a) State and prove Lagrange's Mean Value theorem. (6)

(b) Separate the intervals in which the function

$$f(x) = 2x^3 - 15x^2 + 36x + 1$$

is increasing or decreasing. (6)

(c) Prove that $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$, for $0 \le x \le 1$. (6)

6. (a) State Cauchy's Mean Value theorem. Give its geometrical interpretation. Also verify Cauchy's mean value theorem for the functions $f(x) = \sin x$, $g(x) = \cos x$ in the interval $\left[-\frac{\pi}{2}, 0\right]$. (6½)

(b) Find the minimum and maximum value of the function x^x. (6½)

(c) If $\lim_{x \to 0} \frac{\sin 3x - a \sin x}{x^3}$ is finite, then find the value of a and the limit. (6½)

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 198 **G** **Your Roll No...............**

Unique Paper Code : 235351

Name of the Paper : Integration and Differential Equations

Name of the Course : **B.A. (Prog.) – Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.

1. (a) Find the area enclosed by the curves $y^2 = 4x$ *and* $y = 2x - 4$. (6)

(b) Obtain a reduction formula for $\int sec^n x\, dx$, n being a positive integer and hence evaluate $\int sec^6 x\, dx$. (6)

(c) Evaluate :

$$\int \frac{1}{3sinx - 4cosx}\, dx.$$ (6)

2. (a) Show that

$$\int_0^{\frac{\pi}{2}} cos^m x cos\, nx dx = \frac{m}{m+n}\int_0^{\frac{\pi}{2}} \cos^{m-1} x cos(n-1)x dx.$$

Further show that

$$\int_0^{\frac{\pi}{2}} \cos^n x\, cosnx\, dx = \frac{\pi}{2^{n+1}},$$

where m and n being positive integers. (6½)

(b) Find the exact arc length of the curve

$$24xy = y^4 + 48 \text{ from } y = 2 \text{ to } y = 4.$$ (6½)

(c) Let V_x and V_y be the volume of the solids that result when the region enclosed by

$$y = \frac{1}{x},\ y = 0,\ x = \frac{1}{2} \text{ and } x = b\ (b > 2)$$

is revolved about x – axis and y – axis, respectively. Is there any value of b for which $V_x = V_y$? (6½)

3. (a) Evaluate :

(i) $\int_0^{\pi/2} \log(\tan x + \cot x)\, dx$

(ii) $\int \frac{2x+3}{\sqrt{4x^2+5x+6}}\, dx$ (3+3)

(b) Solve :

$$y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0.$$ (6)

(c) Given that $y = x + 1$ is a solution of

$$(x+1)^2 \frac{d^2y}{dx^2} - 3(x+1)\frac{dy}{dx} + 3y = 0.$$

Find a linearly independent solution by reducing the order. (6)

4. (a) Solve :

$$x^2 \frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 4x - 6.$$ (6)

(b) Using the concept of Wronskian ,show that $e^x \sin x$ and $e^x \cos x$ are linearly independent solution of

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0\,.$$

Find the solution $\mathbf{y}(x)$ satisfying the conditions $\mathbf{y}(0) = 2$ and $\mathbf{y}'(0) = -3$. (6)

(c) Assume that the population of a certain city increases at a rate proportional to the number of inhabitants at any time. If the population doubles in 40 years, in how many years will it triple ? (6)

5. (a) Using method of variation of parameters, find the general solution of

$$\frac{d^2y}{dx^2}+y=tanx\,. \qquad (6½)$$

(b) Solve :

$$a^2\,y^2 z^2\,dx \;+\; b^2\,x^2\,z^2\,dy \;+\; c^2\,x^2\,y^2\,dz \;=\; 0. \qquad (6½)$$

(c) Solve the system of equations :

$$\frac{dx}{dt}+\frac{dy}{dt}+2x+y=0$$

$$\frac{dy}{dt}+5x+3y=0 \qquad (6½)$$

6. (a) (i) Classify the following partial differential equation into elliptic, parabolic or hyperbolic form :

$$r \;+\; 2s \;+\; t \;=\; 0$$

where $r = \dfrac{\partial^2 z}{\partial x^2},\ s = \dfrac{\partial^2 z}{\partial x\,\partial y},\ t = \dfrac{\partial^2 z}{\partial y^2}$. (3)

(ii) Eliminate the arbitrary function f to form the partial differential equation from the following equation

$$z=f\left(\frac{xy}{z}\right). \qquad (3½)$$

(b) Find the general integral of the linear partial differential equation

$$z(xp - yq) = y^2 - x^2.$$ (6½)

(c) Find the complete integral of the partial differential equation

$$p + q = pq.$$ (6½)

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 199 **G** **Your Roll No...............**

Unique Paper Code : 235351

Name of the Paper : Integration and Differential Equations

Name of the Course : **B.A. (Prog.) – Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.

1. (a) Find the area enclosed between the curves $y = x^2$, $y = \sqrt{x}$; $x = 1/4$, $x = 1$. (6)

(b) Evaluate :

$$\int \frac{x^3}{\left(1+x^2\right)^{9/2}}\, dx \quad (6)$$

(c) If $I_n = \int_0^{\frac{\pi}{2}} \theta \sin^n \theta \; d\theta \; (n > 1)$, show that

$$I_n = \frac{n-1}{n} I_{n-2} + \frac{1}{n^2}.$$

Deduce that $I_5 = \dfrac{149}{225}$ (6)

2. (a) Show that

$$\int_0^{\pi/2} Sin^n x \, dx = \frac{n-1}{n} \int_0^{\pi/2} Sin^{n-2} x \, dx \qquad (n \geq 2)$$

Use this result to derive Wallis Sine formulas :

$$\int_0^{\frac{\pi}{2}} Sin^n x\, dx = \frac{\pi}{2}\,\frac{1.3.5 \dots\dots..(n-1)}{2.4.6 \dots\dots\dots\dots n}\;(n \text{ even and } \geq 2)$$

$$\int_0^{\frac{\pi}{2}} Sin^n x\, dx = \frac{2.4.6 \dots\dots..(n-1)}{3.5.7 \dots\dots\dots\dots n}\;(n \text{ odd and } \geq 3)$$ (6½)

(b) Find the volume of the solid that results when the region above x – axis and below ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, \quad (a>0, b>0)$$

is revolved about the x – axis. (6½)

(c) Find the circumference of a circle of radius a from the parametric equation

$x = a \cos t,\; y = a \sin t \quad (0 \leq t \leq 2\pi)$. (6½)

3. (a) Evaluate :

(i) $\int_0^{\pi/2} \log \sin x\, dx$

(ii) $\int \frac{x+2}{\sqrt{4+3x-2x^2}}\, dx$ (3+3)

(b) Solve :

$$(x^2 + y^2 + x)\, dx + xy\, dy = 0.$$ (6)

(c) Given that $y = x$ is a solution of

$$\left(x^2+1\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0.$$

Find a linearly independent solution by reducing the order. (6)

4. (a) Solve :

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=4\sin\log x \qquad (6)$$

(b) Using the concept of Wronskian, show that e^x and e^{2x} are linearly independent solution of

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=0.$$

Find the solution $y(x)$ satisfying the conditions $y(0) = 0$ and $y'(0) = 1$. (6)

(c) Find the orthogonal trajectories of the family of circles

$$x^2 + y^2 = c^2. \qquad (6)$$

5. (a) Using method of variation of parameters, find the general solution of

$$\frac{d^2y}{dx^2}+y=\cot x. \qquad (6½)$$

(b) Solve :

$$(yz + z^2)dx - xzdy + xydz = 0 \qquad (6½)$$

(c) Solve the system of equations :

$$\frac{dx}{dt}+\frac{dy}{dt}-y=2t+1$$

$$2\frac{dx}{dt}+2\frac{dy}{dt}+y=t \qquad (6½)$$

6. (a) (i) Form a partial differential equation by eliminating the constant a and b from the following equation :

$$2z = (ax + y)^2 + b.$$

(ii) Classify the following partial differential equation into hyperbolic, parabolic or elliptic form :

$$(n-1)^2 \frac{\partial^2 z}{\partial x^2} - y^{2n} \frac{\partial^2 z}{\partial y^2} = ny^{2n-1} \frac{\partial z}{\partial y}.$$ (3+3½)

(b) Find the general integral of the following partial differential equation

$$y^2 p - xy\, q = x(z - 2y)$$ (6½)

(c) Find the complete integral of the partial differential equation

$$(p^2 + q^2)y = qz.$$ (6½)

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2319**

Unique Paper Code : **62354343** **GC-3**

Name of the Paper : **Analytical Geometry and Applied Algebra**

Name of the Course : **B.A. (Prog.) Mathematics (CBCS)**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

1. (*a*) Identify and sketch the curve :

$$(y-3)^2 = 6(x-2)$$

and also label the focus, vertex and directrix. 6

(*b*) Describe the graph of the curve :

$$4x^2 + y^2 + 8x - 10y = -13.$$ 6

(*c*) Sketch the hyperbola :

$$16x^2 - y^2 - 32x - 6y = 57.$$

Also find the vertices, foci, asymptotes and the equation of directrices. 6

2. (*a*) Find the equation of the parabola that has its focus at $(0, -3)$ and directrix $y = 3$. Also state the reflection property of parabola. 6

(*b*) Find an equation for the ellipse with length of minor axis 8 and with vertices $(2, 6)$ and $(2, -4)$ and also sketch it. 6

(*c*) Find and sketch the curve of the hyperbola that has vertices at $(2, 4)$ and $(10, 4)$ and foci are 10 units apart. 6

3. (*a*) Consider the equation :

$$3x^2 + 2\sqrt{3}xy + y^2 - 8x + 8\sqrt{3}y = 0.$$

Rotate the coordinate axes to remove the xy-term. Then identify the type of conic represented by the equation and sketch its graph. 6

(*b*) Let an $x'y'$-coordinate system be obtained by rotating an xy-coordinate system through an angle $\theta = 45°$.

