## Academic Year: 2020-21

# DEPARTMENT OF MATHEMATICS 

Syllabus: B. Sc. (Hons) Mathematics and Generic Mathematics for Hons Courses



1. B. Sc. (Hons) Mathematics I \& II Year (Revised CBCS Syllabus)
2. B. Sc. (Hons) Mathematics III Year (Old CBCS Syllabus)
3. Generic Mathematics for I \& II Year (Revised CBCS Syllabus)

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## 1. Introduction

The current focus in higher education is to shift from teacher-centric approach to learnercentric approach. For this as one of the aims, UGC has introduced the learning outcomesbased curriculum framework for undergraduate education. The learning outcomes-based curriculum framework for B.Sc. (Hons.) Mathematics is prepared keeping this in view. The framework is expected to provide a student with knowledge and skills in mathematics along with generic and transferable skills in other areas that help in personal development, employment and higher education in the global world. The programme-learning outcomes and course learning outcomes have been clearly specified to help prospective students, parents and employers understand the nature and extent of the degree programme; to maintain national and international standards, and to help in student mobility.

## 2. Learning Outcomes based approach to Curriculum Planning

The learning outcomes-based curriculum framework for B.Sc. (Hons.) Mathematics is based on the expected learning outcomes and graduate attributes that a graduate in mathematics is expected to attain. The curriculum for B.Sc. (Hons.) Mathematics is prepared keeping in mind the needs and aspirations of students in mathematics as well as the evolving nature of mathematics as a subject. The course learning outcomes and the programme learning outcomes specify the knowledge, understanding, skills, attitudes and values that a student completing this degree is expected to know. The qualification of B.Sc. (Hons.) Mathematics is awarded to a student who can demonstrating the attainment of these outcomes.

### 2.1 Nature and extent of the B.Sc. (Hons.) Mathematics

Mathematics is usually described as the abstract science of number, quantity and space along with their operations. The scope of Mathematics is very broad and it has a wide range of applications in natural sciences, engineering, economics and social sciences. B.Sc. (Hons.) Mathematics Programme aims at developing the ability to think critically, logically and analytically and hence use mathematical reasoning in everyday life. Pursuing a degree in mathematics will introduce the students to a number of interesting and useful ideas in preparations for a number of mathematics careers in education, research, government sector, business sector and industry.

The B.Sc. (Hons.) Mathematics programme covers the full range of mathematics, from classical Calculus to Modern Cryptography, Information Theory, and Network Security. The course lays a structured foundation of Calculus, Real \& Complex analysis, Abstract Algebra, Differential Equations (including Mathematical Modelling), Number Theory, Graph Theory, and C++ Programming exclusively for Mathematics.

An exceptionally broad range of topics covering Pure \& Applied Mathematics: Linear Algebra, metric Spaces, Statistics, Linear Programming, Numerical Analysis, Mathematicl Fi nance, Coding Theory, Mechanics and Biomathematics cater to varied interests and
ambitions. Also hand on sessions in Computer Lab using various Computer Algebra Systems (CAS) softwares such as Mathematica, MATLAB, Maxima, $\mathbf{R}$ to have a deep conceptual understanding of the above tools are carried out to widen the horizon of students' selfexperience. The courses like Biomathematics, Mathematical Finance etc. emphasize on the relation of mathematics to other subjects like Biology, Economics and Finance.

To broaden the interest for interconnectedness between formerly separate disciplines one can choose from the list of Generic electives for example one can opt for economics as one of the GE papers. Skill enhancement Courses enable the student acquire the skill relevant to the main subject. Choices from Discipline Specific Electives provides the student with liberty of exploring his interests within the main subject.

Of key importance is the theme of integrating mathematical and professional skills. The wellstructured programme empowers the student with the skills and knowledge leading to enhanced career opportunities in industry, commerce, education, finance and research.

### 2.2 Aims of Bachelor's degree programme in Mathematics

The overall aims of B.Sc.(Hons) Mathematics Programme are to:

- inculcate strong interest in learning mathematics.
- evolve broad and balanced knowledge and understanding of definitions, key concepts, principles and theorems in Mathematics
- enable learners/students to apply the knowledge and skills acquired by them during the programme to solve specific theoretical and applied problems in mathematics.
- develop in students the ability to apply relevant tools developed in mathematical theory to handle issues and problems in social and natural sciences.
- provide students with sufficient knowledge and skills that enable them to undertake further studies in mathematics and related disciplines
- enable students to develop a range of generic skills which will be helpful in wageemployment, self-employment and entrepreneurship.


## 3. Graduate Attributes in Mathematics

Some of the graduate attributes in mathematics are listed below:
3.1 Disciplinary knowledge: Capability of demonstrating comprehensive knowledge of basic concepts and ideas in mathematics and its subfields, and its applications to other disciplines.
3.2 Communications skills: Ability to communicate various concepts of mathematics in effective and coherent manner both in writing and orally, ability to present the complex mathematical ideas in clear, precise and confident way, ability to explain the development and importance of mathematics and ability to express thoughts and views in mathematically or logically correct statements.
3.3 Critical thinking and analytical reasoning: Ability to apply critical thinking in understanding the concepts in mathematics and allied areas; identify relevant assumptions, hypothesis, implications or conclusions; formulate mathematically correct arguments; ability to analyse and generalise specific arguments or empirical data to get broader concepts.
3.4 Problem solving: Capacity to use the gained knowledge to solve different kinds of non-familiar problems and apply the learning to real world situations; Capability to solve problems in computer graphics using concepts of linear algebra; Capability to apply the knowledge gained in differential equations to solve specific problems or models in operations research, physics, chemistry, electronics, medicine, economics, finance etc.
3.5 Research-related skills: Capability to ask and inquire about relevant/appropriate questions, ability to define problems, formulate hypotheses, test hypotheses, formulate mathematical arguments and proofs, draw conclusions; ability to write clearly the results obtained.
3.6 Information/digital literacy: Capacity to use ICT tools in solving problems or gaining knowledge; capacity to use appropriate softwares and programming skills to solve problems in mathematics,
3.7 Self-directed learning: Ability to work independently, ability to search relevant resources and e-content for self-learning and enhancing knowledge in mathematics.
3.8 Moral and ethical awareness/reasoning: Ability to identify unethical behaviour such as fabrication or misrepresentation of data, committing plagiarism, infringement of intellectual property rights.
3.9 Lifelong learning: Ability to acquire knowledge and skills through self-learning that helps in personal development and skill development suitable for changing demands of work place.

## 4. Qualification descriptors for B.Sc. (Hons.) Mathematics

Students who choose B.Sc. (Hons.) Mathematics Programme, develop the ability to think critically, logically and analytically and hence use mathematical reasoning in everyday life.

Pursuing a degree in mathematics will introduce the students to a number of interesting and useful ideas in preparations for a number of mathematics careers in education, research, government sector, business sector and industry.

The programme covers the full range of mathematics, from classical Calculus to Modern Cryptography, Information Theory, and Network Security. The course lays a structured foundation of Calculus, Real \& Complex analysis, Abstract Algebra, Differential Equations (including Mathematical Modeling), Number Theory, Graph Theory, and C++ Programming exclusively for Mathematics.

An exceptionally broad range of topics covering Pure \& Applied Mathematics: Linear Algebra, Metric Spaces, Statistics, Linear Programming, Numerical Analysis, Mathematical Finance, Coding Theory, Mechanics and Biomathematics cater to varied interests and ambitions. Also hand on sessions in Computer Lab using various Computer Algebra Systems (CAS) softwares such as Mathematica, MATLAB, Maxima, $\mathbf{R}$ to have a deep conceptual understanding of the above tools are carried out to widen the horizon of students' selfexperience.

To broaden the interest for interconnectedness between formerly separate disciplines one can choose from the list of Generic electives for example one can opt for economics as one of the GE papers. Skill enhancement courses enable the student acquire the skill relevant to the main subject. Choices from Discipline Specific Electives provides the student with liberty of exploring his interests within the main subject.

Of key importance is the theme of integrating mathematical and professional skills. The wellstructured programme empowers the student with the skills and knowledge leading to enhanced career opportunities in industry, commerce, education, finance and research. The qualification descriptors for B.Sc. (Hons.) Mathematics may include the following:
i. demonstrate fundamental/systematic and coherent knowledge of the academic field of mathematics and its applications and links to engineering, science, technology, economics and finance; demonstrate procedural knowledge that create different professionals like teachers and researchers in mathematics, quantitative analysts, actuaries, risk managers, professionals in industry and public services.
ii. demonstrate educational skills in areas of analysis, geometry, algebra, mechanics, differential equations etc.
iii. demonstrate comprehensive knowledge about materials, including scholarly, and/or professional literature, relating to essential learning areas pertaining to the field of mathematics, and techniques and skills required for identifying mathematical problems.
iv. Apply the acquired knowledge in mathematics and transferable skills to new/unfamiliar contexts and real-life problems.
v. Demonstrate mathematics-related and transferable skills that are relevant to some of the job trades and employment opportunities.

## 5. Programme Learning Outcomes in B.Sc. (Hons.) Mathematics

The completion of the B.Sc. (Hons.) Mathematics Programme will enable a student to:
i) Communicate mathematics effectively by written, computational and graphic means.
ii) Create mathematical ideas from basic axioms.
iii) Gauge the hypothesis, theories, techniques and proofs provisionally.
iv) Utilize mathematics to solve theoretical and applied problems by critical understanding, analysis and synthesis.
v) Identify applications of mathematics in other disciplines and in the real-world, leading to enhancement of career prospects in a plethora of fields and research.

## 6. Structure of B.Sc. (Hons.) Mathematics

The B.Sc. (Hons.) Mathematics programme is a three-year, six-semesters course. A student is required to complete 148 credits for completion of the course.

|  |  | Semester | Semester |
| :--- | :--- | :--- | :--- |
| Part - I | First Year | Semester I: 22 | Semester II: 22 |
| Part - II | Second Year | Semester III: 28 | Semester IV: 28 |
| Part - III | Third Year | Semester V: 24 | Semester VI: 24 |

Semester wise Details of B.Sc. (Hons.) Mathematics Course \& Credit Scheme


| III | BMATH305: Theory of Real Functions <br> BMATH306: Group <br> Theory-I <br> BMATH307: <br> Multivariate Calculus (including practicals) |  | SEC-1 <br> LaTeX and HTML |  | GE-3 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| L+T/P | $\begin{aligned} 5+1 & =6 ; 5+1=6 ; \\ 4 & +2=6 \end{aligned}$ |  | 4 |  | $5+1=6$ | 28 |
| IV | BMATH408: Partial <br> Differential Equations (including practicals) <br> BMATH409: <br> Riemann Integration <br> and Series of <br> Functions <br> BMATH410: Ring <br> Theory and Linear Algebra-I |  | SEC-2 <br> Computer <br> Algebra <br> Systems and <br> Related <br> Software |  | GE-4 |  |
| L+T/P | $\begin{gathered} 4+2=6 ; 5+1=6 ; \\ 5+1=6 \end{gathered}$ |  | 4 |  | $5+1=6$ | 28 |
| V $\mathrm{L}+\mathrm{T} / \mathrm{P}$ | C11: Metric Spaces <br> C12: Group <br> Theory-II <br>  <br> $5+1=6 ;$ <br> $5+1=6$ |  |  | DSE-1 (including practicals) DSE-2 $\begin{aligned} & 4+2=6 \\ & 5+1=6 \end{aligned}$ |  | 24 |
| Semester | Core Course(14) | Ability Enhancement Compulsory Course (AECC)(2) | Skill <br> Enhancement Course (SEC)(2) | Discipline <br> Specific <br> Elective <br> (DSE)(4) | Generic Elective (GE)(4) | Total Credits |
| VI | C13: <br> Complex Analysis <br> (including practicals) <br> C14: Ring <br> Theory and Linear <br> Algebra-II |  |  | DSE-3 <br> DSE-4 |  |  |
| L+T/P | $4+2=6 ; 5+1=6$ |  |  | $\begin{aligned} & 5+1=6 ; \\ & 5+1=6 \end{aligned}$ |  | 24 |

Legend: L: Lecture Class; T: Tutorial Class; P: Practical Class
Note: One-hour lecture per week equals 1 Credit, 2 Hours practical class per week equals 1 credit. Practical in a group of 15-20 students in Computer Lab and Tutorial in a group of 8-12 students.

## List of Discipline Specific Elective (DSE) Courses:

DSE-1 (Including Practicals): Any one of the following (at least two shall be offered by the college)
(i) Numerical Analysis
(ii) Mathematical Modeling and Graph Theory
(iii) $\mathrm{C}++$ Programming for Mathematics

DSE-2: Any one of the following (at least two shall be offered by the college)
(i) Mathematical Finance
(ii) Discrete Mathematics
(iii) Cryptography and Network Security

DSE-3: Any one of the following (at least two shall be offered by the college)
(i) Probability Theory and Statistics
(ii) Mechanics
(iii) Biomathematics

DSE-4: Any one of the following (at least two shall be offered by the college)
(i) Number Theory
(ii) Linear Programming and Theory of Games
(iii) Applications

## Semester-I

## BMATH101: Calculus

Total Marks: 150 (Theory: 75, Internal Assessment: 25 and Practical: 50)
Workload: 4 Lectures, 4 Practicals (per week) Credits: 6 (4+2)
Duration: 14 Weeks ( 56 Hrs. Theory +56 Hrs. Practical) Examination: 3 Hrs.
Course Objectives: The primary objective of this course is to introduce the basic tools of calculus and geometric properties of different conic sections which are helpful in understanding their applications in planetary motion, design of telescope and to the realworld problems. Also, to carry out the hand on sessions in computer lab to have a deep conceptual understanding of the above tools to widen the horizon of students' selfexperience.

Course Learning Outcomes: This course will enable the students to:
i) Learn first and second derivative tests for relative extrema and apply the knowledge in problems in business, economics and life sciences.
ii) Sketch curves in a plane using its mathematical properties in the different coordinate systems of reference.
iii) Compute area of surfaces of revolution and the volume of solids by integrating over cross-sectional areas.
iv) Understand the calculus of vector functions and its use to develop the basic principles of planetary motion.

## Unit 1: Derivatives for Graphing and Applications

The first-derivative test for relative extrema, Concavity and inflection points, Secondderivative test for relative extrema, Curve sketching using first and second derivative tests; Limits to infinity and infinite limits, Graphs with asymptotes, L'Hôpital's rule; Applications in business, economics and life sciences; Higher order derivatives, Leibniz rule.

## Unit 2: Sketching and Tracing of Curves

Parametric representation of curves and tracing of parametric curves (except lines in $\mathbb{R}^{3}$ ), Polar coordinates and tracing of curves in polar coordinates; Techniques of sketching conics, Reflection properties of conics, Rotation of axes and second degree equations, Classification into conics using the discriminant.

## Unit 3: Volume and Area of Surfaces

Volumes by slicing disks and method of washers, Volumes by cylindrical shells, Arc length, Arc length of parametric curves, Area of surface of revolution; Hyperbolic functions; Reduction formulae.

## Unit 4: Vector Calculus and its Applications

Introduction to vector functions and their graphs, Operations with vector functions, Limits and continuity of vector functions, Differentiation and integration of vector functions; Modeling ballistics and planetary motion, Kepler's second law; Unit tangent, Normal and binormal vectors, Curvature.

## References:

1. Anton, Howard, Bivens, Irl, \& Davis, Stephen (2013). Calculus (10th ed.). John Wiley \& Sons Singapore Pte. Ltd. Indian Reprint (2016) by Wiley India Pvt. Ltd. Delhi.
2. Prasad, Gorakh (2016). Differential Calculus (19th ed.). Pothishala Pvt. Ltd. Allahabad.
3. Strauss, Monty J., Bradley, Gerald L., \& Smith, Karl J. (2007). Calculus (3rd ed.). Dorling Kindersley (India) Pvt. Ltd. (Pearson Education). Delhi. Indian Reprint 2011.

## Additional Reading:

i. Thomas, Jr. George B., Weir, Maurice D., \& Hass, Joel (2014). Thomas' Calculus (13th ed.). Pearson Education, Delhi. Indian Reprint 2017.

## Practical / Lab work to be performed in Computer Lab.

List of the practicals to be done using Mathematica/MATLAB/Maple/Scilab/Maxima etc.

1. Plotting the graphs of the following functions:

$$
\begin{gathered}
a x,[x] \text { (greatest integer function), } \sqrt{a x+b},|a x+b|, c \pm|a x+b|, \\
x^{ \pm n}, x^{\frac{1}{n}}(n \in \mathbb{Z}), \frac{|x|}{x}, \sin \left(\frac{1}{x}\right), x \sin \left(\frac{1}{x}\right), \text { and } e^{ \pm \frac{1}{x}}, \text { for } x \neq 0, \\
e^{a x+b}, \log (a x+b), 1 /(a x+b), \sin (a x+b), \cos (a x+b), \\
|\sin (a x+b)|,|\cos (a x+b)| .
\end{gathered}
$$

Observe and discuss the effect of changes in the real constants $a, b$ and $c$ on the graphs.
2. Plotting the graphs of polynomial of degree 4 and 5 , and their first and second derivatives, and analysis of these graphs in context of the concepts covered in Unit 1.
3. Sketching parametric curves, e.g., trochoid, cycloid, epicycloid and hypocycloid.
4. Tracing of conics in Cartesian coordinates.
5. Obtaining surface of revolution of curves.
6. Graph of hyperbolic functions.
7. Computation of limit, Differentiation, Integration and sketching of vector-valued functions.
8. Complex numbers and their representations, Operations like addition, multiplication, division, modulus. Graphical representation of polar form.
9. Find numbers between two real numbers and plotting of finite and infinite subset of $\mathbb{R}$.
10. Matrix operations: addition, multiplication, inverse, transpose; Determinant, Rank, Eigenvectors, Eigenvalues, Characteristic equation and verification of the Cayley-Hamilton theorem, Solving the systems of linear equations.

## Teaching Plan (Theory of BMATH101: Calculus):

Week 1: The first-derivative test for relative extrema, Concavity and inflection points, Secondderivative test for relative extrema, Curve sketching using first and second derivative tests.
[3] Chapter 4 (Section 4.3).
Week 2: Limits to infinity and infinite limits, Graphs with asymptotes, Vertical tangents and cusps, L'Hôpital's rule.
[3] Chapter 4 (Sections 4.4 and 4.5).
Week 3: Applications of derivatives in business, economics and life sciences. Higher order derivatives and Leibniz rule for higher order derivatives for the product of two functions.
[3] Chapter 4 (Section 4.7).
[2] Chapter 5 (Sections 5.1, 5.2 and 5.4).

Week 4: Parametric representation of curves and tracing of parametric curves (except lines in $\mathbb{R}^{3}$ ), Polar coordinates and the relationship between Cartesian and polar coordinates.
[3] Chapter 9 [Section 9.4 (Pages 471 to 475)].
[1] Chapter 10 (Sections 10.1, and 10.2 up to Example 2, Page 707).
Weeks 5 and 6: Tracing of curves in polar coordinates. Techniques of sketching conics: parabola, ellipse and hyperbola.
[1] Sections 10.2 (Pages 707 to 717), and 10.4 up to Example 10 Page 742)].
Week 7: Reflection properties of conics, Rotation of axes, Second degree equations and their classification into conics using the discriminant.
[1] Sections 10.4 (Pages 742 to 744) and 10.5.
Weeks 8 and 9: Volumes by slicing disks and method of washers, Volumes by cylindrical shells, Arc length, Arc length of parametric curves.
[1] Chapter 5 (Sections 5.2, 5.3 and 5.4).
Week 10: Area of surface of revolution; Hyperbolic functions.
[1] Sections 5.5 and 6.8 .
Week 11: Reduction formulae, and to obtain the iterative formulae for the integrals of the form: $\int \sin ^{n} x d x, \int \cos ^{n} x d x, \int \tan ^{n} x d x, \int \sec ^{n} x d x$ and $\int \sin ^{m} x \cos ^{n} x d x$.
[1] Chapter 7 [Sections 7.2 and 7.3 (Pages 497 to 503)].
Week 12: Introduction to vector functions and their graphs, Operations with vector functions, Limits and continuity of vector functions, Differentiation and tangent vectors.
[3] Chapter 10 (Sections 10.1 and 10.2 up to Page 504).
Week 13: Properties of vector derivatives and integration of vector functions; Modeling ballistics and planetary motion, Kepler's second law.
[3] Chapter 10 [Sections 10.2 (Pages 505 to 511) and 10.3].
Week 14: Unit tangent, Normal and binormal vectors, Curvature.
[1] Sections 12.4 and 12.5 .

## Facilitating the Achievement of Course Learning Outcomes

| $\begin{aligned} & \text { Unit } \\ & \text { No. } \end{aligned}$ | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
| :---: | :---: | :---: | :---: |
| 1. | Learn first and second derivative tests for relative extrema and apply the knowledge in problems in business, economics and life sciences. | (i) Each topic to be explained with illustrations. <br> (ii) Students to be encouraged to discover the relevant concepts. <br> (iii) Students be given homework/assignments. <br> (iv) Discuss and solve the theoretical and practical problems in the class. <br> (v) Students to be encouraged to apply concepts to real world problems. | - Presentations and class discussions. <br> - Assignments and class tests. <br> - Student presentations. <br> - Mid-term examinations. <br> - Practical and viva-voce examinations. <br> - End-term examinations. |
| 2. | Sketch curves in a plane using its mathematical properties in the different coordinate systems of reference. |  |  |
| 3. | Compute area of surfaces of revolution and the volume of solids by integrating over crosssectional areas. |  |  |
| 4. | Understand the calculus of vector functions and its use to develop the basic principles of planetary motion. |  |  |

Keywords: Concavity, Extrema, Inflection point, Hyperbolic functions, Leibniz rule, L'Hôpital's rule, Polar and parametric coordinates, Vector functions.

## BMATH102: Algebra

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) Credits: 6 (5+1)
Duration: 14 Weeks ( 70 Hrs.) Examination: 3 Hrs.
Course Objectives: The primary objective of this course is to introduce the basic tools of theory of equations, complex numbers, number theory and matrices to understand their connection with the real-world problems. Perform matrix algebra with applications to computer graphics.
Course Learning Outcomes: This course will enable the students to:
i) Employ De Moivre's theorem in a number of applications to solve numerical problems.
ii) Learn about equivalent classes and cardinality of a set.
iii) Use modular arithmetic and basic properties of congruences.
iv) Recognize consistent and inconsistent systems of linear equations by the row echelon form of the augmented matrix.
v) Find eigenvalues and corresponding eigenvectors for a square matrix.

## Unit 1: Theory of Equations and Complex Numbers

Polynomials, The remainder and factor theorem, Synthetic division, Factored form of a polynomial, Fundamental theorem of algebra, Relations between the roots and the coefficients of polynomial equations, Theorems on imaginary, integral and rational roots; Polar representation of complex numbers, De Moivre's theorem for integer and rational indices and their applications. The $n$th roots of unity.

## Unit 2: Equivalence Relations and Functions

Equivalence relations, Functions, Composition of functions, Invertibility and inverse of functions, One-to-one correspondence and the cardinality of a set.

## Unit 3: Basic Number Theory

Well ordering principle, The division algorithm in $\mathbb{Z}$, Divisibility and the Euclidean algorithm, Fundamental theorem of arithmetic, Modular arithmetic and basic properties of congruences; Principle of mathematical induction.

## Unit 4: Row Echelon Form of Matrices and Applications

Systems of linear equations, Row reduction and echelon forms, Vector equations, The matrix equation $A \mathbf{x}=b$, Solution sets of linear systems, The inverse of a matrix; Subspaces, Linear independence, Basis and dimension, The rank of a matrix and applications; Introduction to linear transformations, The matrix of a linear transformation; Applications to computer graphics, Eigenvalues and eigenvectors, The characteristic equation and Cayley-Hamilton theorem.

## References:

1. Andreescu, Titu \& Andrica Dorin. (2014). Complex Numbers from A to...Z. (2nd ed.). Birkhäuser.
2. Dickson, Leonard Eugene (2009). First Course in the Theory of Equations. The Project Gutenberg EBook (http://www.gutenberg.org/ebooks/29785)
3. Goodaire, Edgar G., \& Parmenter, Michael M. (2005). Discrete Mathematics with Graph Theory (3rd ed.). Pearson Education Pvt. Ltd. Indian Reprint 2015.
4. Kolman, Bernard, \& Hill, David R. (2001). Introductory Linear Algebra with Applications (7th ed.). Pearson Education, Delhi. First Indian Reprint 2003.
5. Lay, David C., Lay, Steven R., \& McDonald, Judi J. (2016). Linear Algebra and its Applications (5th ed.). Pearson Education.

## Additional Readings:

i. Andrilli, Stephen, \& Hecker, David (2016). Elementary Linear Algebra (5th ed.). Academic Press, Elsevier India Private Limited.
ii. Burton, David M. (2012). Elementary Number Theory (7th ed.). McGraw-Hill Education Pvt. Ltd. Indian Reprint.

## Teaching Plan (BMATH102: Algebra):

Weeks 1 and 2: Polynomials, The remainder and factor theorem, Synthetic division, Factored form of a polynomial, Fundamental theorem of algebra, Relations between the roots and the coefficients of polynomial equations, Theorems on imaginary, integral and rational roots.
[2] Chapter II (Sections 12 to 16, 19 to 21, 24 and 27, Statement of the Fundamental theorem of algebra).
Weeks 3 and 4: Polar representation of complex numbers, De Moivre's theorem for integer and rational indices and their applications, The $n$th roots of unity.
[1] Chapter 2 [Section 2.1(2.1.1 to 2.1.3), Section 2.2 (2.2.1, 2.2 .2 (up to Page 45, without propositions), 2.2.3].
Weeks 5 and 6. Equivalence relations, Functions, Composition of functions, Invertibility and inverse of functions, One-to-one correspondence and the cardinality of a set.
[3] Chapter 2 (Section 2.4 (2.4.1 to 2.4.4)), and Chapter 3.
Weeks 7 and 8: Well ordering principle, The division algorithm in $\mathbb{Z}$, Divisibility and the Euclidean algorithm, Modular arithmetic and basic properties of congruences, Statements of the fundamental theorem of arithmetic and principle of mathematical induction.
[3] Chapter 4 [Sections 4.1 (4.1.2,4.1.5,4.1.6), 4.2 (4.2.1 to 4.2.11, up to problem 11), 4.3 (4.3.7 to 4.3.9), 4.4 (4.4.1 to 4.4.8)], and Chapter 5 (Section 5.1.1).