(*i*) Find the $x'y'$-coordinate of the point whose xy-coordinates are $\left(\sqrt{2}, \sqrt{2}\right)$.

(*ii*) Find an equation of the curve $x^2 - xy + y^2 - 6 = 0$ in $x'y'$-coordinates. 6

(*c*) Find the equation of the sphere through the four points $(4, -1, 2)$, $(0, -2, 3)$, $(1, -5, -1)$, $(2, 0, 1)$. 6

4. (*a*) Let $u = i - 3j + 2k$, $v = i + j$ and $w = 2i + 2j - 4k$. Find the length of $3u - 5v + 2w$. Also find the volume of the parallelepiped with adjacent edges u, v and w. 6½

(*b*) Prove that :

$$u.v = \frac{1}{4}\left(\|u + v\|^2 - \|u - v\|^2\right).$$ 6½

(*c*) (*i*) Using vector, find the area of triangle with vertices A(2, 2, 0), B(–1, 0, 2) and C(0, 4, 3).

(*ii*) Sketch the surface $z = \cos x$ in 3-space. 3+3½

5. (*a*) (*i*) Find the parametric equation of line that is tangent to the circle $x^2 + y^2 = 25$ at the point (3, –4).

(*ii*) Show that the lines L_1 and L_2 intersect and find their point of intersection :

$$L_1 : x = 1 + 4t, \quad y - 3 = t, \; z - 1 = 0$$

$$L_2 : x + 13 = 12t, \quad y - 1 = 6t, \; z - 2 = 3t.$$ 3+3½

(*b*) Find the distance between the skew lines :

$$L_1 : x = 1 + 4t, \quad y = 5 - 4t, \; z = -1 + 5t, \; -\infty < t < \infty$$

$$L_2 : x = 2 + 8t, \quad y = 4 - 3t, \; z = 5 + t, \; -\infty < t < \infty.$$ 3+3½

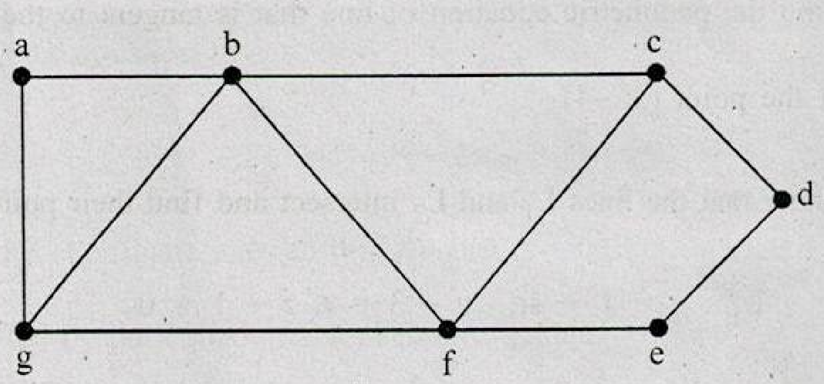
(*c*) (*i*) Find the equation of the plane through (–1, 4, 2) that contains the line of intersection of the planes $4x - y + z - 2 = 0$ and $2x + y - 2z - 3 = 0$.

(*ii*) Do the points (1, 0, –1), (0, 2, 3), (–2, 1, 1) and (4, 2, 3) lie in the same plane. Justify your answer. 3+3½

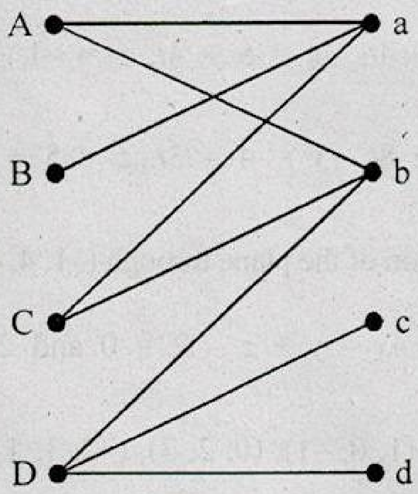
6. (*a*) Define a Latin square. Given an example of a Latin square of order 3. Is it unique ? Justify. 6½

(*b*) Three Pitcher of sizes 7L, 4L and 3L (L = litre) are given. Only 7L pitcher is full. Find a minimum sequence of pouring to make the quantity in three pitchers as 2L, 2L, 3L. 6½

(*c*) (*i*) In the following figure find all sets of two vertices whose removal disconnects the graph :



(*ii*) Find a matching or explain why none exists for the following graph :



3+3½

This question paper contains 3 printed pages. **Your Roll No. ....**

Sl. No. of Ques. Paper	: 2320	**GC-3**
Unique Paper Code	: 62354343	
Name of Paper	: **Analytical Geometry and Applied Algebra**	
Name of Course	: **B.A. (Prog.) Mathematics (CBCS)**	
Semester	: **III**	
Duration :	: **3 hours**	
Maximum Marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.
Attempt any two parts from each question.

SET-C

1. (a) Identify and sketch the curve:
$$x = y^2 - 4y + 2$$
and also label the focus, vertex and directrix. 6

(b) Describe the graph of the curve:
$$3(x+2)^2 + 4(y+1)^2 = 12$$
Also find its centre and foci. 6

(c) Describe the graph of the hyperbola:
$$x^2 - y^2 - 4x + 8y - 21 = 0$$
And sketch its graph. 6

2. (a) Find the equation of the parabola that has its vertex at $(1,2)$ and focus at $(4,2)$. Also state the reflection property of parabola. 6

(b) Find the equation of the ellipse whose length of major axis is 26 and foci $(\pm 5,0)$ and also sketch it. 6

(c) Find and sketch the curve of the hyperbola whose foci are $(6,4)$ and $(-4,4)$ and eccentricity is 2. 6

3. (a) Consider the equation:
$$3x^2 + 2xy + 3y^2 = 19.$$

Rotate the coordinate axes to remove the xy-term. Then identify the type of conic represented by the equation and sketch its graph. 6

(b) Let an x'y' - coordinate system be obtained by rotating an xy - coordinate system through an angle $\theta = 30°$.

(i) Find the x'y' - coordinate of the point whose xy - coordinates are (2, 4).

(ii) Find an equation of the curve $2x^2 + 2\sqrt{3}xy = 3$ in x'y' - coordinates . 6

(c) Find the equation of two spheres that are centered at the origin and are tangent to the sphere of radius 1 centered at $(0,0,7)$. 6

4(a) (i) Find a vector of length 9 and oppositely directed to $\mathbf{v} = -5\mathbf{i} + 4\mathbf{i} + 8\mathbf{k}$.

(ii) Sketch the surface $2x + z = 3$ in 3-space. $3+3\frac{1}{2}$

(b) (i) Find the vector component of $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$ orthogonal to $\mathbf{b} = \mathbf{i} + 2\mathbf{j} + 28\mathbf{k}$.

(ii) Find the area of triangle with vertices P(2, 0, −3), Q(1,4, 5), R(7, 2, 9). $3+3\frac{1}{2}$

(c) Prove that

$$\|\mathbf{u}+\mathbf{v}\|^2 + \|\mathbf{u}-\mathbf{v}\|^2 = 2\|\mathbf{u}\|^2 + 2\|\mathbf{v}\|^2$$

and interpret the result geometrically. $6\frac{1}{2}$

5 (a) Let L_1 and L_2 be the lines whose parametric equations are

L_1 : $x = 4t$ $y = 1-2t$ $z = 2 + 2t$

L_2 : $x = 1 + t$ $y = 1 - t$ $z = -1 + 4t$

(i) Show that the lines L_1 and L_2 intersect at the point (2, 0, 3).

(ii) Find the parametric equation of line that is perpendicular to L_1 and L_2 and passes through their point of intersection. $3+3\frac{1}{2}$

(b) (i) Determine whether the points P_1 (6, 9, 7) , P_2 (9, 2, 0) and P_3 (0, −5, −3) lie on the same line.

(ii) Where does the line

$x = 2 - t,\quad y = 3t,\quad z = -1 + 2t$

intersect the plane $2y + 3z = 6$. $3+3\frac{1}{2}$

(c) (i) Find the equation of the plane through (1, 4, 3) that is perpendicular to the line

$x = 2 + t,\quad y + 3 = 2t,\quad z = -t.$

(ii) Determine whether the planes

$3x - 2y + z = 1$, $4x + y - 2z = 4$

are parallel, perpendicular or neither. $3 + 3\frac{1}{2}$

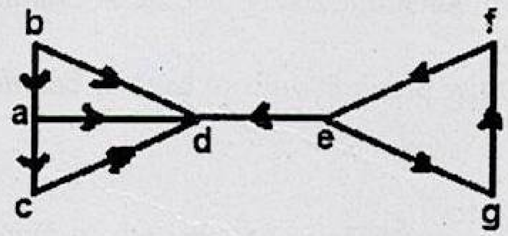
6. (a) Given three containers 3, 7, and 10 liters respectively with the largest being full of water, determine a minimum sequence of pouring method of dividing this quantity of water into two equal amounts of 5 liters using the three containers and no other measuring devices. $6\frac{1}{2}$

(b) Is the following square a Latin square? Can it be a group with the multiplication operation defined?

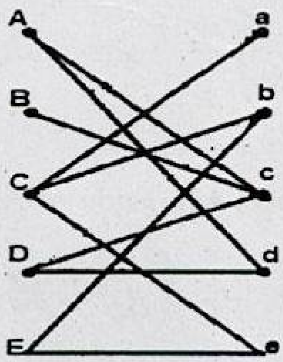
*	1	2	3	4	5
1	1	2	3	4	5
2	2	1	4	5	3
3	3	4	5	2	1
4	4	5	1	3	2
5	5	3	2	1	4

$6\frac{1}{2}$

(c) (i) Given the influence model. Find the sets of minimum number of vertices which can influence every other vertex in the graph.