Weeks 9 and 10: Systems of linear equations, Row reduction and echelon forms, Vector equations, The matrix equation $A \mathbf{x}=b$, Solution sets of linear systems, The inverse of a matrix.
[5] Chapter 1 (Sections 1.1 to 1.5) and Chapter 2 (Section 2.2).
Week 11 and 12: Subspaces, Linear independence, Basis and dimension, The rank of a matrix and applications.
[4] Chapter 6 (Sections 6.2, 6.3, 6.4, and 6.6).
Weeks 13: Introduction to linear transformations, Matrix of a linear transformation; Applications to computer graphics.
[5] Chapter 1 (Sections 1.8 and 1.9), and Chapter 2 (Section 2.7).
Week 14: Eigenvalues and eigenvectors, The characteristic equation and Cayley-Hamilton theorem.
[5] Chapter 5 (Sections 5.1 and 5.2, Supplementary Exercises 5 and 7, Page 328).
Facilitating the Achievement of Course Learning Outcomes

| Unit <br> No. | Course Learning Outcomes | Teaching and Learning <br> Activity | Assessment Tasks |
| :---: | :--- | :--- | :--- |
| 1. | Employ De Moivre's theorem in a <br> number of applications to solve | (i) Each topic to be explained <br> with examples. | • Student |


|  | numerical problems. | (ii) Students to be involved in |  |
| :---: | :--- | :---: | :---: |
| discussions and encouraged |  |  |  |
| to ask questions. |  |  |  |
| (iii) Students to be given |  |  |  |
| homework/assignments. |  |  |  |
| (iv) Students to be encouraged |  |  |  |
| to give short presentations. | Learn about equivalent classes and <br> cardinality of a set. | presentations. <br> • Participation in <br> discussions. |  |
| $\bullet$ | Assignments and <br> class tests. <br> - Mid-term <br> examinations. <br> properties of congruences. |  |  |
| 4. | Recognize consistent and <br> inconsistent systems of linear <br> equations by the row echelon form <br> of the augmented matrix. | End-term <br> Find eigenvalues and corresponding <br> eigenvectors for a square matrix. |  |

Keywords: Cardinality of a set, Cayley-Hamilton theorem, De Moivre's theorem, Eigenvalues and eigenvectors, Equivalence relations, Modular arithmetic, Row echelon form, The Fundamental theorem of algebra.

## Semester-II

## BMATH203: Real Analysis

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) Credits: 6 (5+1)
Duration: 14 Weeks ( 70 Hrs.) Examination: 3 Hrs.
Course Objectives: The course will develop a deep and rigorous understanding of real line $\mathbb{R}$. and of defining terms to prove the results about convergence and divergence of sequences and series of real numbers. These concepts have wide range of applications in real life scenario.

Course Learning Outcomes: This course will enable the students to:
i) Understand many properties of the real line $\mathbb{R}$, including completeness and Archimedean properties.
ii) Learn to define sequences in terms of functions from $\mathbb{N}$ to a subset of $\mathbb{R}$.
iii) Recognize bounded, convergent, divergent, Cauchy and monotonic sequences and to calculate their limit superior, limit inferior, and the limit of a bounded sequence.
iv) Apply the ratio, root, alternating series and limit comparison tests for convergence and absolute convergence of an infinite series of real numbers.

## Unit 1: Real Number System $\mathbb{R}$

Algebraic and order properties of $\mathbb{R}$, Absolute value of a real number; Bounded above and bounded below sets, Supremum and infimum of a nonempty subset of $\mathbb{R}$.

## Unit 2: Properties of $\mathbb{R}$

The completeness property of $\mathbb{R}$, Archimedean property, Density of rational numbers in $\mathbb{R}$; Definition and types of intervals, Nested intervals property; Neighborhood of a point in $\mathbb{R}$, Open and closed sets in $\mathbb{R}$.

## Unit 3: Sequences in $\mathbb{R}$

Convergent sequence, Limit of a sequence, Bounded sequence, Limit theorems, Monotone sequences, Monotone convergence theorem, Subsequences, Bolzano-Weierstrass theorem for sequences, Limit superior and limit inferior for bounded sequence, Cauchy sequence, Cauchy's convergence criterion.

## Unit 4: Infinite Series

Convergence and divergence of infinite series of real numbers, Necessary condition for convergence, Cauchy criterion for convergence; Tests for convergence of positive term series: Integral test, Basic comparison test, Limit comparison test, D'Alembert's ratio test, Cauchy's $n$th root test; Alternating series, Leibniz test, Absolute and conditional convergence.

## References:

1. Bartle, Robert G., \& Sherbert, Donald R. (2015). Introduction to Real Analysis (4th ed.). Wiley India Edition. New Delhi.
2. Bilodeau, Gerald G., Thie, Paul R., \& Keough, G. E. (2010). An Introduction to Analysis (2nd ed.). Jones \& Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.
3. Denlinger, Charles G. (2011). Elements of Real Analysis. Jones \& Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

## Additional Readings:

i. Ross, Kenneth A. (2013). Elementary Analysis: The Theory of Calculus (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian Reprint.
ii. Thomson, Brian S., Bruckner, Andrew. M., \& Bruckner, Judith B. (2001). Elementary Real Analysis. Prentice Hall.

## Teaching Plan (BMATH203: Real Analysis):

Weeks 1 and 2: Algebraic and order properties of $\mathbb{R}$. Absolute value of a real number; Bounded above and bounded below sets, Supremum and infimum of a nonempty subset of $\mathbb{R}$.
[1] Chapter 2 [Sections 2.1, 2.2 (2.2.1 to 2.2.6) and 2.3 (2.3.1 to 2.3.5)]
Weeks 3 and 4: The completeness property of $\mathbb{R}$, Archimedean property, Density of rational numbers in $\mathbb{R}$, Definition and types of intervals, Nested intervals property; Neighborhood of a point in $\mathbb{R}$, Open and closed sets in $\mathbb{R}$.
[1] Sections 2.3 (2.3.6), 2.4 (2.4.3 to 2.4.9), and 2.5 up to Theorem 2.5.3.
[1] Chapter 11 [Section 11.1 (11.1.1 to 11.1.3)].
Weeks 5 and 6: Sequences and their limits, Bounded sequence, Limit theorems.
[1] Sections 3.1, 3.2.
Week 7: Monotone sequences, Monotone convergence theorem and applications.
[1] Section 3.3.
Week 8: Subsequences and statement of the Bolzano-Weierstrass theorem. Limit superior and limit inferior for bounded sequence of real numbers with illustrations only.
[1] Chapter 3 [Section 3.4 (3.4.1 to 3.4.12), except 3.4.4, 3.4.7, 3.4.9 and 3.4.11].
Week 9: Cauchy sequences of real numbers and Cauchy's convergence criterion.
[1] Chapter 3 [Section 3.5 (3.5.1 to 3.5.6)].
Week 10: Convergence and divergence of infinite series, Sequence of partial sums of infinite series, Necessary condition for convergence, Cauchy criterion for convergence of series.
[3] Section 8.1.
Weeks 11 and 12: Tests for convergence of positive term series: Integral test statement and convergence of $p$-series, Basic comparison test, Limit comparison test with applications, D'Alembert's ratio test and Cauchy's $n$th root test.
[3] Chapter 8 (Section 8.2 up to 8.2.19).
Weeks 13 and 14: Alternating series, Leibniz test, Absolute and conditional convergence.
[2] Chapter 6 (Section 6.2).

## Facilitating the Achievement of Course Learning Outcomes

| Unit <br> No. | Course Learning Outcomes | Teaching and Learning <br> Activity | Assessment Tasks |
| :---: | :--- | :--- | :--- |
| $1 . \&$ | Understand many properties of the real <br> line $\mathbb{R}$ including, completeness and <br> Archimedean properties. | (i) Each topic to be <br> explained with <br> examples. | • Presentations and <br> participation in <br> discussions. |
| 3. | Learn to define sequences in terms of <br> functions from $\mathbb{N}$ to a subset of $\mathbb{R}$. | (ii) Students to be involved <br> in discussions and <br> encouraged to ask <br> Recognize bounded, convergent, <br> divergent, Cauchy and monotonic <br> sequences and to calculate their limit <br> superior, limit inferior, and the limit of <br> a bounded sequence. | questions. <br> class tests. |
| (iii) Students to be given |  |  |  |
| homework/assignments. Mid-term |  |  |  |
| (iv) Students to be |  |  |  |$\quad$| examinations. |
| :--- |
| • End-term |
| examinations. |


| 4. | Apply the ratio, root, alternating series <br> and limit comparison tests for <br> convergence and absolute convergence <br> of an infinite series of real numbers. | encouraged to give short <br> presentations. <br> (v) Illustrate the concepts <br> through CAS. | ${ }^{\text {the }}$ |
| :---: | :--- | :--- | :--- |$\quad$

Keywords: Archimedean property, Absolute and conditional convergence of series, Bolzano-Weierstrass theorem, Cauchy sequence, Convergent sequence, Leibniz test, Limit of a sequence, Nested intervals property, Open and closed sets in $\mathbb{R}$.

# BMATH204: Differential Equations 

Total Marks: 150 (Theory: 75, Internal Assessment: 25 and Practical: 50)
Workload: 4 Lectures, 4 Practicals (per week) Credits: 6 (4+2)
Duration: 14 Weeks ( 56 Hrs. Theory +56 Hrs. Practical) Examination: 3 Hrs.
Course Objectives: The main objective of this course is to introduce the students to the exciting world of differential equations, mathematical modeling and their applications.
Course Learning Outcomes: The course will enable the students to:
i) Learn basics of differential equations and mathematical modeling.
ii) Formulate differential equations for various mathematical models.
iii) Solve first order non-linear differential equations and linear differential equations of higher order using various techniques.
iv) Apply these techniques to solve and analyze various mathematical models.

## Unit 1: Differential Equations and Mathematical Modeling

Differential equations and mathematical models, Order and degree of a differential equation, Exact differential equations and integrating factors of first order differential equations, Reducible second order differential equations, Applications of first order differential equations to acceleration-velocity model, Growth and decay model.

## Unit 2: Population Growth Models

Introduction to compartmental models, Lake pollution model (with case study of Lake Burley Griffin), Drug assimilation into the blood (case of a single cold pill, case of a course of cold pills, case study of alcohol in the bloodstream), Exponential growth of population, Limited growth of population, Limited growth with harvesting.

## Unit 3: Second and Higher Order Differential Equations

General solution of homogeneous equation of second order, Principle of superposition for a homogeneous equation; Wronskian, its properties and applications, Linear homogeneous and non-homogeneous equations of higher order with constant coefficients, Euler's equation, Method of undetermined coefficients, Method of variation of parameters, Applications of second order differential equations to mechanical vibrations.

## Unit 4: Analysis of Mathematical Models

Interacting population models, Epidemic model of influenza and its analysis, Predator-prey model and its analysis, Equilibrium points, Interpretation of the phase plane, Battle model and its analysis.

## References:

1. Barnes, Belinda \& Fulford, Glenn R. (2015). Mathematical Modelling with Case Studies, Using Maple and MATLAB (3rd ed.). CRC Press, Taylor \& Francis Group.
2. Edwards, C. Henry, Penney, David E., \& Calvis, David T. (2015). Differential Equation and Boundary Value Problems: Computing and Modeling (5th ed.). Pearson Education.
3. Ross, Shepley L. (2004). Differential Equations (3rd ed.). John Wiley \& Sons. India

## Additional Reading:

i. Ross, Clay C. (2004). Differential Equations: An Introduction with Mathematica ${ }^{\circledR}$ (2nd ed.). Springer.

## Practical / Lab work to be performed in a Computer Lab:

Modeling of the following problems using Mathematica/MATLAB/Maple/Maxima/Scilab etc.

1. Plotting of second and third order respective solution family of differential equation.
2. Growth and decay model (exponential case only).
3. (i) Lake pollution model (with constant/seasonal flow and pollution concentration).
(ii) Case of single cold pill and a course of cold pills.
(iii) Limited growth of population (with and without harvesting).
4. (i) Predatory-prey model (basic Volterra model, with density dependence, effect of DDT, two prey one predator).
(ii) Epidemic model of influenza (basic epidemic model, contagious for life, disease with carriers).
(iii) Battle model (basic battle model, jungle warfare, long range weapons).
5. Plotting of recursive sequences, and study of the convergence.
6. Find a value $m \in \mathbb{N}$ that will make the following inequality holds for all $n>m$ :
(i) $|\sqrt[n]{0.5}-1|<10^{-3}$,
(ii) $|\sqrt[n]{n}-1|<10^{-3}$,
(iii) $(0.9)^{n}<10^{-3}$,
(iv) $\frac{2^{n}}{n!}<10^{-7}$, etc.
7. Verify the Bolzano-Weierstrass theorem through plotting of sequences and hence identify convergent subsequences from the plot.
8. Study the convergence/divergence of infinite series of real numbers by plotting their sequences of partial sum.
9. Cauchy's root test by plotting $n$th roots.
10. D'Alembert's ratio test by plotting the ratio of $n$th and $(n+1)$ th term of the given series of positive terms.
11. For the following sequences $\left\langle a_{n}\right\rangle$, given $\varepsilon=\frac{1}{2^{k}}, p=10^{j}, k=0,1,2, \ldots ; j=1,2,3, \ldots$ Find $m \in \mathbb{N}$ such that
(i) $\left|a_{m+p}-a_{m}\right|<\varepsilon$,
(ii) $\left|a_{2 m+p}-a_{2 m}\right|<\varepsilon$,
where $a_{n}$ is given as:
(a) $\frac{n+1}{n}$,
(b) $\frac{1}{n}$,
(c) $1-\frac{1}{2}+\frac{1}{3}-\cdots+\frac{(-1)^{n-1}}{n}$
(d) $\frac{(-1)^{n}}{n}$,
(e) $2^{-n} n^{2}$,
(f) $1+\frac{1}{2!}+\cdots+\frac{1}{n!}$.
12. For the following series $\sum a_{n}$, calculate
(i) $\left|\frac{a_{n+1}}{a_{n}}\right|$,
(ii) $\left|a_{n}\right|^{\frac{1}{n}}$, for $n=10^{j}, j=1,2,3, \ldots$,
and identify the convergent series, where $a_{n}$ is given as:
(a) $\left(\frac{1}{n}\right)^{1 / n}$,
(b) $\frac{1}{n}$,
(c) $\frac{1}{n^{2}}$,
(d) $\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3 / 2}}$,
(e) $\frac{n!}{n^{n}}$,
(f) $\frac{n^{3}+5}{3^{n}+2}$,
(g) $\frac{1}{n^{2}+n}$,
(■) $\frac{1}{\sqrt{n+1}}$,
(i) $\cos n$,
(j) $\frac{1}{n \log n}$,
(k) $\frac{1}{n(\log n)^{2}}$.

## Teaching Plan (Theory of BMATH204: Differential Equations):

Weeks 1 and 2: Differential equations and mathematical models, Order and degree of a differential equation, Exact differential equations and integrating factors of first order differential equations, Reducible second order differential equations.
[2] Chapter 1 (Sections 1.1 and 1.6).
[3] Chapter 2.
Week 3: Application of first order differential equations to acceleration-velocity model, Growth and decay model.
[2] Chapter 1 (Section 1.4, Pages 35 to 38), and Chapter 2 (Section 2.3).
[3] Chapter 3 (Section 3.3, A and B with Examples 3.8, 3.9).
Week 4: Introduction to compartmental models, Lake pollution model (with case study of Lake Burley Griffin).
[1] Chapter 2 (Sections 2.1, 2.5 and 2.6).
Week 5: Drug assimilation into the blood (case of a single cold pill, case of a course of cold pills, Case study of alcohol in the bloodstream).
[1] Chapter 2 (Sections 2.7 and 2.8).
Week 6: Exponential growth of population, Density dependent growth, Limited growth with harvesting.
[1] Chapter 3 (Sections 3.1 to 3.3).
Weeks 7 to 9: General solution of homogeneous equation of second order, Principle of superposition for a homogeneous equation; Wronskian, its properties and applications; Linear homogeneous and non-homogeneous equations of higher order with constant coefficients; Euler's equation.
[2] Chapter 3 (Sections 3.1 to 3.3).
Weeks 10 and 11: Method of undetermined coefficients, Method of variation of parameters; Applications of second order differential equations to mechanical vibrations.
[2] Chapter 3 (Sections 3.4 (Pages 172 to 177) and 3.5).
Weeks 12 to 14: Interacting population models, Epidemic model of influenza and its analysis, Predator-prey model and its analysis, Equilibrium points, Interpretation of the phase plane, Battle model and its analysis.
[1] Chapter 5 (Sections 5.1, 5.2, 5.4 and 5.9), and Chapter 6 (Sections 6.1 to 6.4).

## Facilitating the Achievement of Course Learning Outcomes

|  |  |  | Assessment Tasks |
| :---: | :---: | :---: | :---: |
| 1. | Learn basics of differential equations and mathematical modeling. | (i) Each topic to be explained with examples and illustrated on computers using Mathematica /MATLAB /Maple/Maxima/Scilab. <br> (ii) Students to be involved in discussions and encouraged to ask questions. <br> (iii) Students to be given homework/assignments. <br> (iv) Students to be encouraged to give short presentations. | - Presentations and participation in discussions. <br> - Assignments and class tests. <br> - Mid-term examinations. <br> - Practical and viva-voce examinations. <br> - End-term examinations. |
| 2. |  |  |  |
| 3. | Solve first order non-linear differential equations and linear differential equations of higher order using various techniques. |  |  |
| 4. | Apply th and analy models. |  |  |

Keywords: Battle model, Epidemic model, Euler's equation, Exact differential equation, Integrating factor, Lake pollution model, Mechanical vibrations, Phase plane, Predator-prey model, Wronskian and its properties.

## Semester-III

## BMATH305: Theory of Real Functions

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) Credits: 6 (5+1)
Duration: 14 Weeks ( 70 Hrs.) Examination: 3 Hrs.
Course Objectives: It is a basic course on the study of real valued functions that would develop an analytical ability to have a more matured perspective of the key concepts of calculus, namely, limits, continuity, differentiability and their applications.
Course Learning Outcomes: This course will enable the students to:
i) Have a rigorous understanding of the concept of limit of a function.
ii) Learn about continuity and uniform continuity of functions defined on intervals.
iii) Understand geometrical properties of continuous functions on closed and bounded intervals.
iv) Learn extensively about the concept of differentiability using limits, leading to a better understanding for applications.
v) Know about applications of mean value theorems and Taylor's theorem.

## Unit 1: Limits of Functions

Limits of functions ( $\varepsilon-\delta$ approach), Sequential criterion for limits, Divergence criteria, Limit theorems, One-sided limits, Infinite limits and limits at infinity.

## Unit 2: Continuous Functions and their Properties

Continuous functions, Sequential criterion for continuity and discontinuity, Algebra of continuous functions, Properties of continuous functions on closed and bounded intervals; Uniform continuity, Non-uniform continuity criteria, Uniform continuity theorem.

## Unit 3: Derivability and its Applications

Differentiability of a function, Algebra of differentiable functions, Carathéodory's theorem, Chain rule; Relative extrema, Interior extremum theorem, Rolle's theorem, Mean- value theorem and applications, Intermediate value property of derivatives, Darboux's theorem.

## Unit 4: Taylor's Theorem and its Applications

Taylor polynomial, Taylor's theorem with Lagrange form of remainder, Application of Taylor's theorem in error estimation; Relative extrema, and to establish a criterion for convexity; Taylor's series expansions of $e^{x}, \sin x$ and $\cos x$.

## Reference:

1. Bartle, Robert G., \& Sherbert, Donald R. (2015). Introduction to Real Analysis (4th ed.). Wiley India Edition. New Delhi.

## Additional Readings:

i. Ghorpade, Sudhir R. \& Limaye, B. V. (2006). A Course in Calculus and Real Analysis. Undergraduate Texts in Mathematics, Springer (SIE). First Indian reprint.
ii. Mattuck, Arthur. (1999). Introduction to Analysis, Prentice Hall.
iii. Ross, Kenneth A. (2013). Elementary Analysis: The Theory of Calculus (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian Reprint.

## Teaching Plan (BMATH305: Theory of Real Functions):

Week 1: Definition of the limit, Sequential criterion for limits, Criterion for non-existence of limit.
[1] Chapter 4 (Section 4.1).
Week 2: Algebra of limits of functions with illustrations and examples, Squeeze theorem.
[1] Chapter 4 (Section 4.2).
Week 3: Definition and illustration of the concepts of one-sided limits, Infinite limits and limits at infinity.
[1] Chapter 4 (Section 4.3).
Weeks 4 and 5: Definitions of continuity at a point and on a set, Sequential criterion for continuity, Algebra of continuous functions, Composition of continuous functions.
[1] Sections 5.1 and 5.2.
Weeks 6 and 7: Various properties of continuous functions defined on an interval, viz., Boundedness theorem, Maximum-minimum theorem, Statement of the location of roots theorem, Intermediate value theorem and the preservation of intervals theorem.
[1] Chapter 5 (Section 5.3).
Week 8: Definition of uniform continuity, Illustration of non-uniform continuity criteria, Uniform continuity theorem.
[1] Chapter 5 [Section 5.4 (5.4.1 to 5.4.3)].
Weeks 9 and 10: Differentiability of a function, Algebra of differentiable functions, Carathéodory's theorem and chain rule.
[1] Chapter 6 [Section 6.1 (6.1.1 to 6.1.7)].
Weeks 11 and 12: Relative extrema, Interior extremum theorem, Mean value theorem and its applications, Intermediate value property of derivatives - Darboux's theorem.
[1] Section 6.2.
Weeks 13 and 14: Taylor polynomial, Taylor's theorem and its applications, Taylor's series expansions of $e^{x}, \sin x$ and $\cos x$.
[1] Chapter 6 (Sections 6.4.1 to 6.4.6), and Chapter 9 (Example 9.4.14, Page 286).

## Facilitating the Achievement of Course Learning Outcomes

| Unit No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
| :---: | :---: | :---: | :---: |
| 1. | Have a rigorous understanding of the concept of limit of a function. | (i) Each topic to be explained with examples. <br> (ii) Students to be involved in discussions and encouraged to ask questions. <br> (iii) Students to be given homework/ assignments. <br> (iv) Students to be encouraged to give short presentations. <br> (v) Illustrate the concepts through CAS. | - Presentations and participation in discussions. <br> - Assignments and class tests. <br> - Mid-term examinations. <br> - End-term examinations. |
| 2. | Learn about continuity and uniform continuity of functions defined on intervals. <br> Understand geometrical properties of continuous functions on closed and bounded intervals. |  |  |
| 3. | Learn extensively about the concept of differentiability using limits, leading to a better understanding for applications. |  |  |
| 4. | Know about applications of mean value theorems and Taylor's theorem. |  |  |

Keywords: Continuity, Convexity, Differentiability, Limit, Relative extrema, Rolle's theorem, Taylor's theorem, Uniform continuity.

## BMATH306: Group Theory-I

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) Credits: 6 (5+1)
Duration: 14 Weeks (70 Hrs.) Examination: 3 Hrs.
Course Objectives: The objective of the course is to introduce the fundamental theory of groups and their homomorphisms. Symmetric groups and group of symmetries are also studied in detail. Fermat's Little theorem as a consequence of the Lagrange's theorem on finite groups.

Course Learning Outcomes: The course will enable the students to:
i) Recognize the mathematical objects that are groups, and classify them as abelian, cyclic and permutation groups, etc.
ii) Link the fundamental concepts of groups and symmetrical figures.
iii) Analyze the subgroups of cyclic groups and classify subgroups of cyclic groups.
iv) Explain the significance of the notion of cosets, normal subgroups and factor groups.
v) Learn about Lagrange's theorem and Fermat's Little theorem.
vi) Know about group homomorphisms and group isomorphisms.

## Unit 1: Groups and its Elementary Properties

Symmetries of a square, Dihedral groups, Definition and examples of groups including permutation groups and quaternion groups (illustration through matrices), Elementary properties of groups.

## Unit 2: Subgroups and Cyclic Groups

Subgroups and examples of subgroups, Centralizer, Normalizer, Center of a group, Product of two subgroups; Properties of cyclic groups, Classification of subgroups of cyclic groups.

## Unit 3: Permutation Groups and Lagrange's Theorem

Cycle notation for permutations, Properties of permutations, Even and odd permutations, Alternating groups; Properties of cosets, Lagrange's theorem and consequences including Fermat's Little theorem; Normal subgroups, Factor groups, Cauchy's theorem for finite abelian groups.

## Unit 4: Group Homomorphisms

Group homomorphisms, Properties of homomorphisms, Group isomorphisms, Cayley's theorem, Properties of isomorphisms, First, Second and Third isomorphism theorems for groups.

## Reference:

1. Gallian, Joseph. A. (2013). Contemporary Abstract Algebra (8th ed.). Cengage Learning India Private Limited, Delhi. Fourth impression, 2015.

## Additional Reading:

i. Rotman, Joseph J. (1995). An Introduction to The Theory of Groups (4th ed.). Springer-Verlag, New York.

## Teaching Plan (BMATH306: Group Theory-I):

Week 1: Symmetries of a square, Dihedral groups, Definition and examples of groups including permutation groups and quaternion groups (illustration through matrices).
[1] Chapter 1.
Week 2: Definition and examples of groups, Elementary properties of groups.
[1] Chapter 2.
Week 3: Subgroups and examples of subgroups, Centralizer, Normalizer, Center of a Group, Product of two subgroups.
[1] Chapter 3.
Weeks 4 and 5: Properties of cyclic groups. Classification of subgroups of cyclic groups.
[1] Chapter 4
Weeks 6 and 7: Cycle notation for permutations, Properties of permutations, Even and odd permutations, Alternating group.
[1] Chapter 5 (up to Page 110).
Weeks 8 and 9: Properties of cosets, Lagrange's theorem and consequences including Fermat's Little theorem.
[1] Chapter 7 (up to Example 6, Page 150).
Week 10: Normal subgroups, Factor groups, Cauchy's theorem for finite abelian groups.
[1] Chapters 9 (Theorem 9.1, 9.2, 9.3 and 9.5, and Examples 1 to 12).
Weeks 11 and 12: Group homomorphisms, Properties of homomorphisms, Group isomorphisms, Cayley's theorem.
[1] Chapter 10 (Theorems 10.1 and 10.2, Examples 1 to 11).
[1] Chapter 6 (Theorem 6.1, and Examples 1 to 8).
Weeks 13 and 14: Properties of isomorphisms, First, Second and Third isomorphism theorems.
[1] Chapter 6 (Theorems 6.2 and 6.3), Chapter 10 (Theorems 10.3, 10.4, Examples 12 to 14, and Exercises 41 and 42 for second and third isomorphism theorems for groups).

## Facilitating the Achievement of Course Learning Outcomes

| Unit No. | C | Teaching and Learning Activity | Assessment Tasks |
| :---: | :---: | :---: | :---: |
| 1. | Recognize the mathematical objects that are groups, and classify them as abelian, cyclic and permutation groups, etc. <br> Link the fundamental concepts of groups and symmetrical figures. | (i) Each topic to be explained with examples. <br> (ii) Students to be involved in discussions and encouraged to ask questions. <br> (iii) Students to be given homework/assignments. <br> (iv) Students to be encouraged to give short presentations. | - Presentations and participation in discussions. <br> - Assignments and class tests. <br> - Mid-term examinations. <br> - End-term examinations. |
| 2. | Analyze the subgroups of cyclic groups and classify subgroups of cyclic groups. |  |  |
| 3. | Explain the significance of the notion of cosets, normal subgroups and factor groups. <br> Learn about Lagrange's theorem and Fermat's Little theorem. |  |  |
| 4. | Know about group homomorphisms and group isomorphisms. |  |  |

Keywords: Cauchy's theorem for finite Abelian groups, Cayley's theorem, Centralizer, Cyclic group, Dihedral group, Group homomorphism, Lagrange's theorem, Normalizer, Permutations.

## BMATH307: Multivariate Calculus

Total Marks: 150 (Theory: 75, Internal Assessment: 25 and Practical: 50)
Workload: 4 Lectures, 4 Practicals (per week) Credits: 6 (4+2)
Duration: 14 Weeks ( 56 Hrs. Theory +56 Hrs. Practical) Examination: 3 Hrs.
Course Objectives: To understand the extension of the studies of single variable differential and integral calculus to functions of two or more independent variables. Also, the emphasis will be on the use of Computer Algebra Systems by which these concepts may be analyzed and visualized to have a better understanding. This course will facilitate to become aware of applications of multivariable calculus tools in physics, economics, optimization, and understanding the architecture of curves and surfaces in plane and space etc.