(ii) Find a matching or explain why none exists for the following graph.



$3 + 3\frac{1}{2}$

1000

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2136**

Unique Paper Code : **32353301** **GC-3**

Name of the Paper : **Latex and HTML**

Name of the Course : **B.Sc. (Hons.) Mathematics—CBCS : Skill Enhancement Course**

Semester : **III**

Duration : **2 Hours** Maximum Marks : **50**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

1. Fill in the blanks : 5×1=5

(*i*) In LaTeX, items can be listed using .... environment.

(*ii*) Line breaks in a LaTeX document are produced by .... command.

(*iii*) .... command is used to give comments in a latex document.

(*iv*) .... command is used to draw a circle with center (x, y) and radius r in pstricks.

(*v*) .... attribute of the img tag in HTML is used to specify the source of the image.

2. Answer any *eight* parts from the following : 8×2½=20

(*i*) Give the command in LaTeX to obtain the expression $\left(\frac{a+b}{x+y}\right)^{\frac{1}{3}}$.

P.T.O.

(*ii*) Write the difference between the commands **\vdots** and **\ddots**.

(*iii*) Write the output of the command :

$\frac{d}{dx}\left(\int_{0}^{x}f(t)\, dt\right) = f(x)$.

(*iv*) What is wrong with the following HTML construction :

<p> This is   bold and italics </p> .

(*v*) Give any *three* attributes of the ***font*** tag in HTML.

(*vi*) What is wrong with the following input :

<p> Also checkout the

 University of Delhi </p>

(*vii*) Write a code in LaTeX for typesetting $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$.

(*viii*) Write a LaTeX code to produce $\mathbb R$ in the output.

(*ix*) Write the command in LaTeX to generate the expression $x^{x^{x^{x}}}$.

(*x*) What is the output of the command \psarc(1, 1) {2} {0} {70} in pstricks.

3. Answer any *five* questions from the following : 5×5=25

(*i*) Write the code in LaTeX to plot the curves $y = \sqrt{x}$ and $y = x^2$ on the same coordinate. Show the square root function as a dotted curve and the square function as a dashed curve.

(*ii*) Find the errors in the following LaTeX source, write a corrected version and write its output :

```
\Documentclass{article}

\usepackage{amsmath}

\begin{document}

We have following options

\begin{itemize}

\item $$x\ge y$

\item $x\le y$

\item x=y

\end{document}
```

(*iii*) Write a code in LaTeX for typesetting the following expression :

$$\frac{\frac{5}{a^2b}-\frac{2}{ab^2}}{\frac{3}{a^2b^2}+\frac{4}{ab}}=\frac{\frac{5b-2a}{a^2b^2}}{\frac{3+4ab}{a^2b^2}}.$$

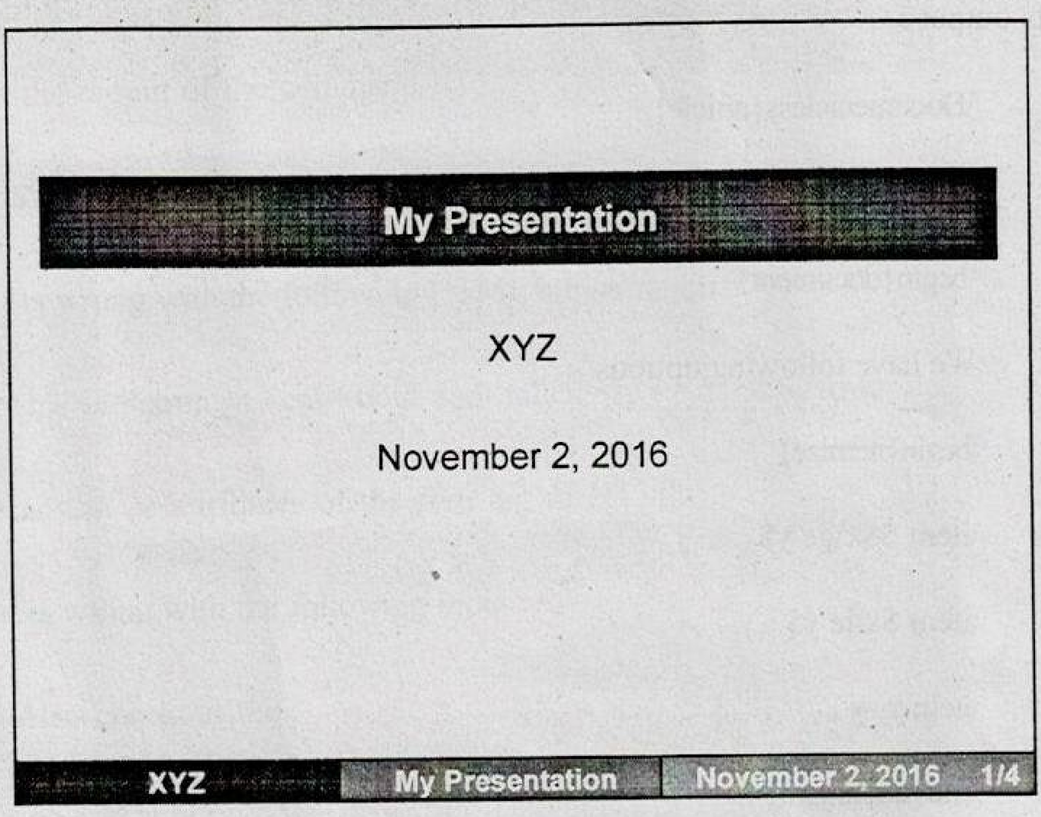
(*iv*) Write a code in LaTeX to typeset the following :

A system of linear equations in 3 variables x_1, x_2 and x_3 can be represented as :

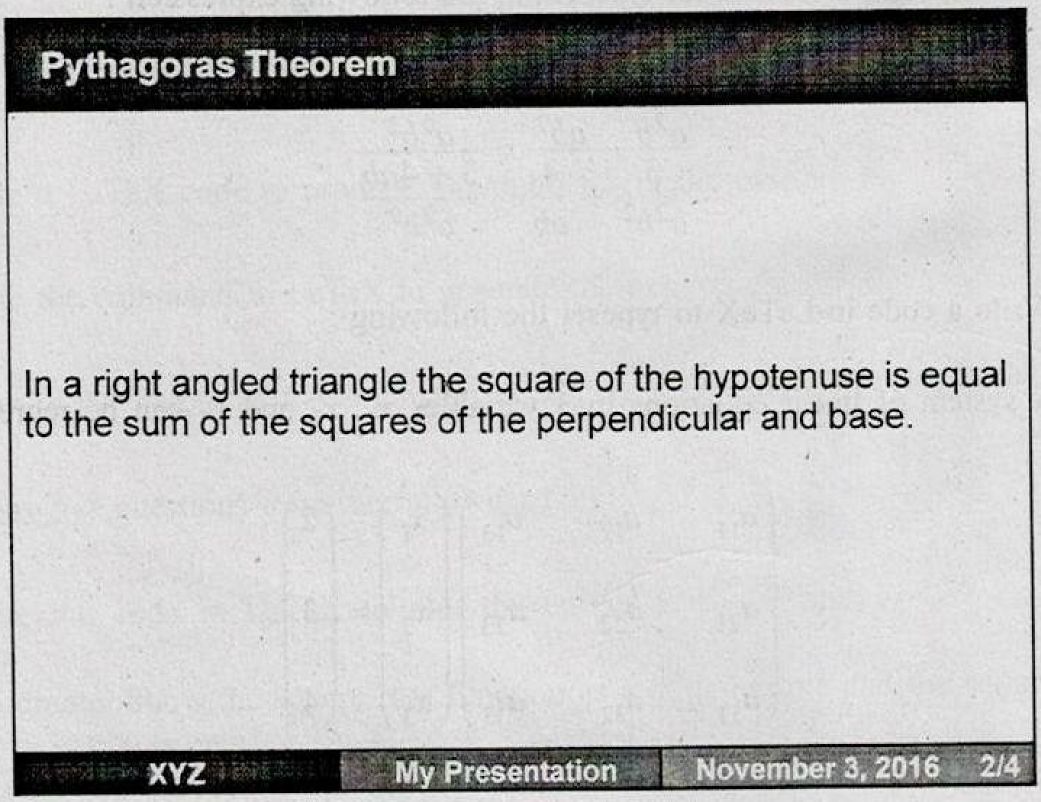
$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}.$$

(*v*) Write LaTeX code in beamer to prepare the following presentation :

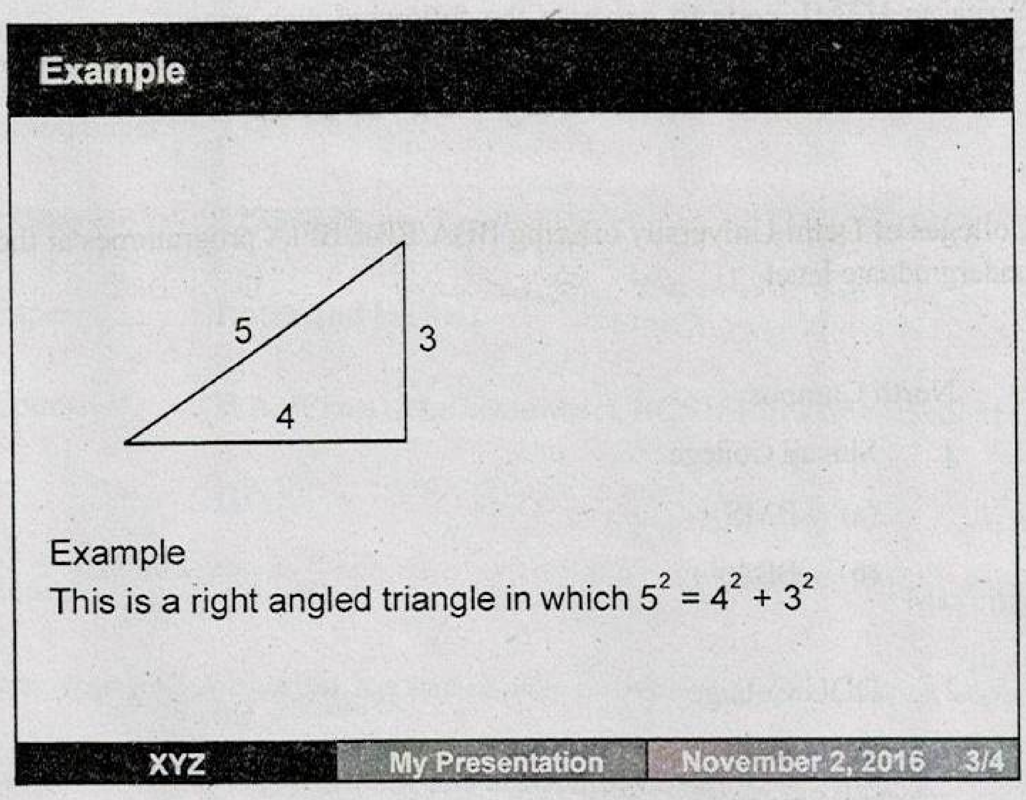
Slide 1



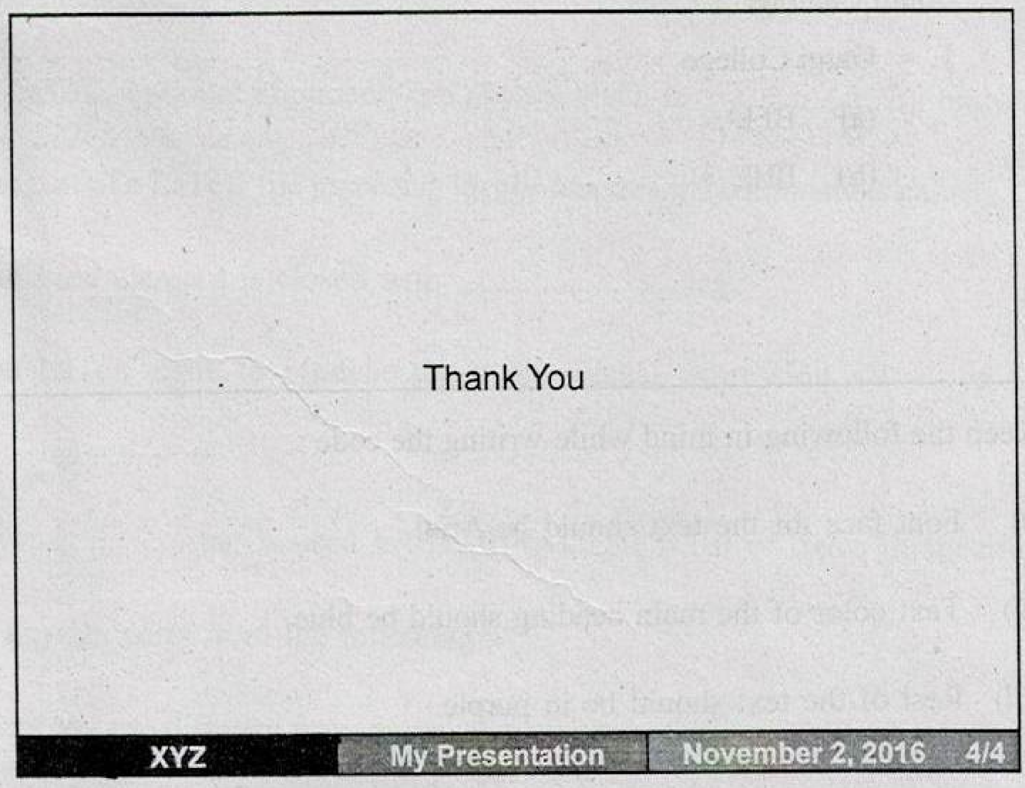
Slide 2



Slide 3



Slide 4



(*vi*) Write an HTML code to generate the following web page :

University of Delhi

Colleges of Delhi University offering BBA/BBE/BFIA programmes at the undergraduate level

- North Campus
  1. Shivaji College
     (a) BMS
     (b) BBA

  2. DDU College
     (a) BBE
     (b) BMS
- South Campus
  1. Gargi College
     (a) BFIA
     (b) BBE

Keep the following in mind while writing the code :

(*i*) Font face for the text should be Arial.

(*ii*) Text color of the main heading should be blue.

(*iii*) Rest of the text should be in purple.

This question paper contains **4+1** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2349**

Unique Paper Code : **62353325** **GC-3**

Name of the Paper : **Latex and HTML**

Name of the Course : **B.A. (Prog.) Mathematics (CBCS) Skill Enhancement Course**

Semester : **III**

Duration : **2 Hours** Maximum Marks : **50**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

1. Fill in the blanks : 5×1=5

   (*a*) In LaTeX, optional arguments are always given in .... brackets.

   (*b*) The part of a LaTeX file preceding ***\begin {document}*** command is called ....

   (*c*) The ***html*** element is closed with .... tag.

   (*d*) The LaTeX code to produce the mathematical expression $e^{i\theta} = \cos\theta + i\sin\theta$ is ....

   (*e*) .... tag is used in HTML to create a list of items in specified order.

2. Answer any *ten* parts from the following : 10×2=20

   (1) Write any *two* different ways of including mathematical expressions in LaTeX document.

P.T.O.

(2) Write the difference between the commands ***\ldots*** and ***\cdots***.

(3) What is the output of \pscircle (3, 2){1} in pstricks ?

(4) Write the output of the command **$\sqrt[n]{5}$**.

(5) What is the command for writing the set {0, 1} in LaTeX ?

(6) Explain the difference in the outputs of the following two LaTeX source codes :

(*i*) \begin{document}

Suppose that \$ x = 25 \$

\end{document}

(*ii*) \begin{document}

Suppose that \$\$ x = 25 \$\$

\end{document}

(7) Write a set of commands to be put in the main document in LaTeX to produce :

$$\lim_{x\to 0} \frac{\sin x}{x} = 1.$$

(8) Write the output of :

\documentclass{beamer}

\title{Skill Enhancement Course}

\author{ABC}

\institute{University of Delhi}

```
\begin{document}

\begin{frame}

\titlepage

\end{frame}

\end{document}
```

(9) Write a code in LaTeX to produce the output :

$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}.$$

(10) Write the following postfix expressions in the standard form :

x 1 *add* 2 *exp* 1 *x sub div*.

(11) What is the output of the command **\psline (1, 1) (5, 1) (1, 4) (1, 1)** in pstricks ?

Also, give a rough sketch of the same.

(12) What is wrong with the following input ? What is the right way to do it ?

If $theta = pi$ then $sin theta = 0$.

Answer any *five* parts from the following : 5×5=25

(*i*) Write a code in LaTeX to plot the function :

$$f(x) = \begin{cases} x^2, & 0 \leq x \leq 2 \\ -x^2, & -2 \leq x < 0 \end{cases}.$$

(*ii*) Write the code for the following in LaTeX environment :

Let $x = (x_1, x_2, \ldots\ldots, x_n)$, where the x_i are non-negative real numbers. Set :

$$\mathrm{M}_r(x) = \left(\frac{x_1^r + x_2^r + \ldots\ldots + x_n^r}{n}\right)^{\frac{1}{r}}, \quad r \in \mathbf{R}\backslash\{0\}$$

and

$$\mathrm{M}_0(x) = (x_1 x_2 \ldots\ldots x_n)^{1/n}.$$

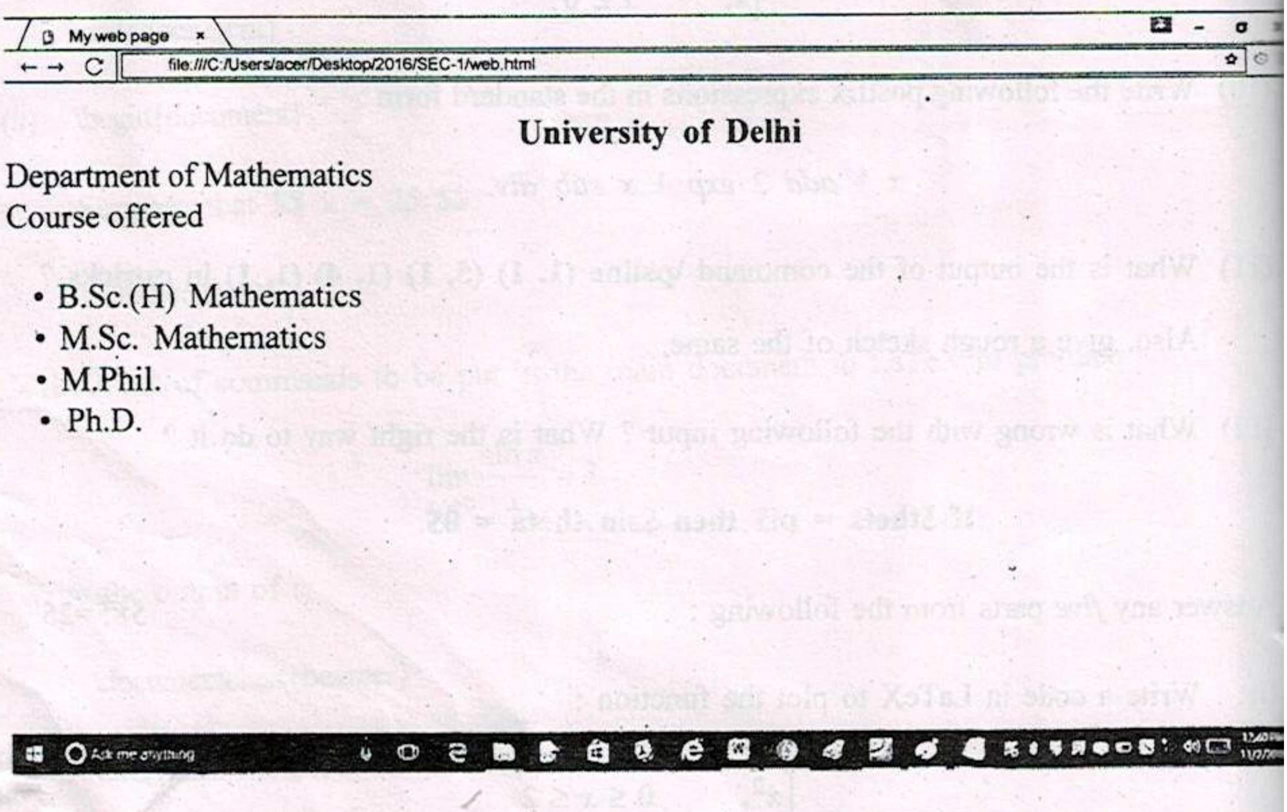
(*iii*) Write a presentation in beamer with the following content :