Course Learning Outcomes: This course will enable the students to:
i) Learn the conceptual variations when advancing in calculus from one variable to multivariable discussion.
ii) Understand the maximization and minimization of multivariable functions subject to the given constraints on variables.
iii) Learn about inter-relationship amongst the line integral, double and triple integral formulations.
iv) Familiarize with Green's, Stokes' and Gauss divergence theorems.

## Unit 1: Calculus of Functions of Several Variables

Functions of several variables, Level curves and surfaces, Limits and continuity, Partial differentiation, Higher order partial derivative, Tangent planes, Total differential and differentiability, Chain rule, Directional derivatives, The gradient, Maximal and normal property of the gradient, Tangent planes and normal lines.

## Unit 2: Extrema of Functions of Two Variables and Properties of Vector Field

Extrema of functions of two variables, Method of Lagrange multipliers, Constrained optimization problems; Definition of vector field, Divergence and curl.

## Unit 3: Double and Triple Integrals

Double integration over rectangular and nonrectangular regions, Double integrals in polar coordinates, Triple integral over a parallelopiped and solid regions, Volume by triple integrals, Triple integration in cylindrical and spherical coordinates, Change of variables in double and triple integrals.

## Unit 4: Green's, Stokes' and Gauss Divergence Theorem

Line integrals, Applications of line integrals: Mass and Work, Fundamental theorem for line integrals, Conservative vector fields, Green's theorem, Area as a line integral, Surface integrals, Stokes' theorem, Gauss divergence theorem.

## Reference:

1. Strauss, Monty J., Bradley, Gerald L., \& Smith, Karl J. (2007). Calculus (3rd ed.). Dorling Kindersley (India) Pvt. Ltd. (Pearson Education). Delhi. Indian Reprint 2011.

## Additional Reading:

i. Marsden, J. E., Tromba, A., \& Weinstein, A. (2004). Basic Multivariable Calculus. Springer (SIE). First Indian Reprint.

## Practical / Lab work to be performed in Computer Lab.

List of practicals to be done using Mathematica / MATLAB / Maple/Maxima/Scilab, etc.

1. Let $f(x)$ be any function and $L$ be any real number. For given $a$ and $\varepsilon>0$ find a $\delta>$ 0 such that for all $x$ satisfying $0<|x-a|<\delta$, the inequality $0<|f(x)-l|<\varepsilon$ holds. For example:
(i) $f(x)=x+1, L=5, a=4, \varepsilon=0.01$.
(ii) $f(x)=\sqrt{x+1}, L=1, a=4, \varepsilon=0.1$.
(iii) $f(x)=x^{2}, L=4, a=-2, \varepsilon=0.5$.
(iv) $f(x)=\frac{1}{x}, L=-1, a=-1, \varepsilon=0.1$.
2. Discuss the limit of the following functions when $x$ tends to 0 :

$$
\begin{aligned}
& \pm \frac{1}{x}, \sin \left(\frac{1}{x}\right), \cos \left(\frac{1}{x}\right), x \sin \left(\frac{1}{x}\right), x \cos \left(\frac{1}{x}\right), x^{2} \sin \left(\frac{1}{x}\right), \\
& \frac{1}{x^{n}}(n \in \mathbb{N}),[x] \text { greatest integer function, } \frac{1}{x} \sin \left(\frac{1}{x}\right) .
\end{aligned}
$$

3. Discuss the limit of the following functions when $x$ tends to infinity:

$$
e^{ \pm \frac{1}{x}}, \sin \left(\frac{1}{x}\right), \frac{1}{x} e^{ \pm x}, \frac{x}{x+1}, x^{2} \sin \left(\frac{1}{x}\right), \frac{a x+b}{c x^{2}+d x+e}(a \neq 0, c \neq 0) .
$$

4. Discuss the continuity of the functions at $x=0$ in the Practical 2 .
5. Illustrate the geometric meaning of Rolle's theorem of the following functions on the given interval:
(i) $x^{3}-4 x$ on $[-2,2]$;
(ii) $(x-3)^{4}(x-5)^{3}$ on [3, 5] etc.
6. Illustrate the geometric meaning of Lagrange's mean value theorem of the following functions on the given interval:
(i) $\log x$ on $[1 / 2,2]$; (ii) $x(x-1)(x-2)$ on [0, 1/2]; (iii) $2 x^{2}-7 x+10$ on [2,5] etc.
7. Draw the following surfaces and find level curves at the given heights:
(i) $f(x, y)=10-x^{2}-y^{2} ; z=1, z=6, z=9$.
(ii) $f(x, y)=x^{2}+y^{2} ; z=1, z=6, z=9$.
(iii) $f(x, y)=x^{3}-y ; z=1, z=6$.
(iv) $f(x, y)=x^{2}+\frac{y^{2}}{4} ; z=1, z=5, z=8$.
(v) $f(x, y)=4 x^{2}+y^{2} ; z=0, z=6, z=9$.
8. Draw the following surfaces and discuss whether limit exits or not as $(x, y)$ approaches to the given points. Find the limit, if it exists:
(i) $f(x, y)=\frac{x+y}{x-y} ;(x, y) \rightarrow(0,0)$ and $(x, y) \rightarrow(1,3)$.
(ii) $f(x, y)=\frac{x-y}{\sqrt{x^{2}+y^{2}}} ;(x, y) \rightarrow(0,0)$ and $(x, y) \rightarrow(2,1)$.
(iii) $f(x, y)=(x+y) e^{x y} ;(x, y) \rightarrow(1,1)$ and $(x, y) \rightarrow(1,0)$.
(iv) $f(x, y)=e^{x y} ;(x, y) \rightarrow(0,0)$ and $(x, y) \rightarrow(1,0)$.
(v) $f(x, y)=\frac{x+y^{2}}{x^{2}+y^{2}} ;(x, y) \rightarrow(0,0)$.
(vi) $f(x, y)=\frac{x^{2}-y^{2}}{x^{2}+y^{2}} ;(x, y) \rightarrow(0,0)$ and $(x, y) \rightarrow(2,1)$.
9. Draw the tangent plane to the following surfaces at the given point:
(i) $f(x, y)=\sqrt{x^{2}+y^{2}}$ at $(3,1, \sqrt{10})$.
(ii) $f(x, y)=10-x^{2}-y^{2}$ at $(2,2,2)$.
(iii) $x^{2}+y^{2}+z^{2}=9$ at $(3,0,0)$.
(iii) $z=\tan ^{-1} x$ at $\left(1, \sqrt{3}, \frac{\pi}{3}\right)$ and $\left(2,2, \frac{\pi}{4}\right)$.
(iii) $z=\log \left|x+y^{2}\right|$ at $(-3,-2,0)$.
10. Use an incremental approximation to estimate the following functions at the given point and compare it with calculated value:
(i) $f(x, y)=3 x^{4}+2 y^{4}$ at $(1.01,2.03)$.
(ii) $f(x, y)=x^{5}-2 y^{3}$ at $(0.98,1.03)$.
(iii) $f(x, y)=e^{x y}$ at $(1.01,0.98)$.
11. Find critical points and identify relative maxima, relative minima or saddle points to the following surfaces, if it exists:
(i) $z=x^{2}+y^{2}$; (ii) $z=1-x^{2}-y^{2}$; (iii) $z=y^{2}-x^{2}$; (iv) $z=x^{2} y^{4}$.
12. Draw the following regions D and check whether these regions are of Type I or Type II:
(i) $D=\left\{(x, y): 0 \leq x \leq 2,1 \leq y \leq e^{x}\right\}$.
(ii) $D=\left\{(x, y): \log y \leq x \leq 2,1 \leq y \leq e^{2}\right\}$.
(iii) $D=\left\{(x, y): 0 \leq x \leq 1, x^{3} \leq y \leq 1\right\}$.
(iv) The region $D$ bounded by $y=x^{2}-2$ and the line $y=x$.
(v) $D=\left\{(x, y): 0 \leq x \leq \frac{\pi}{4}, \sin x \leq y \leq \cos x\right\}$.

## Teaching Plan (Theory of BMATH307: Multivariate Calculus):

Week 1: Definition of functions of several variables, Graphs of functions of two variables - Level curves and surfaces, Limits and continuity of functions of two variables.
[1] Sections 11.1 and 11.2.
Week 2: Partial differentiation, and partial derivative as slope and rate, Higher order partial derivatives. Tangent planes, incremental approximation, Total differential.
[1] Chapter 11 (Sections 11.3 and 11.4).
Week 3: Differentiability, Chain rule for one parameter, Two and three independent parameters.
[1] Chapter 11 (Sections 11.4 and 11.5).
Week 4: Directional derivatives, The gradient, Maximal and normal property of the gradient, Tangent and normal lines.
[1] Chapter 11 (Section 11.6).
Week 5: First and second partial derivative tests for relative extrema of functions of two variables, and absolute extrema of continuous functions.
[1] Chapter 11 [Section 11.7 (up to page 605)].
Week 6: Lagrange multipliers method for optimization problems with one constraint, Definition of vector field, Divergence and curl.
[1] Sections 11.8 (Pages 610-614)] and 13.1.
Week 7: Double integration over rectangular and nonrectangular regions.
[1] Sections 12.1 and 12.2.
Week 8: Double integrals in polar co-ordinates, and triple integral over a parallelopiped.
[1] Chapter 12 (Sections 12.3 and 12.4).
Week 9: Triple integral over solid regions, Volume by triple integrals, and triple integration in cylindrical coordinates.
[1] Chapter 12 (Sections 12.4 and 12.5).
Week 10: Triple integration in spherical coordinates, Change of variables in double and triple integrals.
[1] Chapter 12 (Sections 12.5 and 12.6).
Week 11: Line integrals and its properties, applications of line integrals: mass and work.
[1] Chapter 13 (Section 13.2).

Week 12: Fundamental theorem for line integrals, Conservative vector fields and path independence.
[1] Chapter 13 (Section 13.3).
Week 13: Green's theorem for simply connected region, Area as a line integral, Definition of surface integrals.
[1] Chapter 13 [Sections 13.4 (Pages 712 to 716), 13.5 (Pages 723 to 726)].
Week 14: Stokes' theorem and the divergence theorem.
[1] Chapter 13 [Sections 13.6 (Pages 733 to 737), 13.7 (Pages 742 to 745)].
Note. To improve the problem solving ability, for similar kind of examples based upon the above contents, the Additional Reading (i) may be consulted.

Facilitating the Achievement of Course Learning Outcomes

| $\begin{aligned} & \text { Unit } \\ & \text { No. } \\ & \hline \end{aligned}$ | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
| :---: | :---: | :---: | :---: |
| 1. | Learn the conceptual variations when advancing in calculus from one variable to multivariable discussion. | (i) Each topic to be explained with illustrations. <br> (ii) Students to be encouraged to discover the relevant concepts. <br> (iii) Students to be given homework/assignments. <br> (iv) Discuss and solve the theoretical and practical problems in the class. <br> (v) Students to be encouraged to apply concepts to real world problems. | - Presentations and class discussions. <br> - Assignments and class tests. <br> - Mid-term examinations. <br> - Practical and vivavoce examinations. <br> - End-term examinations. |
| 2. | Understand the maximization and minimization of multivariable functions subject to the given constraints on variables. |  |  |
| 3. | Learn about inter-relationship amongst the line integral, double and triple integral formulations. |  |  |
| 4. | Familiarize with Green's, Stokes' and Gauss divergence theorems. |  |  |

Keywords: Directional derivatives, Double integral, Gauss divergence theorem, Green's theorem, Lagrange's multipliers, Level curves, Stokes' theorem, Volume integral, Vector field.

# Skill Enhancement Paper 

SEC-1: LaTeX and HTML
Total Marks: 100 (Theory: 38, Internal Assessment: 12, and Practical: 50)
Workload: 2 Lectures, 4 Practicals (per week) Credits: 4 (2+2)
Duration: 14 Weeks ( 28 Hrs. Theory +56 Hrs. Practical) Examination: 2 Hrs.
Course Objectives: The purpose of this course is to acquaint students with the latest typesetting skills, which shall enable them to prepare high quality typesetting, beamer presentation and webpages.
Course Learning Outcomes: After studying this course the student will be able to:
i) Create and typeset a LaTeX document.
ii) Typeset a mathematical document using LaTex.
iii) Learn about pictures and graphics in LaTex.
iv) Create beamer presentations.
v) Create web page using HTML.

## Unit 1: Getting Started with LaTeX

Introduction to TeX and LaTeX, Typesetting a simple document, Adding basic information to a document, Environments, Footnotes, Sectioning and displayed material.

## Unit 2: Mathematical Typesetting with LaTeX

Accents and symbols, Mathematical typesetting (elementary and advanced): Subscript/ Superscript, Fractions, Roots, Ellipsis, Mathematical Symbols, Arrays, Delimiters, Multiline formulas, Spacing and changing style in math mode.

## Unit 3: Graphics and Beamer Presentation in LaTeX

Graphics in LaTeX, Simple pictures using PSTricks, Plotting of functions, Beamer presentation.

## Unit 4: HTML

HTML basics, Creating simple web pages, Images and links, Design of web pages.

## References:

1. Bindner, Donald \& Erickson, Martin. (2011). A Student's Guide to the Study, Practice, and Tools of Modern Mathematics. CRC Press, Taylor \& Francis Group, LLC.
2. Lamport, Leslie (1994). LaTeX: A Document Preparation System, User's Guide and Reference Manual (2nd ed.). Pearson Education. Indian Reprint.

## Additional Readings:

i. Dongen, M. R. C. van (2012). LaTeX and Friends. Springer-Verlag.
ii. Robbins, J. N. (2018). Learning Web Design: A Beginner's Guide to HTML (5th ed.). O'Reilly Media Inc.

## Practical / Lab work to be performed in Computer Lab.

[1] Chapter 9 (Exercises 4 to 10), Chapter 10 (Exercises 1 to 4 and 6 to 9),
Chapter 11 (Exercises 1, 3, 4, and 5), and Chapter 15 (Exercises 5, 6 and 8 to 11).

## Teaching Plan (Theory of SEC-1: LaTeX and HTML):

Weeks 1 to 3: Introduction to TeX and LaTeX, Typesetting a simple document, Adding basic information to a document, Environments, Footnotes, Sectioning and displayed material.
[1] Chapter 9 ( 9.1 to 9.5 ).
[2] Chapter 2 (2.1 to 2.5).
Weeks 4 to 6: Accents of symbols, Mathematical typesetting (elementary and advanced): Subscript/Superscript, Fractions, Roots, Ellipsis, Mathematical symbols, Arrays, Delimiters, Multiline formulas, Spacing and changing style in math mode.
[1] Chapter 9 (9.6 and 9.7).
[2] Chapter 3 (3.1 to 3.3).
Weeks 7 and 8: Graphics in LaTeX, Simple pictures using PSTricks, Plotting of functions.
[1] Chapter 9 (Section 9.8). Chapter 10 (10.1 to 10.3).
[2] Chapter 7 (7.1 and 7.2).
Weeks 9 and 10: Beamer presentation.
[1] Chapter 11 (Sections 11.1 to 11.4).
Weeks 11 and 12: HTML basics, Creating simple web pages.
[1] Chapter 15 (Sections 15.1 and 15.2).
Weeks 13 and 14: Adding images and links, Design of web pages.
[1] Chapter 15 (Sections 15.3 to 15.5).
Facilitating the Achievement of Course Learning Outcomes

| Unit <br> No. | Course Learning Outcomes | Teaching and Learning <br> Activity | Assessment Tasks |
| :---: | :--- | :--- | :--- |
| 1. | Create and typeset a LaTeX <br> document. | (i) Each topic to be explained <br> with illustrations on <br> computers. | • Presentations and <br> class discussions. |
| 2. | Typeset a mathematical <br> document using LaTex. | (ii) Students be given <br> homework/ assignments. <br> class tests. |  |
| 3. | Learn about pictures and <br> graphics in LaTex. <br> Mid-term <br> ereate beamer presentations. | (iii) Students be encouraged to <br> create simple webpages. | examations. <br> End-term <br> examinations. |
| 4. | Create web page using HTML. |  |  |

Keywords: LaTex, Mathematical typesetting, PSTricks, Beamer, HTML.

## Semester-IV

## BMATH408: Partial Differential Equations

Total Marks: 150 (Theory: 75, Internal Assessment: 25 and Practical: 50)
Workload: 4 Lectures, 4 Practicals (per week) Credits: 6 (4+2)
Duration: 14 Weeks ( 56 Hrs. Theory +56 Hrs. Practical) Examination: 3 Hrs.
Course Objectives: The main objectives of this course are to teach students to form and solve partial differential equations and use them in solving some physical problems.

Course Learning Outcomes: The course will enable the students to:
i) Formulate, classify and transform first order PDEs into canonical form.
ii) Learn about method of characteristics and separation of variables to solve first order PDE's.
iii) Classify and solve second order linear PDEs.
iv) Learn about Cauchy problem for second order PDE and homogeneous and nonhomogeneous wave equations.
v) Apply the method of separation of variables for solving many well-known second order PDEs.

## Unit 1: First Order PDE and Method of Characteristics

Introduction, Classification, Construction and geometrical interpretation of first order partial differential equations (PDE), Method of characteristic and general solution of first order PDE, Canonical form of first order PDE, Method of separation of variables for first order PDE.

Unit 2: Mathematical Models and Classification of Second Order Linear PDE
Gravitational potential, Conservation laws and Burger's equations, Classification of second order PDE, Reduction to canonical forms, Equations with constant coefficients, General solution.

## Unit 3: The Cauchy Problem and Wave Equations

Mathematical modeling of vibrating string and vibrating membrane, Cauchy problem for second order PDE, Homogeneous wave equation, Initial boundary value problems, Nonhomogeneous boundary conditions, Finite strings with fixed ends, Non-homogeneous wave equation, Goursat problem.

## Unit 4: Method of Separation of Variables

Method of separation of variables for second order PDE, Vibrating string problem, Existence and uniqueness of solution of vibrating string problem, Heat conduction problem, Existence and uniqueness of solution of heat conduction problem, Non-homogeneous problem.

## Reference:

1. Myint-U, Tyn \& Debnath, Lokenath. (2007). Linear Partial Differential Equation for Scientists and Engineers (4th ed.). Springer, Third Indian Reprint, 2013.

## Additional Readings:

i. Sneddon, I. N. (2006). Elements of Partial Differential Equations, Dover Publications. Indian Reprint.
ii. Stavroulakis, Ioannis P \& Tersian, Stepan A. (2004). Partial Differential Equations: An Introduction with Mathematica and MAPLE (2nd ed.). World Scientific.

## Practical / Lab work to be performed in a Computer Lab:

Modeling of the following similar problems using Mathematica/MATLAB/Maple/Maxima/Scilab etc.

1. Solution of Cauchy problem for first order PDE.
2. Plotting the characteristics for the first order PDE.
3. Plot the integral surfaces of a given first order PDE with initial data.
4. Solution of wave equation $\frac{\partial^{2} u}{\partial t^{2}}=c^{2} \frac{\partial^{2} u}{\partial x^{2}}$ for any two of the following associated conditions:
(i) $u(x, 0)=\phi(x), u(x, 0)=\psi(x), x \in \mathbb{R}, t>0$.
(ii) $u(x, 0)=\phi(x), u_{t}(x, 0)=\psi(x), u(0, t)=0, x>0 t>0$.
(iii) $u(x, 0)=\phi(x), u_{t}(x, 0)=\psi(x), u_{x}(0, t)=0, x>0, t>0$.
(iv) $u(x, 0)=\phi(x), u(x, 0)=\psi(x), u(0, t)=0, u(l, t)=0,0<x<l, t>0$.
5. Solution of one-dimensional heat equation $u_{t}=k u_{x x}$, for a homogeneous rod of length $l$. That is - solve the IBVP:

$$
\begin{gathered}
u_{t}=k u_{x x}, \quad 0<x<l, \quad t>0, \\
u(0, t)=0, \quad u(l, t)=0, \quad t \geq 0 \\
u(0, t)=f(x), 0 \leq x \leq l .
\end{gathered}
$$

6. Solving systems of ordinary differential equations.
7. Draw the following sequence of functions on the given interval and discuss the pointwise convergence:
(i) $f_{n}(x)=x^{n}$ for $x \in \mathbb{R}$,
(ii) $f_{n}(x)=\frac{x}{n}$ for $x \in \mathbb{R}$,
(iii) $f_{n}(x)=\frac{x^{2}+n x}{n}$ for $x \in \mathbb{R}$,
(iv) $f_{n}(x)=\frac{\sin n x+n}{n}$ for $x \in \mathbb{R}$
(v) $f_{n}(x)=\frac{x^{n}}{x+n}$ for $x \in \mathbb{R} x \geq 0$,
(vi) $f_{n}(x)=\frac{n_{x}^{n}}{1+n^{2} x^{2}}$ for $x \in \mathbb{R}$
(vii) $f_{n}(x)=\frac{n x}{1+n x}$ for $x \in \mathbb{R}, x \geq 0$,
(viii) $f_{n}(x)=\frac{x^{n}}{1+x^{n}}$ for $x \in \mathbb{R}, x \geq 0$
8. Discuss the uniform convergence of sequence of functions (i) to (viii) given above in (7).

## Teaching Plan (Theory of BMATH408: Partial Differential Equations):

Week 1: Introduction, Classification, Construction of first order partial differential equations (PDE). [1] Chapter 2 (Sections 2.1 to 2.3).
Week 2: Method of characteristics and general solution of first order PDE.
[1] Chapter 2 (Sections 2.4 and 2.5).
Week 3: Canonical form of first order PDE, Method of separation of variables for first order PDE.
[1] Chapter 2 (Sections 2.6 and 2.7).
Week 4: The vibrating string, Vibrating membrane, Gravitational potential, Conservation laws.
[1] Chapter 3 (Sections 3.1 to 3.3, 3.5 and 3.6).
Weeks 5 and 6: Reduction to canonical forms, Equations with constant coefficients, General solution. [1] Chapter 4 (Sections 4.1 to 4.5).
Weeks 7 and 8: The Cauchy problem for second order PDE, Homogeneous wave equation.
[1] Chapter 5 (Sections 5.1, 5.3 and 5.4).

Weeks 9 and 10: Initial boundary value problem, Non-homogeneous boundary conditions, Finite string with fixed ends, Non-homogeneous wave equation, Goursat problem.
[1] Chapter 5 (Sections 5.5 to 5 . and 5.9).
Weeks 11 and 12: Method of separation of variables for second order PDE, Vibrating string problem.
[1] Chapter 7 (Sections 7.1 to 7.3).
Weeks 13 and 14: Existence (omit proof) and uniqueness of vibrating string problem. Heat conduction problem. Existence (omit proof) and uniqueness of the solution of heat conduction problem. Non-homogeneous problem.
[1] Chapter 7 (Sections 7.4 to 7.6 and 7.8).

## Facilitating the Achievement of Course Learning Outcomes

| $\begin{aligned} & \hline \text { Unit } \\ & \text { No. } \end{aligned}$ | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
| :---: | :---: | :---: | :---: |
| 1. | Formulate, classify and transform first order PDEs into canonical form. <br> Learn about method of characteristics and separation of variables to solve first order PDEs. | (i) Each topic to be explained with examples. <br> (ii) Students to be encouraged to discover the relevant concepts. <br> (iii) Students to be given homework/ assignments. <br> (iv) Discuss and solve the theoretical and practical problems in the class. <br> (v) Students to be encouraged to apply concepts to real world problems. | - Presentations and class discussions. <br> - Assignments and class tests. <br> - Mid-term examinations. <br> - Practical and viva-voce examinations. <br> - End-term examinations. |
| 2. | Classify and solve second order linear PDEs. |  |  |
| 3. | Learn about Cauchy problem for second order PDE and homogeneous and non-homogeneous wave equations. |  |  |
| 4. | Apply the method of separation of variables for solving many wellknown second order PDEs. |  |  |

Keywords: Cauchy problem, Characteristics, Conservation laws and Burger's equations, Heat equation, Vibrating membrane, Wave equation.

## BMATH409: Riemann Integration \& Series of Functions

Total Marks: 100 (Theory: 75 and Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) Credits: 6 (5+1)
Duration: 14 Weeks ( 70 Hrs.) Examination: 3 Hrs.
Course Objectives: To understand the integration of bounded functions on a closed and bounded interval and its extension to the cases where either the interval of integration is infinite, or the integrand has infinite limits at a finite number of points on the interval of integration. The sequence and series of real valued functions, and an important class of series of functions (i.e., power series).

Course Learning Outcomes: The course will enable the students to:
i) Learn about some of the classes and properties of Riemann integrable functions, and the applications of the Fundamental theorems of integration.
ii) Know about improper integrals including, beta and gamma functions.
iii) Learn about Cauchy criterion for uniform convergence and Weierstrass M-test for uniform convergence.
iv) Know about the constraints for the inter-changeability of differentiability and integrability with infinite sum.
v) Approximate transcendental functions in terms of power series as well as, differentiation and integration of power series.

## Unit 1: Riemann Integration

Definition of Riemann integration, Inequalities for upper and lower Darboux sums, Necessary and sufficient conditions for the Riemann integrability, Definition of Riemann integration by Riemann sum and equivalence of the two definitions, Riemann integrability of monotone functions and continuous functions, Properties of Riemann integrable functions, Definitions of piecewise continuous and piecewise monotone functions and their Riemann integrability, intermediate value theorem for integrals, Fundamental theorems (I and II) of calculus, and the integration by parts.

## Unit 2: Improper Integral

Improper integrals of Type-I, Type-II and mixed type, Convergence of beta and gamma functions, and their properties.

## Unit 3: Sequence and Series of Functions

Pointwise and uniform convergence of sequence of functions, Theorem on the continuity of the limit function of a sequence of functions, Theorems on the interchange of the limit and derivative, and the interchange of the limit and integrability of a sequence of functions. Pointwise and uniform convergence of series of functions, Theorems on the continuity, derivability and integrability of the sum function of a series of functions, Cauchy criterion and the Weierstrass M-test for uniform convergence.

## Unit 4: Power Series

Definition of a power series, Radius of convergence, Absolute convergence (Cauchy-Hadamard theorem), Uniform convergence, Differentiation and integration of power series, Abel's theorem.

## References:

1. Bartle, Robert G., \& Sherbert, Donald R. (2015). Introduction to Real Analysis (4th ed.). Wiley India Edition. Delhi.
2. Denlinger, Charles G. (2011). Elements of Real Analysis. Jones \& Bartlett (Student Edition). First Indian Edition. Reprinted 2015.
3. Ghorpade, Sudhir R. \& Limaye, B. V. (2006). A Course in Calculus and Real Analysis. Undergraduate Texts in Mathematics, Springer (SIE). First Indian reprint.
4. Ross, Kenneth A. (2013). Elementary Analysis: The Theory of Calculus (2nd ed.). Undergraduate Texts in Mathematics, Springer.