Slide – 1 contains **the title of the presentation, author's name and affiliation**

Slide – 2 contains the **list of subjects taught in B.A. (Prog.) course**

Slide – 3 contains **Thank You**

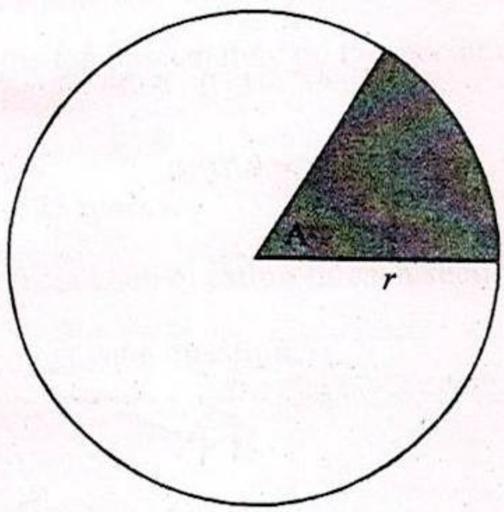
(*iv*) Write an ***html*** code to generate the following web page :



(*v*) Write a code in LaTeX to get the following matrix :

$$A = \begin{bmatrix} a & c & e \\ b & d & f \\ g & i & k \\ h & j & l \end{bmatrix}.$$

(*vi*) Write the code in LaTeX to draw the following circle with shaded sector :



[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 238 **G** **Your Roll No...............**

Unique Paper Code : 235551

Name of the Paper : Analysis

Name of the Course : **B.A. Programme – Mathematics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. There are **3** sections.
3. Each section consists of **25** marks.
4. Attempt any **two** parts from each question in each section.
5. Marks are indicated against each question.

SECTION I

1. (a) Define supremum and infimum of the set $S \subseteq R$. Find the supremum and infimum of the set

$$S = \left\{1, 1+\frac{1}{2}, 1+\frac{1}{2}+\frac{1}{2^2}, ..., 1+\frac{1}{2}+\frac{1}{2^2}+.....+\frac{1}{2^{n-1}}, ...\right\} \quad (6)$$

(b) State the properties which make the set R of real umbers, a complete ordered field. (6)

(c) Define limit point of a set. Show that the set N of natural numbers has no limit point. (6)

2. (a) State Bolzano-Weierstrass theorem for sets. Prove that the set $\left\{3^n + \frac{1}{3^n}; n \in N\right\}$ has no limit point. How does it contradict Bolzano-Weierstrass theorem ? (6.5)

(b) Test the continuity of the function

$$f(x) = \begin{cases} \frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases} \text{ at } x = 0$$ (6.5)

(c) Define uniform continuity of a function f on an interval I. Show that the function defined by $f(x) = x^3$ is uniformly continuous on $[-3, 3]$. (6.5)

3. (a) If $a_n \leq b_n \leq c_n$ for all n and $\langle a_n \rangle$ and $\langle c_n \rangle$ converge to l then $\langle b_n \rangle$ also converges to l. (6.5)

(b) State Monotone convergence theorem for sequence and hence prove that the sequence $\langle a_n \rangle$ defined as

$a_1 = 1,\ a_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \ldots\ldots.. + \frac{1}{(n-1)!}\ (n \geq 2)$ is convergent. (6.5)

(c) State Cauchy's General Principle of convergence for sequence and show that the sequence $\langle a_n \rangle$ where $a_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots.. + \frac{1}{n}$ is not convergent. (6.5)

4. (a) Let $\sum_{1}^{\infty} u_n$ and $\sum_{1}^{\infty} v_n$ be two positive terms series such that $u_n \leq kv_n$ $\forall$ n, k being a fixed positive number then prove that

(i) $\sum u_n$ converges if $\sum v_n$ is convergent

(ii) $\sum v_n$ diverges if $\sum u_n$ is divergent (6)

(b) Test the convergence of the following series :

(i) $\frac{\sqrt{2}-\sqrt{1}}{1}-\frac{\sqrt{3}-\sqrt{2}}{2}+\frac{\sqrt{4}-\sqrt{3}}{3}-\text{.........}$

(ii) $\sum_{1}^{\infty}(-1)^n \frac{n+2}{2^n+5}$

(iii) $\sum_{1}^{\infty}(-1)^{n-1}\frac{1}{n}$ (6)

(c) State Cauchy's General Principle of convergence for an infinite series $\sum_{1}^{\infty} u_n$

and show that the series $\sum_{1}^{\infty}\frac{1}{n}$ does not converge. (6)

5. (a) A bounded function f is integrable on [a, b] iff for every $\varepsilon > 0$, there exists partition P of [a, b] such that $U(P, f) - L(P, f) < \varepsilon$. (6.5)

(b) Show that $\int_{-\infty}^{\infty} e^{-x^2}\,dx$ converges. (6.5)

(c) Show that $\int_0^2 x^4\left(8-x^2\right)^{-\frac{1}{2}}dx = \frac{16}{3}\beta\left(\frac{5}{3}, \frac{2}{3}\right)$. (6.5)

6. (a) Find the Fourier series of the function f where

$$f(x)=\begin{cases}-1, & \text{for } -\pi \le x < 0 \\ 1, & \text{for } 0 \le x \le \pi\end{cases}$$ (6)

(b) State Cauchy's uniform convergence criteria for a sequence of functions.

Test the sequence $\langle f_n(x) \rangle$ for uniform convergence, where

$$f_n(x) = \frac{\sin nx}{\sqrt{n}}, \quad 0 \le x \le 2\pi$$ (6)

(c) (i) Find the radius of convergence of the power series

$$\frac{1}{2}x + \frac{1.3}{2.5}x^2 + \frac{1.3.5}{2.5.8}x^3 + \ldots\ldots\ldots\ldots$$

(ii) If f is defined in [0, 1] by the condition

$f(x) = (-1)^{r-1}$, when $\frac{1}{r+1} < x \le \frac{1}{r}$, $(r = 1, 2, 3, \ldots\ldots)$,

$f(0) = 0$. Show that $\int_0^1 f(x)\,dx = \log 4 - 1$ (6)

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 239 G **Your Roll No...............**

Unique Paper Code : 235551

Name of the Paper : Analysis

Name of the Course : **B.A. Programme – Mathematics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. There are **3** sections.
3. Each section consists of **25** marks.
4. Attempt any **two** parts from each question in each section.
5. Marks are indicated against each question.

SECTION I

1. (a) Let A and B be non empty subsets of R and let

$$C = \{x + y : x \in A, y \in B\}$$

If each of the sets A and B has a supremum, show that C has a supremum and Sup C = Sup A + Sup B. (6)

(b) Give example of a set $S \subseteq R$ which has

(i) Exactly one limit point

(ii) Infinite number of limit points

(iii) Two limit points (6)

(c) Show that function defined as $f(x) = \begin{cases} x & \text{if } x \text{ is irrational} \\ -x, & \text{if } x \text{ is rational} \end{cases}$

is continuous only at $x = 0$. (6)

2. (a) Define an open set $S \subseteq R$. Show that intersection of two open sets is again an open set but the intersection of infinite number of open sets need not be open. Justify your answer by an example (6.5)

(b) Show that the function $f(x) = \lim_{n\to\infty} \frac{x^n}{1+x^n}$ for all $x \geq 0$,

is continuous at all points except at $x = 1$. (6.5)

(c) Show that the function f defined by $f(x) = x^2$ is uniformly continuous on [−2, 2]. (6.5)

SECTION II

3. (a) Show that $\lim_{n\to\infty} (n)^{1/n} = 1$. (6.5)

(b) If $\langle a_n \rangle$ and $\langle b_n \rangle$ be two sequences such that

$\lim_{n\to\infty} a_n = a,\ \lim_{n\to\infty} b_n = b,$ then show that

$\lim_{n\to\infty} (a_n b_n) = \left(\lim_{n\to\infty} a_n\right)\left(\lim_{n\to\infty} b_n\right) = ab.$ (6.5)

(c) If $\lim_{n\to\infty} a_n = l$, then show that $\lim_{n\to\infty} \frac{a_1 + a_2 + \dots\dots + a_n}{n} = l$. (6.5)

4. (a) State and prove D'Alembert's Ratio Test for a positive terms series $\sum_{1}^{\infty} u_n$.

(6)

(b) Test the convergence of the following series :

(i) $\frac{x}{1.3}+\frac{x^2}{2.4}+\frac{x^3}{3.5}+\frac{x^4}{4.6}+$........ for all values of x.