## Additional Reading:

i. Bilodeau, Gerald G., Thie, Paul R., \& Keough, G. E. (2010). An Introduction to Analysis (2nd ed.). Jones \& Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

## Teaching Plan (BMATH409: Riemann Integration \& Series of Functions):

Week 1: Definition of Riemann integration, Inequalities for upper and lower Darboux sums.
[4] Chapter 6 [Section 32 (32.1 to 32.4)].
Week 2: Necessary and sufficient conditions for the Riemann integrability, Definition of Riemann integration by Riemann sum and equivalence of the two definitions.
[4] Chapter 6 [Section 32 ( 32.5 to 32.10)].
Week 3: Riemann integrability of monotone functions and continuous functions, Algebra and properties of Riemann integrable functions.
[4] Chapter 6 [Section 33 (33.1 to 33.6)].
Week 4: Definitions of piecewise continuous and piecewise monotone functions and their Riemann integrability, Intermediate value theorem for integrals.
[4] Chapter 6 [Section 33 (33.7 to 33.10)].
Week 5: First and second fundamental theorems of integral calculus, and the integration by parts.
[4] Chapter 6 [Section 34 (34.1 to 34.3)].
Week 6: Improper integrals of Type-I, Type-II and mixed type.
[2] Chapter 7 [Section 7.8 (7.8.1 to 7.8.18)].
Week 7: Convergence of beta and gamma functions, and their properties.
[3] Pages 405-408.
Week 8: Definitions and examples of pointwise and uniformly convergent sequence of functions.
[1] Chapter 8 [Section 8.1 (8.1.1 to 8.1.10)].
Week 9: Motivation for uniform convergence by giving examples, Theorem on the continuity of the limit function of a sequence of functions.
[1] Chapter 8 [Section 8.2 (8.2.1 to 8.2.2)].
Week 10: The statement of the theorem on the interchange of the limit function and derivative, and its illustration with the help of examples, The interchange of the limit function and integrability of a sequence of functions.
[1] Chapter 8 [Section 8.2 (Theorems 8.2.3 and 8.2.4)].
Week 11: Pointwise and uniform convergence of series of functions, Theorems on the continuity, derivability and integrability of the sum function of a series of functions.
[1] Chapter 9 [Section 9.4 (9.4.1 to 9.4.4)].
Week 12: Cauchy criterion for the uniform convergence of series of functions, and the Weierstrass M-test for uniform convergence.
[2] Chapter 9 [Section 9.4 (9.4.5 to 9.4.6)].
Week 13: Definition of a power series, Radius of convergence, Absolute and uniform convergence of a power series.
[4] Chapter 4 (Section 23).

Week 14: Differentiation and integration of power series, Statement of Abel's theorem and its illustration with the help of examples.
[4] Chapter 4 [Section 26 (26.1 to 26.6)].

## Facilitating the Achievement of Course Learning Outcomes

| Unit No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
| :---: | :---: | :---: | :---: |
| 1. | Learn about some of the classes and properties of Riemann integrable functions, and the applications of the fundamental theorems of integration. | (i) Each topic to be explained with examples. <br> (ii) Students to be involved in discussions and encouraged to ask questions. <br> (iii) Students to be given homework/assignments. <br> (iv) Students to be encouraged to give short presentations. | - Presentations and participation in discussions. <br> - Assignments and class tests. <br> - Mid-term examinations. <br> - End-term examinations. |
| 2. | Know about improper integrals including, beta and gamma functions. |  |  |
| 3. | Learn about Cauchy criterion for uniform convergence and Weierstrass M-test for uniform convergence. Know about the constraints for the inter-changeability of differentiability and integrability with infinite sum. |  |  |
| 4. | Approximate transcendental functions in terms of power series as well as, differentiation and integration of power series. |  |  |

Keywords: Beta function, Gamma function, Improper integral, Power series, Radius of convergence, Riemann integration, Uniform convergence, Weierstrass M-test.

# BMATH410: Ring Theory \& Linear Algebra-I 

Total Marks: 100 (Theory: 75 and Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) Credits: 6 (5+1)
Duration: 14 Weeks ( 70 Hrs.) Examination: 3 Hrs.
Course Objectives: The objective of this course is to introduce the fundamental theory of two objects, namely - rings and vector spaces, and their corresponding homomorphisms.
Course Learning Outcomes: The course will enable the students to:
i) Learn about the fundamental concept of rings, integral domains and fields.
ii) Know about ring homomorphisms and isomorphisms theorems of rings.
iii) Learn about the concept of linear independence of vectors over a field, and the dimension of a vector space.
iv) Basic concepts of linear transformations, dimension theorem, matrix representation of a linear transformation, and the change of coordinate matrix.

## Unit 1: Introduction of Rings

Definition and examples of rings, Properties of rings, Subrings, Integral domains and fields, Characteristic of a ring, Ideals, Ideal generated by a subset of a ring, Factor rings, Operations on ideals, Prime and maximal ideals.

## Unit 2: Ring Homomorphisms

Ring homomorphisms, Properties of ring homomorphisms, First, Second and Third Isomorphism theorems for rings, The Field of quotients.

## Unit 3: Introduction of Vector Spaces

Vector spaces, Subspaces, Algebra of subspaces, Quotient spaces, Linear combination of vectors, Linear span, Linear independence, Basis and dimension, Dimension of subspaces.

## Unit 4: Linear Transformations

Linear transformations, Null space, Range, Rank and nullity of a linear transformation, Matrix representation of a linear transformation, Algebra of linear transformations, Isomorphisms, Isomorphism theorems, Invertibility and the change of coordinate matrix.

## References:

1. Gallian, Joseph. A. (2013). Contemporary Abstract Algebra (8th ed.). Cengage Learning India Private Limited. Delhi. Fourth impression, 2015.
2. Friedberg, Stephen H., Insel, Arnold J., \& Spence, Lawrence E. (2003). Linear Algebra (4th ed.). Prentice-Hall of India Pvt. Ltd. New Delhi.

## Additional Readings:

i. Dummit, David S., \& Foote, Richard M. (2016). Abstract Algebra (3rd ed.). Student Edition. Wiley India.
ii. Herstein, I. N. (2006). Topics in Algebra (2nd ed.). Wiley Student Edition. India.
iii. Hoffman, Kenneth, \& Kunze, Ray Alden (1978). Linear Algebra (2nd ed.). PrenticeHall of India Pvt. Limited. Delhi. Pearson Education India Reprint, 2015.

## Teaching Plan (BMATH410: Ring Theory \& Linear Algebra-I):

Week 1: Definition and examples of rings, Properties of rings, Subrings.
[1] Chapter 12.
Week 2: Integral domains and fields, Characteristic of a ring.
[1] Chapter 13.
Week 3 and 4: Ideals, Ideal generated by a subset of a ring, Factor rings, Operations on ideals, Prime and maximal ideals.
[1] Chapter 14.
Week 5: Ring homomorphisms, Properties of ring homomorphisms.
[1] Chapter 15 (up to Theorem 15.2).
Week 6: First, Second and Third Isomorphism theorems for rings, The field of quotients.
[1] Chapter 15 (Theorems 15.3 to 15.6, Examples 10 to 12), and Exercises 3 and 4 on Page 347.

Week 7: Vector spaces, Subspaces, Algebra of subspaces.
[2] Chapter 1 (Sections 1.2 and 1.3).
Week 8: Linear combination of vectors, Linear span, Linear independence.
[2] Chapter 1 (Sections 1.4 and 1.5).
Weeks 9 and 10: Bases and dimension. Dimension of subspaces.
[2] Chapter 1 (Section 1.6).
Week 11: Linear transformations, Null space, Range, Rank and nullity of a linear transformation.
[2] Chapter 2 (Section 2.1).
Weeks 12 and 13: Matrix representation of a linear transformation, Algebra of linear transformations.
[2] Chapter 2 (Sections 2.2 and 2.3).
Week 14: Isomorphisms, Isomorphism theorems, Invertibility and the change of coordinate matrix.
[2] Chapter 2 (Sections 2.4 and 2.5).
Facilitating the achievement of Course Learning Outcomes

| Unit <br> No. | Course Learning Outcomes | Teaching and Learning <br> Activity | Assessment Tasks |
| :---: | :--- | :--- | :--- |
| 1. | Learn about the fundamental <br> concept of rings, integral domains <br> and fields. | (i) Each topic to be explained <br> with examples. <br> (ii) Students to be involved in <br> discussions and encouraged <br> to ask questions. | - Presentations and <br> participation in <br> discussions. <br> (iii) Students to be given <br> homework/assignments. <br> hssignments and <br> class tests. <br> - Mid-term <br> examinations. <br> - End-term <br> examinations. |
| 2. | Know about ring homomorphisms <br> and isomorphisms theorems of <br> rings. | (iv) Students to be encouraged <br> to give short presentations. | Learn about the concept of linear <br> independence of vectors over a <br> field, and the dimension of a vector <br> space. |
| 4. | Basic concepts of linear <br> transformations, dimension <br> theorem, matrix representation of a <br> linear transformation, and the <br> change of coordinate matrix. |  |  |

Keywords: Basis and dimension of a vector space, Characteristic of a ring, Integral domain, Isomorphism theorems for rings, Linear transformations, Prime and maximal ideals, Quotient field, Vector space.

# Skill Enhancement Paper 

## SEC-2: Computer Algebra Systems and Related Software

Total Marks: 100 (Theory: 38, Internal Assessment: 12, and Practical: 50)
Workload: 2 Lectures, 4 Practicals (per week) Credits: 4 (2+2)
Duration: 14 Weeks ( 28 Hrs. Theory +56 Hrs. Practical) Examination: 2 Hrs.
Course Objectives: This course aims at familiarizing students with the usage of computer algebra systems (/Mathematica/MATLAB/Maxima/Maple) and the statistical software R. The basic emphasis is on plotting and working with matrices using CAS. Data entry and summary commands will be studied in $\mathbf{R}$. Graphical representation of data shall also be explored.
Course Learning Outcomes: This course will enable the students to:
i) Use of computer algebra systems (Mathematica/MATLAB/Maxima/Maple etc.) as a calculator, for plotting functions and animations
ii) Use of CAS for various applications of matrices such as solving system of equations and finding eigenvalues and eigenvectors.
iii) Understand the use of the statistical software $\mathbf{R}$ as calculator and learn to read and get data into $\mathbf{R}$.
iv) Learn the use of $\mathbf{R}$ in summary calculation, pictorial representation of data and exploring relationship between data.
v) Analyze, test, and interpret technical arguments on the basis of geometry.

## Unit 1: Introduction to CAS and Applications

Computer Algebra System (CAS), Use of a CAS as a calculator, Computing and plotting functions in 2D, Plotting functions of two variables using Plot3D and ContourPlot, Plotting parametric curves surfaces, Customizing plots, Animating plots, Producing tables of values, working with piecewise defined functions, Combining graphics.

## Unit 2: Working with Matrices

Simple programming in a CAS, Working with matrices, Performing Gauss elimination, operations (transpose, determinant, inverse), Minors and cofactors, Working with large matrices, Solving system of linear equations, Rank and nullity of a matrix, Eigenvalue, eigenvector and diagonalization.

## Unit 3: R-The Statistical Programming Language

$\mathbf{R}$ as a calculator, Explore data and relationships in R. Reading and getting data into R: Combine and scan commands, Types and structure of data items with their properties, Manipulating vectors, Data frames, Matrices and lists, Viewing objects within objects, Constructing data objects and conversions.

## Unit 4: Data Analysis with $R$

Summary commands: Summary statistics for vectors, Data frames, Matrices and lists, Summary tables, Stem and leaf plot, Histograms, Plotting in R: Box-whisker plots, Scatter plots, Pairs plots, Line charts, Pie charts, Cleveland dot charts and bar charts, Copy and save graphics to other applications.

## References:

1. Bindner, Donald \& Erickson, Martin. (2011). A Student's Guide to the Study, Practice, and Tools of Modern Mathematics. CRC Press, Taylor \& Francis Group, LLC.
2. Torrence, Bruce F., \& Torrence, Eve A. (2009). The Student's Introduction to Mathematica ${ }^{\circledR}$ : A Handbook for Precalculus, Calculus, and Linear Algebra (2nd ed.). Cambridge University Press.
3. Gardener, M. (2012). Beginning R: The Statistical Programming Language, Wiley.

## Additional Reading:

i. Verzani, John (2014). Using $R$ for Introductory Statistics (2nd ed.). CRC Press, Taylor \& Francis Group.

Note: Theoretical and Practical demonstration should be carried out only in one of the CAS: Mathematica/MATLAB/Maxima/Scilab or any other.

## Practical / Lab work to be performed in Computer Lab.

[1] Chapter 12 (Exercises 1 to 4 and 8 to 12), Chapter 14 (Exercises 1 to 3)
[2] Chapter 3 [Exercises 3.2(1 and 2), 3.3(1, 2 and 4), 3.4(1 and 2), 3.5(1 to 4), 3.6(2 and 3)].
[2] Chapter 6 (Exercises 6.2 and 6.3) and Chapter 7 [Exercises 7.1(1), 7.2, 7.3(2), 7.4(1) and 7.6].

Note: Relevant exercises of [3] Chapters 2 to 5 and 7 (The practical may be done on the database to be downloaded from http://data.gov.in/).