(ii) $\sum_{1}^{\infty}\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$

(iii) $\sum_{1}^{\infty}\frac{\sqrt{n+1}-\sqrt{n-1}}{n}$ (6)

(c) Define absolute convergence and conditional convergence of an infinite series. Show that every absolutely convergent series is convergent. With the help of an example, show that the converse need. (6)

SECTION III

5. (a) Define Riemann integrability of a bounded function over [a, b]. Show that every monotonic function on [a, b] is integrable on [a, b]. (6)

(b) Discuss the convergence of the improper integral $\int_0^{\frac{\pi}{2}} \log \sin x \, dx$. (6)

(c) (i) Define Beta and Gamma functions. What is the relation between Beta and Gamma functions ?

(ii) Show that $\sqrt{\frac{1}{2}} = \sqrt{\pi}$ (6)

6. (a) Find the Fourier series in $[-\pi, \pi]$ for the function :

$$f(x) = \begin{cases} x, & \text{if } -\pi < x \leq 0 \\ 2x, & \text{if } 0 < x \leq \pi \end{cases} \quad (6.5)$$

(b) State Weierstrass's M-test for the series of the functions f_n defined on [a, b]. Show that the series $\sum_{1}^{\infty} \frac{\sin nx}{n^p}$ is uniformly convergent for all real values of x if p > 1. (6.5)

(c) (i) Find the radius of convergence of the power series

$$\sum_{0}^{\infty} \frac{(n+1)}{(n+2)(n+3)} x^n .$$

(ii) Discuss the Riemann integrability of the function f on [0, 3] where f(x) = [x], [x] is the greatest integer ≤ x. (6.5)

'5 Gp R

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 2257 **GC-3** **Your Roll No...............**

Unique Paper Code : 32355101

Name of the Paper : GE – I Calculus

Name of the Course : **Generic Elective for Hons. Courses**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. Do any **five** questions from each of the **three** sections.
3. Each question is for **five** marks.

SECTION I

1. Use $\varepsilon - \delta$ definition to show that

$$\lim_{x \to 3}(3x-7)=2.$$

2. Find the equations of the asymptotes for the curve

$$f(x)=\frac{x^3+1}{x^2}.$$

3. Find the Linearization of

$$f(x) = \sin x \quad \text{at} \quad x = \pi.$$

4. For $f(x) = x^3 - 3x + 3$

(i) Identify where the extrema of 'f' occur.

(ii) Find where the graph is concave up and where it is concave down.

5. Use L'Hôpital's rule to find

$$\lim_{x \to \pi/2} \frac{1-\sin x}{1+\cos 2x}.$$

6. The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

7. Find the length of the astroid

$$x = \cos^3 t, \quad y = \sin^3 t, \quad 0 \le t \le 2\pi.$$

SECTION II

8. State Limit comparison test. Using the limit comparison test, discuss the convergence of

$$\int_1^\infty \frac{dx}{1+x^2}.$$

9. Identify the symmetries of the curve and then sketch the graph of

$$r = \sin 2\theta.$$

10. Solve the initial value problem for $\vec{r}$ as a vector function of t

Differential equation : $\frac{d^2 \vec{r}}{dt^2} = 32\hat{k}$

Initial Conditions : $\vec{r}(0) = 100\hat{k}$

and : $\left(\frac{d\vec{r}}{dt}\right)_{t=0} = 8\hat{i} + 8\hat{j}$

11. Find the curvature for the helix

$$\vec{r}(t) = (a\cos t)\hat{i} + (a\sin t)\hat{j} + bt\hat{k},\ a, b \geq 0\ \ a^2 + b^2 \neq 0$$

12. Write the acceleration vector $\vec{a} = a_T\hat{T} + a_N\hat{N}$ at the given value of t without finding $\hat{T}$ and $\hat{N}$ for the position vector given by

$$\vec{r}(t) = (t\cos t)\hat{i} + (t\sin t)\hat{j} + t^2\hat{k},\quad t = 0$$

13. Show that $f(x,y) = \begin{cases} \dfrac{2xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

is continuous at every point except at the origin.

14. If $f(x,y) = \dfrac{x - y + 2\sqrt{x} - 2\sqrt{y}}{\sqrt{x} - \sqrt{y}}$

(i) Find the domain of the given function f(x,y).

(ii) Evaluate $\lim\limits_{(x,y)\to(0,0)} f(x,y)$.

SECTION III

15. If $z = 5\tan^{-1} x$ and $x = e^u + \ln v$,

find $\dfrac{\partial z}{\partial u}$ and $\dfrac{\partial z}{\partial v}$ using chain rule, when $u = \ln 2$, $v = 1$.

16. Find the directions in which the given function f increase and decrease most rapidly at the given point p_0. Then, find the derivative of the function in those directions.

$$f(x,y,z) = \frac{x}{y} - yz,\ p_0(4,1,1)$$

17. Find parametric equations for the line tangent to the curve of intersection of the given surfaces at the given point.

$$\text{Surfaces}: x + y^2 + 2z = 4, \quad x = 1$$

$$\text{Point}: (1, 1, 1).$$

18. Find equations for the

(a) Tangent plane and

(b) Normal line at the point p_0 on the given surface

$$z^2 - 2x^2 - 2y^2 - 12 = 0; \quad p_0(1, -1, 4).$$

19. Find the absolute maxima and minima of the function $f(x, y) = x^2 + y^2$ on the closed triangular plate bounded by the lines $x = 0$, $y = 0$, $y + 2x = 2$ in the first quadrant.

20. If $f(x,y) = x\cos y + ye^x$, find $\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y \partial x}, \frac{\partial^2 f}{\partial y^2}$ and $\frac{\partial^2 f}{\partial x \partial y}$.

21. If $f(x,y) = x - y$ and $g(x,y) = 3y$

Show that

(i) $\nabla(fg) = g\nabla f + f\nabla g$

(ii) $\nabla\left(\frac{f}{g}\right) = \frac{g\nabla f - f\nabla g}{g^2}$

12/13 (30)

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 2594 **GC-3** **Your Roll No..............**

Unique Paper Code : 32355301

Name of the Paper : Differential Equations

Name of the Course : **Generic Elective – 3 for Hons. Courses, Under CBCS**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.

2. Attempt **all** questions by selecting any **two** parts from each question.

1. (a) Find an integrating factor and solve the differential equation :

$$\left(e^{x+y} - y\right)dx + \left(xe^{x+y} + 1\right)dy = 0.$$ (6.5)

(b) Solve the equation : $y' + (x+1)y = e^{x^2} y^3$, $y(0) = 0.5$. (6.5)

(c) Find the orthogonal trajectories of the family of parabolas $y = ce^{-3x}$. (6.5)

2. (a) Show that e^{3x} and xe^{3x} form a basis of the following differential equation $y'' - 6y + 9y - 0$. Find also the solution that satisfies the conditions $y(0) = -1.4$, $y'(0) = 4.6$. (6)

(b) Solve the initial value problem : (6)

$$x^2y'' + 3xy' + y = 0,\ y(1) = 4,\ y'(1) = -2.$$

(c) Find the radius of convergence of the series : $\sum_{m=2}^{\infty} \frac{(-1)^m (x-1)^{2m}}{4^m}$ (6)

3. (a) Find a general solution of the following nonhomogeneous differential equation :

$$y'' + 3y' + 2y = 30e^{2x}$$

using variation of parameters. (6.5)

(b) Use the method of undetermined coefficients to find the particular solution of the differential equation : $y'' - 4y' + 4y = 2e^{2x}$. (6.5)

(c) Find a homogeneous linear ordinary differential equation for which two functions x^3 and x^{-2} are solutions. Show also linear independence by considering their Wronskian. (6.5)

4. (a) Find the general solution of the partial differential equation

$$yzu_x - xzu_y + xy(x^2 + y^2)u_z = 0.$$ (6)

(b) Find a general solution of the differential equation :

$$(x^2D^2 + xD - 4I)y = 0, \text{ where } D = \frac{d}{dx}.$$ (6)

(c) Find the particular solution of the linear system that satisfies the stated initial conditions :

$$\frac{dy_1}{dt} = -5y_1 + 2y_2,\ y_1(0) = 1$$

$$\frac{dy_2}{dt} = 2y_1 - 2y_2,\ y_2(0) = -2$$ (6)

5. (a) Find a power series solution of the following differential equation, in powers of x

$$y'' + xy' - 2y = 0.$$ (6.5)

(b) Find the solution of the quasi-linear partial differential equation :

$$u(x + y)u_x + u[x - y)u_y = x^2 + y^2$$

with the Cauchy data $u = 0$ on $y = 2x$. (6.5)

(c) Reduce the equation : $yu_x + u_y = x$

to canonical form, and obtain the general solution. (6.5)

6. (a) Solve the initial-value problem :

$$u_x + 2u_y = 0, \quad \mu(0, y) = 4e^{-2y}$$

using the method of separation of variables. (6)

(b) Obtain the canonical form of the equation : $x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} = 0$, and hence find the general solution. (6)

(c) Reduce the following partial differential equation with constant coefficients,

$$u_{xx} + 2u_{xy} + 5u_{yy} + u_x = 0.$$

into canonical form. (6)

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 15

Unique Paper Code : 235166 G

Name of the Paper : **Maths-I Calculus and Matrices (MAPT-101)**

Name of the Course : **B.Sc. (Hons.) Computer Science/B.Sc. (Mathematical Sciences)/ B.Sc. (Physical Sciences)**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* questions from each section.

Section I

1. (*a*) Verify that the set $\{(1, -1), (1, 1)\}$ is a basis of R^2.

   (*b*) Examine which of the following is a subspace of R^2 :

   $$U = \{(a, b^2);\ a, b \in R\}$$

   $$V = \{(a, b);\ a > 0,\ a, b \in R\}.$$

   (*c*) Which of the following transformations are linear ? Also justify :

   (1) $T : R^2 \rightarrow R^2$ defined as $T(x, y) = (1, 2)$

   (2) $T : R^3 \rightarrow R^2$ defined as $T(x, y, z) = (x, 4y)$. 4,4,4

2. (*a*) Solve the following system of equations using elementary row operations :

$$2x + 3y + z = 9$$

$$x + 2y + 3z = 6$$

$$3x + y + 2z = 8.$$

(*b*) Reduce the matrix :

$$\begin{pmatrix} 2 & 3 & 3 \\ 3 & 6 & 12 \\ 2 & 4 & 8 \end{pmatrix}$$

to a riangular form by elementary row operations and hence determine its rank.

(*c*) Find Eigen values and Eigen vectors corresponding to one of them for the matrix : 4,4,4

$$\begin{pmatrix} 1 & 3 & 1 \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{pmatrix}.$$

3. (*a*) For what values of λ and μ do the following system of equations :

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu$$

have an infinite number of solutions.

(*b*) Find the inverse of the matrix using E-row operations :

$$\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}.$$

(*c*) Find the value of c for which the vectors $\{(1, 4), (c, -8)\}$ are linearly dependent.

4,4,4

Section II

4. (*a*) Use the definition to show that the sequence $\left(2 + \left(-\frac{1}{2}\right)^n\right)$ converges to 2.

(*b*) Sketch the graph of $y = \sin(2x + 1)$. Mention the transformation used at each step.

(*c*) Find the *n*th derivative of :

$$y = \frac{x}{1 + 3x + 2x^2}.$$

6,6,6

5. (*a*) Assume that the rate at which radioactive nuclei decay is proportional to the number of nuclei present in a given sample. In a certain sample, 10% of the original number of radioactive nuclei has undergone disintegration in a period of 200 years. Find the percentage of the original radioactive nuclei that will remain after 1000 years.

(*b*) Show that :

$$w(x, t) = 4\cos(2x + 2ct) + e^{x+ct}$$

satisfies wave equation.

(*c*) If $u = f(r)$, where

$$r = \sqrt{x^2 + y^2},$$

prove that : 6,6,6

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r}f'(r).$$

6. (*a*) If

$$y = e^{m \cos^{-1} x},$$

show that :

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0.$$

(*b*) Find the Maclaurin's series expansion for $f(x) = \sin x$, assuming that :

$$\lim_{n\to\infty} R_n(x) = 0.$$

(*c*) Sketch the level curves of the function $f(x, y) = 10 - x^2 - y^2$ of height 1, 6 and 10. 6,6,6

Section III

7. (*a*) Prove that for any *two* complex numbers z_1 and z_2 :

$$|z_1 + z_2| \le |z_1| + |z_2|.$$

(*b*) Let z_1, z_2, z_3 be complex numbers such that :

$$z_1 + z_2 + z_3 = 0 \text{ and } |z_1| = |z_2| = |z_3| = 1.$$

Prove that :

$$z_1^2 + z_2^2 + z_3^2 = 0.$$

4,3½

8. (*a*) Use De Moivre's theorem to solve the equation :

$$z^7 - z^4 + z^3 - 1 = 0.$$

(*b*) Prove that :

$$\left(\frac{1 + \sin\phi + i\cos\phi}{1 + \sin\phi - i\cos\phi}\right)^n = \cos\left(\frac{n\pi}{2} - n\phi\right) + i\sin\left(\frac{n\pi}{2} - n\phi\right).$$

4,3½

9. (*a*) Form an equation in the lowest degree with real coefficients which has $2 - 3i$ and $3 + 2i$ as two of its roots.

(*b*) Find all the values of $\left(\sqrt{3} - i\right)^{2/5}$. 4,3½

This question paper contains **4+2** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **17**

Unique Paper Code : **236163/234506** **G**

Name of the Paper : **Operational Research—I (Concurrent)**

Name of the Course : **B.Sc. Mathematical Sciences & B.Sc. (H) Computer Science**

Semester : **I/V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *Five* questions in all.

Question No. **1** is compulsory.

Use of non-programmable calculator is allowed.

1. (*i*) Explain the nature of O.R. and its limitations.

(*ii*) What is shadow price ? Write the dual of the following LPP :

Minimize : $Z = x_1 + x_2 + x_3$

Subject to :

$$x_1 - 3x_2 + 4x_3 = 5$$

$$x_1 - 2x_2 + 7x_3 \leq 3$$

$$2x_2 - x_3 \geq 4$$

$$x_1 \geq 0 \; x_2 \text{ and } x_3 \text{ unrestricted in sign}$$

P.T.O.

(*iii*) Consider the following LP with two variables :

Maximize : $Z = 3x_1 + 2x_2$

Subject to :

$$x_1 + x_2 \leq 4$$

$$x_1 - x_2 \leq 2$$

$$x_1, x_2 \geq 0.$$

(*a*) Determine all the basic solutions of the problem, and classify them as feasible and infeasible.

(*b*) Verify optimal solution graphically. Show how infeasible basic solutions are represented on graphical solútion space.

(*iv*) Consider the following LPP :

Maximize : $Z = 5x_1 - 6x_2 + 3x_3 - 5x_4 + 12x_5$

Subject to :

$$x_1 + 3x_2 + 5x_3 + 6x_4 + 3x_5 \leq 30$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0.$$

Solve the problem by inspection (do not use Gauss-Jordan row operations), and justify the answers in terms of the basic solutions of the simplex method.

(*v*) What do you understand by the convex set. Show that the following set is convex :

$$C = \{(x_1, x_2) \mid x_1^2 + x_2^2 \leq 3, x_1 \geq 0, x_2 \geq 0\}.$$

(*vi*) A manufacturing company purchases 9000 parts of a machine for its annual requirements, ordering one month usage at a time. Each part costs Rs. 20. The ordering cost per order is Rs. 15 and the carrying charges are 15% of the average inventory per year. You have been assigned to suggest a more economical purchasing policy for the company. What advice would you offer and how much would it save the company per year ?

(*vii*) Explain ABC analysis. What are its advantages and limitations, if any ? 7×5=35

2. In the Ma-and-Pa grossery store, shelf space is limited and must be used effectively to increase profit. Two cereal items grano and wheat compete for a total shelf space of 60 sq. feet. A box of grano occupies 0.2 sq. feet and a box of wheat needs 0.4 sq. feet. The maximum daily demand of grano and wheat are 200 and 120 boxes respectively. A box of grano nets \$ 1 in profit and a box of wheat \$ 1.35. Ma-and-Pa thinks that because the unit profit of wheat is 35% higher than that of grano, wheat should be allocated 35% more space than grano which amounts to allocating about 57% to wheat and 43% to grano. What do you think ? Formulate the above problem as linear programming problem and solve it graphically. 10

3. Solve the following LPP using two-phase method :

Maximize : $Z = 2x_1 + 2x_2 + 4x_3$

Subject to :

$$2x_1 + x_2 + x_3 \le 2$$

$$3x_1 + 4x_2 + 2x_3 \ge 8$$

$$x_1, x_2, x_3 \ge 0$$

(*a*) Show that phase I will terminate with an artificial variable in the basis.

(*b*) Remove artificial variable prior to the start of phase II, then carry out phase II. 10

4. (*a*) The following table represents a simplex iteration. All variables are non-negative. The table is not optimal for either a maximisation or minimization problem. Thus, when a non-basic variable enters the solution it can either increase or decrease z or leave it unchanged, depending on the parameters of the entering non-basic variable.

Basic	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	**Solution**
x_8	0	3	0	–2	–3	–1	5	1	12
x_3	0	1	1	3	1	0	3	0	6
x_1	1	–1	0	0	6	–4	0	0	0
z-row	0	–5	0	4	–1	–10	0	0	620

Categorize the variables as basic and non-basic and provide the current values of all the variables. Assuming that the problem is of the maximisation type, identify the non-basic variables that have the potential to improve the value of z. If each such variable enters the basic solution, determine the associated leaving variable, if any, and the associated change in z. Do not use the Gauss-Jordan row operation.

(*b*) Consider the following Linear Programming Problem :

Maximize : $Z = 5x_1 + 2x_2 + 3x_3$

Subject to :

$$x_1 + 5x_2 + 2x_3 = 30$$

$$x_1 - 5x_2 - 6x_3 \le 40$$

$$x_1, x_2, x_3 \ge 0$$

Given that the artificial variable x_4 and the slack variable x_5 form the starting basic variables and that M equals 100 when solving the problem, the optimal table is given in the following table. Write the associated dual problem and determine its optimal solution. 2×5=10

Basic	x_1	x_2	x_3	x_4	x_5	**Solution**
x_1	1	5	2	1	0	30
x_5	0	−10	−8	−1	1	10
z-row	0	23	7	105	0	150

5. Solve the following LPP using simplex algorithm and hence list out five optimal solutions using alternative optimal solution : 10

Maximize : $Z = 2x_1 + 4x_2$

Subject to :

$$x_1 + 2x_2 \leq 5$$

$$x_1 + x_2 \leq 4$$

$$x_1, x_2 \geq 0.$$

6. (*a*) A company manufactures two products A and B. The unit revenues are \$ 2 and \$ 3, respectively. Two raw materials M1 and M2, used in the manufacture of the two products have daily availability of 8 and 18 units, respectively. One unit of A uses 2 units of M1 and 2 units of M2 and the respective values for B are 3 and 6.

(*i*) Determine the dual prices of M1 and M2 and their feasibility ranges graphically.

(*ii*) Is it advisable to arrange 2 additional units of M1 at the cost of 25 cents per unit ?

(*b*) Write a short note on degeneracy of LPP. 7,3

7. (*a*) Derive the expression for EOQ under the following assumptions :

(*i*) demand is known and uniform

(*ii*) shortages are not allowed

(*iii*) lead time is zero.

(*b*) Find the optimum order quantity for a product for which the price breaks are as follows :

Quantity	**Unit Cost** (in Rs.)
$0 \leq Q_1 < 800$	Re. 1.00
$800 \leq Q_2$	Re. 0.98

The yearly demand for the product is 1600 units per year, cost of placing an order is Rs. 5, the cost of storage is 10% per year. 2×5=10

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **24**

Unique Paper Code : **235366** **G**

Name of the Paper : **Mathematics–III (Algebra) MAPT-303**

Name of the Course : **B.Sc. Mathematical Sciences/B.Sc. App. Physical Science/ Analytical Chemistry/Industrial Chemistry/B.Sc. Physical Science**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

All questions are compulsory.

Marks are indicated.

Unit I

1. (*a*) Show that :

$$G = \left\{ \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} : a \in R \right\}$$

is a group under matrix multiplication. 6

(*b*) Prove that the group G is abelian if and only if :

$$(ab)^{-1} = a^{-1}b^{-1} \quad \forall a, b \in G.$$ 6

(*c*) Let H be a non-empty finite subset of a group G. Then H is a subgroup of G if and only if H is closed under the operation of G. 6

P.T.O.

2. (*a*) Find the inverse of the element $\begin{bmatrix} 2 & 6 \\ 3 & 5 \end{bmatrix}$ in $GL(2, Z_{11})$. 6

(*b*) Show that :

$$U(14) = \langle 3 \rangle = \langle 5 \rangle.$$

Is $U(14) = \langle 11 \rangle$? 6

(*c*) Let

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{pmatrix},$$

Write :

(*i*) α as a product of disjoint cycles.

(*ii*) α as a product of 2-cycles.

(*iii*) Determine whether α is an even odd permutation. 6

3. (*a*) State and prove Lagrange's theorem for subgroups. 6

(*b*) Prove that the center :

$$Z(G) = \{x \in G : xg = gx \ \forall g \in G\}$$

of a group G is a normal subgroup of G. 6

(*c*) Let H be a subgroup of G and $a, b \in G$. Then show that : 6

$$|aH| = |bH|.$$

Unit II

4. (*a*) Let

$$Z[\sqrt{2}] = \{a + b\sqrt{2} : a, b \in Z\}$$

Prove that $Z[\sqrt{2}]$ is ring under the ordinary addition and multiplication of real numbers. 6½

(*b*) Prove that a finite integral domain is a field. 6½

(*c*) Let $M_2(Z)$ be the rings of all 2 × 2 matrices over the integers and let :

$$R = \left\{\begin{bmatrix} a & a-b \\ a-b & b \end{bmatrix} : a, b \in Z\right\}.$$

Prove or disprove that R is a subring of $M_2(Z)$. 6½

Unit III

5. (*a*) Determine whether or not the set :

$$\{(2, -1, 0), (1, 2, 5), (7, -1, 5)\}$$

is linearly independent over R. 6½

(*b*) Let

$$V = R^3 \text{ and } W = \{(a, b, c) \in V : a^2 + b^2 = c^2\}.$$

Is W a subspace of V ? If so, what is its dimension ? 6½

(*c*) For the vector space :

$$V = \left\{\begin{bmatrix} a & b \\ b & c \end{bmatrix} : a, b, c \in Q\right\}$$

find a basis for V over Q. 6½

6. (*a*) Which of the following functions T from R^2 into R^2 are linear transformations :

(*i*) $T(x_1, x_2) = (1 + x_1, x_2)$

(*ii*) $T(x_1, x_2) = (x_1^2, x_2)$. 6½

(*b*) Let T be a function R^2 to R^3 defined by :

$$T(x_1, x_2) = (x_1 + x_2, x_1 - x_2, x_2).$$

Find the range, rank, kernel and nullity of T. 6½

(*c*) Let T be linear operator on C^2 defined by :

$$T(x_1, x_2) = (x_1, 0)$$

Let β be the standard ordered basis for C^2 and let $\beta' = \{\alpha_1, \alpha_2\}$ be the ordered basis defined by :

$$\alpha_1 = (1, i), \ \alpha_2 = (-i, 2),$$

What is the matrix of T relative to pair β, β' ? 6½

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **63**

Unique Paper Code : **235161** **G**

Name of the Paper : **Mathematics and Statistics (MACT-303)**

Name of the Course : **B.Sc. (Hons.) Botany/Zoology/Biological Sciences/B.Sc. Life Sciences**

Semester : **III/I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

There are *three* sections in this question paper.

Attempt any *two* questions from each section.

Students are allowed to use simple calculator.

Section I

1. (*a*) For what value of k is the function : 5½

$$f(x) = \begin{cases} x^2 + 5, & x \leq 2 \\ k - 4x, & x > 2 \end{cases}$$

continuous at $x = 2$?

(*b*) Out of 300 people in an office, 140 drink coffee and 180 drink tea. If 45 drink neither, find out how many drink both. Also illustrate this fact by a Venn diagram. 5½

(*c*) Let x be a multiple of 3 (i.e. 0, 3, 6, 9, 12 ....) and $f(x)$ be the remainder obtained on dividing x by 5. Is $f(x)$ a periodic function of x ? If yes, what is the period ? Plot the graph of the function. 5½

2. (*a*) Is the sequence

$$<\frac{\sin^2 n}{n}>$$

convergent ? If yes, what is its limit ? 5½

(*b*) Find the sum of the infinite geometric series : 5½

$$5-\frac{10}{3}+\frac{20}{9}-\frac{40}{27}+\ldots\ldots.$$

(*c*) (*i*) Differentiate $\log_e(\log_e(x))$ w.r.t. x.

(*ii*) If

$$y = 2 - \sin x + \sec x,$$

find $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$. 5½

3. (*a*) Expand $e^{-x} + e^{2x}$ in ascending powers of x. 5½

(*b*) Write the first five terms of the sequence given by the recursion formula : 5½

$$a_1 = -2,\ a_2 = 1,\ a_{n+2} = \frac{a_{n+1}}{a_n}.$$

(*c*) Evaluate : 5½

$$\int_2^{10} \frac{1}{(2x+7)\sqrt{2x+7}}\,dx.$$

4. (*a*) Evaluate any *two* of the following : 6

(*i*) $\int_e^{e^3} \frac{1}{x \log x}\,dx$

(*ii*) $\int \cos\sqrt{x}\, dx$

(*iii*) $\int e^x \left(x^2 + 2x\right)dx$.

(*b*) If

$$A = \{x : x \geq 8\} \text{ and } B = \{x : x < 24\},$$

then find

$$A \cup B^C,\ A \cap B \text{ and } (A \cup B)\backslash(A \cap B)$$

where B^C denotes the complement of B. 5

(*c*) Assume that a given population grows according to the rule

$$P(t) = 50000 + 50\, t^2,$$

where the time t is measured in hours. Find the average growth rate during the time interval $t = 2$ to $t = 8$. 5½

Section II

5. (*a*) If

$$A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix},\ B = \begin{bmatrix} p & 1 \\ q & -1 \end{bmatrix} \text{ and } (A + B)^2 = A^2 + B^2;$$

determine the values of p and q. 6

(*b*) If

$$g(x) = x^2 + 5x - 7 \text{ and } A = \begin{bmatrix} 3 & 1 \\ -1 & -2 \end{bmatrix},$$

find $g(A)$. 5

6. (*a*) If

$$A = \begin{bmatrix} 14 & 1 & -1 \\ 3 & 2 & 15 \\ 0 & 8 & 14 \end{bmatrix} \text{ and } B = \begin{bmatrix} -4 & 21 & 6 \\ 13 & 4 & -1 \\ 2 & 0 & 2 \end{bmatrix},$$

find matrix X such that

$$A + B - 2X = 0.$$ 5

(*b*) If

$$A = \begin{bmatrix} 1 & 2 & 5 \\ -4 & 1 & 6 \\ 8 & 4 & 7 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix},$$

compute AB and BA. Is it true that AB = BA ? 6

7. (*a*) Find the image of the point (–2, 4) under rotation through an angle 45° in the counterclockwise direction. 3

(*b*) If 3

$$A = \begin{bmatrix} 0 & 3 \\ -2 & 5 \end{bmatrix},$$

find k so that

$$kA^2 = 5A - 6I_2.$$

(*c*) Express the following as a set of linear equations : 5

(*i*) $$\begin{bmatrix} 1 & -y & 2 \\ 3 & 15 & 3x \\ z & 3 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ 11 \\ 2 \end{bmatrix}$$

(*ii*) $\begin{bmatrix} 1 & y & -9 \\ 3 & 0 & x \end{bmatrix} \begin{bmatrix} z & 0 \\ 1 & -1 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 6 & 8 \end{bmatrix}$.

Section III

8. (*a*) In a botanical lab there are 16 plants. A carcinogen is sprayed uniformly. Find the probability of getting half of the total plants mutated, assuming that the chances of mutation in each plant is 50%. 5

(*b*) Find the mean and standard deviation of the two observations $a + b$ and $a - b$. 5

9. (*a*) The mean of a certain number of observations is 40. If two more items with values 50 and 64 are added to this data, the mean rises to 42. Find the number of items in the original data. 5

(*b*) The marks of 5 students are given in Mathematics and Physics. Find the coefficient of correlation between them : 5

Marks in Mathematics	2	8	5	4	6
Marks in Physics	1	7	6	2	4

10. (*a*) Fit a straight line to the following data with x as independent variable and y as dependent variables : 5

x	1	2	3	4	5	6
y	2.4	3	3.6	4	5	6

(*b*) In an intelligence test administered to 1000 children, the average score is 42 and S.D. is 24. Assuming normal distribution, find the number of children whose scores lie between 20 and 40. 5

11. (*a*) The means of two samples of 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the population of standard deviation 2.5 inches ? 5

(*b*) In a certain factory manufacturing razor blades, there is a small chance 1/500 for any blade to be defective. The blades are in the packets of 10. Use Poisson distribution to find the probability that the packet contains : 5

(*i*) No defective blade

(*ii*) At least one defective blade.