## Teaching Plan (Theory of SEC-1: Computer Algebra Systems and Related Software):

Weeks 1 to 3: Computer Algebra System (CAS), Use of a CAS as a calculator, Computing and plotting functions in 2D, Producing tables of values, Working with piecewise defined functions, Combining graphics. Simple programming in a CAS.
[1] Chapter 12 (Sections 12.1 to 12.5).
[2] Chapter 1, and Chapter 3 (Sections 3.1 to 3.6 and 3.8).
Weeks 4 and 5: Plotting functions of two variables using Plot3D and contour plot, Plotting parametric curves surfaces, Customizing plots, Animating plots.
[2] Chapter 6 (Sections 6.2 and 6.3).
Weeks 6 to 8: Working with matrices, Performing Gauss elimination, Operations (Transpose, Determinant, Inverse), Minors and cofactors, Working with large matrices, Solving system of linear equations, Rank and nullity of a matrix, Eigenvalue, Eigenvector and diagonalization.
[2] Chapter 7 (Sections 7.1 to 7.8).
Weeks 9 to 11: R as a calculator, Explore data and relationships in R. Reading and getting data into R: Combine and scan commands, Types and structure of data items with their properties. Manipulating vectors, Data frames, Matrices and lists. Viewing objects within objects. Constructing data objects and conversions.
[1] Chapter 14 (Sections 14.1 to 14.4).
[3] Chapter 2, and Chapter 3.
Weeks 12 to 14: Summary commands: Summary statistics for vectors, Data frames, Matrices and lists. Summary tables. Stem and leaf plot, histograms. Plotting in R: Box-whisker plots, Scatter plots, Pairs plots, Line charts, Pie charts, Cleveland dot charts and Bar charts. Copy and save graphics to other applications.
[1] Chapter 14 (Section 14.7).
[3] Chapter 5 (up to Page 157), and Chapter 7.

## Facilitating the Achievement of Course Learning Outcomes

| Unit No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
| :---: | :---: | :---: | :---: |
| 1. | Use of computer algebra systems (Mathematica/MATLAB/Maxima/Maple etc.) as a calculator, for plotting functions and animations | (i) Each topic to be explained with illustrations using CAS or $\mathbf{R}$. <br> (ii) Students to be given homework/ assignments. <br> (iii) Students to be encouraged to do look for new applications. | - Presentations and class discussions. <br> - Assignments and class tests. <br> - Mid-term examinations. <br> - End-term examinations. |
| 2. | Use of CAS for various applications of matrices such as solving system of equations and finding eigenvalues and eigenvectors. |  |  |
| 3. | Understand the use of the statistical software $\mathbf{R}$ as calculator and learn to read and get data into R. |  |  |
| 4. | Learn the use of $\mathbf{R}$ in summary calculation, pictorial representation of data and exploring relationship between data. <br> Analyze, test, and interpret technical arguments on the basis of geometry. |  |  |

Keywords: Plot3D, ContourPlot, Calculator, Summary commands, Histograms.

Choice Based Credit System (CBCS)

## UNIVERSITY OF DELHI

## DEPARTMENT OF MATHEMATICS

## UNDERGRADUATE PROGRAMME <br> (Courses effective from Academic Year 2015-16)



## SYLLABUS OF COURSES TO BE OFFERED <br> Core Courses, Elective Courses \& Ability Enhancement Courses

Disclaimer: The CBCS syllabus is uploaded as given by the Faculty concerned to the Academic Council. The same has been approved as it is by the Academic Council on 13.7.2015 and Executive Council on 14.7.2015. Any query may kindly be addressed to the concerned Faculty.

Undergraduate Programme Secretariat

## C 11 Metric Spaces

Total marks: 100
Theory: 75
Internal Assessment: 25
5 Lectures, 1 Tutorial (per week per student)
Metric spaces: definition and examples. Sequences in metric spaces, Cauchy sequences. Complete Metric Spaces.
[1] Chapter1, Section 1.2 (1.2.1 to 1.2.6 ). Section 1.3, Section 1.4 (1.4.1 to 1.4.4), Section 1.4 (1.4.5 to 1.4.14 (ii)).

Open and closed balls, neighbourhood, open set, interior of a set, Limit point of a set, closed set, diameter of a set, Cantor's Theorem, Subspaces, dense sets, separable spaces.
[1] Chapter2, Section 2.1 (2.1.1 to 2.1.16), Section 2.1 (2.1.17 to 2.1.44), Section 2.2, Section 2.3 (2.3.12 to 2.3.16)

Continuous mappings, sequential criterion and other characterizations of continuity, Uniform continuity, Homeomorphism, Contraction mappings, Banach Fixed point Theorem.
[1] Chapter3, Section 3.1, Section3.4 (3.4.1 to 3.4.8), Section 3.5 (3.5.1 to 3.5.7(iv) ), Section 3.7 ( 3.7.1 to 3.7.5)

Connectedness, connected subsets of $\mathbf{R}$, connectedness and continuous mappings. [1] Chapter4, Section 4.1 (4.1.1 to 4.1.12)

Compactness, compactness and boundedness, continuous functions on compact spaces.
[1] Chapter5, Section 5.1 (5.1.1 to 5.1.6), Section 5.3 (5.3.1 to 5.3.11)

## REFERENCES:

[1] Satish Shirali \& Harikishan L. Vasudeva, Metric Spaces, Springer Verlag London (2006) (First Indian Reprint 2009)

## SUGGESTED READINGS:

[1] S. Kumaresan, Topology of Metric Spaces, Narosa Publishing House, Second Edition 2011.
[2] G. F. Simmons, Introduction to Topology and Modern Analysis, Mcgraw-Hill, Edition 2004.

## C 12 Group Theory-II

Total Marks: 100
Theory: 75
Internal Assessment: 25
5 Lectures, 1 Tutorial (per week per student)

Automorphism, inner automorphism, automorphism groups, automorphism groups of finite and infinite cyclic groups, applications of factor groups to automorphism groups, Characteristic subgroups, Commutator subgroup and its properties.
[1]: Chapter 6, Chapter 9 (Theorem 9.4), Exersices1-4 on page168, Exercises 52, 58 on page Pg 188.

Properties of external direct products, the group of units modulo n as an external direct product, internal direct products, Fundamental Theorem of finite abelian groups. [1]: Chapter 8, Chapter 9 (Section on internal direct products), Chapter 11.
Group actions, stabilizers and kernels, permutation representation associated with a given group action, Applications of group actions: Generalized Cayley's theorem, Index theorem. Groups acting on themselves by conjugation, class equation and consequences, conjugacy in $S_{n}, p$-groups, Sylow's theorems and consequences, Cauchy's theorem, Simplicity of $A_{n}$ for $n \geq 5$, non-simplicity tests.
[2]: Chapter 1 (Section 1.7), Chapter 2 (Section 2.2), Chapter 4 (Section 4.1-4.3, 4.54.6).
[1]: Chapter 25.

## REFERENCES:

1. Joseph A. Gallian, Contemporary Abstract Algebra (4th Ed.), Narosa Publishing House, 1999.
2. David S. Dummit and Richard M. Foote, Abstract Algebra (3rd Edition), John Wiley and Sons (Asia) Pvt. Ltd, Singapore, 2004

## C13 Complex Analysis (including practicals)

Total marks: 150
Theory: 75
Internal Assessment: 25
Practical: 50
5 Lectures, Practical 4 (in group of 15-20)

Limits, Limits involving the point at infinity, continuity.
Properties of complex numbers, regions in the complex plane, functions of complex variable, mappings. Derivatives, differentiation formulas, Cauchy-Riemann equations, sufficient conditions for differentiability.
[1]: Chapter 1 (Section 11), Chapter 2 (Section 12, 13) Chapter 2 (Sections 15, 16, 17, $18,19,20,21,22)$

Analytic functions, examples of analytic functions, exponential function, Logarithmic function, trigonometric function, derivatives of functions, definite integrals of functions. [1]: Chapter 2 (Sections 24, 25), Chapter 3 (Sections 29, 30, 34),Chapter 4 (Section 37, 38)

Contours, Contour integrals and its examples, upper bounds for moduli of contour integrals.
[1]: Chapter 4 (Section 39, 40, 41, 43)

Antiderivatives, proof of antiderivative theorem, Cauchy-Goursat theorem, Cauchy integral formula. An extension of Cauchy integral formula, consequences of Cauchy integral formula, Liouville's theorem and the fundamental theorem of algebra.
[1]: Chapter 4 (Sections 44, 45, 46, 50) , Chapter 4 (Sections 51, 52, 53)

Convergence of sequences and series, Taylor series and its examples. Laurent series and its examples, absolute and uniform convergence of power series, uniqueness of series representations of power series.
[1]: Chapter 5 (Sections 55, 56, 57, 58, 59, 60, 62, 63, 66)
Isolated singular points, residues, Cauchy's residue theorem, residue at infinity. Types of isolated singular points, residues at poles and its examples, definite integrals involving sines and cosines.
[1]: Chapter 6 (Sections 68, 69, 70, 71, 72, 73, 74), Chapter 7 (Section 85).

## REFERENCES:

1. James Ward Brown and Ruel V. Churchill, Complex Variables and Applications (Eighth Edition), McGraw - Hill International Edition, 2009.

## SUGGESTED READING:

2. Joseph Bak and Donald J. Newman, Complex analysis (2nd Edition), Undergraduate Texts in Mathematics, Springer-Verlag New York, Inc., New York, 1997.

## LAB WORK TO BE PERFORMED ON A COMPUTER <br> (MODELING OF THE FOLLOWING PROBLEMS USING MATLAB/ MATHEMATICAI MAPLE ETC.)

1. Declaring a complex number and graphical representation.
e.g. $Z_{1}=3+4 i, Z_{2}=4-7 i$
2. Program to discuss the algebra of complex numbers.
e.g., if $Z_{1}=3+4 i, Z_{2}=4-7 i$, then find $Z_{1}+Z_{2}, Z_{1}-Z_{2}, Z_{1} * Z_{2}$, and $Z_{1} / Z_{2}$
3. To find conjugate, modulus and phase angle of an array of complex numbers.
e.g., $Z=\left[\begin{array}{llll}2+3 i & 4-2 i & 6+11 i & 2-5 i\end{array}\right]$
4. To compute the integral over a straight line path between the two specified end points.
e. g., $\quad$, where $C$ is the straight line path from $-1+i$ to $2-i$.
5. To perform contour integration.
e.g., (i) , where $C$ is the Contour given by $x=y^{2}+1$;
(ii) , where C is the contour given by , which can be
parameterized by $x=\cos (t), y=\sin (t)$ for
6. To plot the complex functions and analyze the graph .
e.g., (i) $f(z)=Z$
(ii) $f(z)=Z^{3}$
7. $f(z)=\left(Z^{4}-1\right)^{1 / 4}$
8. 

etc.
7. To perform the Taylor series expansion of a given function $f(z)$ around a given point $z$.

The number of terms that should be used in the Taylor series expansion is given for each function. Hence plot the magnitude of the function and magnitude of its Taylors series expansion.
e.g., (i) $f(z)=\exp (z)$ around $z=0, n=40$.
(ii) $f(z)=\exp \left(z^{2}\right)$ around $z=0, n=160$.
8. To determines how many terms should be used in the Taylor series expansion of a given function $f(z)$ around $z=0$ for a specific value of $z$ to get a percentage error of less than 5 \%.
e.g., For $f(z)=\exp (z)$ around $z=0$, execute and determine the number of necessary terms to get a percentage error of less than $5 \%$ for the following values of $z$ :
(i) $z=30+30 i$
(ii)
9. To perform Laurents series expansion of a given function $f(z)$ around a given point $z$.
e.g., (i) $f(z)=(\sin z-1) / z^{4}$ around $z=0$
(ii) $f(z)=\cot (z) / z^{4}$ around $z=0$.
10. To compute the poles and corresponding residues of complex functions.
e.g.,
11. To perform Conformal Mapping and Bilinear Transformations.

## C 14 Ring Theory and Linear Algebra - II

Total Marks : 100
Theory: 75
Internal Assessment: 25
5 Lectures, 1 Tutorial (per week per student)

Polynomial rings over commutative rings, division algorithm and consequences, principal ideal domains, factorization of polynomials, reducibility tests, irreducibility tests, Eisenstein criterion, unique factorization in $\mathrm{Z}[x]$.
Divisibility in integral domains, irreducibles, primes, unique factorization domains, Euclidean domains.
[1]: Chapter 16, Chapter 17, Chapter 18.
Dual spaces, dual basis, double dual, transpose of a linear transformation and its matrix in the dual basis, annihilators, Eigenspaces of a linear operator, diagonalizability, invariant subspaces and Cayley-Hamilton theorem, the minimal polynomial for a linear operator.
[2]: Chapter 2 (Section 2.6), Chapter 5 (Sections 5.1-5.2, 5.4), Chapter 7(Section 7.3).
Inner product spaces and norms, Gram-Schmidt orthogonalization process, orthogonal complements, Bessel's inequality, the adjoint of a linear operator, Least Squares Approximation, minimal solutions to systems of linear equations, Normal and self-adjoint operators, Orthogonal projections and Spectral theorem.
[2]: Chapter 6 (Sections 6.1-6.4, 6.6).

## REFERENCES:

1. Joseph A. Gallian, Contemporary Abstract Algebra (4th Ed.), Narosa Publishing House, 1999.
2. Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence, Linear Algebra (4th Edition), Prentice-Hall of India Pvt. Ltd., New Delhi, 2004.

## SUGGESTED READING:

## (Linear Algebra)

1. S Lang, Introduction to Linear Algebra ( $2^{\text {nd }}$ edition), Springer, 2005
2. Gilbert Strang, Linear Algebra and its Applications, Thomson, 2007
3. S. Kumaresan, Linear Algebra- A Geometric Approach, Prentice Hall of India, 1999. 4. Kenneth Hoffman, Ray Alden Kunze, Linear Algebra $2^{\text {nd }}$ Ed., Prentice-Hall Of India Pvt. Limited, 1971
(Ring theory and group theory)
4. John B.Fraleigh, A first course in Abstract Algebra, $7^{\text {th }}$ Edition, Pearson Education India, 2003.
5. Herstein, Topics in Algebra (2 ${ }^{\text {nd }}$ edition), John Wiley \& Sons, 2006
6. $M$ ichael Artin, Algebra ( $2^{\text {nd }}$ edition), Pearson Prentice Hall, 2011
7. Robinson, Derek John Scott., An introduction to abstract algebra, Hindustan book agency, 2010.

# DSE-1 (including practicals): Any one of the following (at least two shall be offered by the college): 

## DSE-1(i) Numerical Methods

Total marks: 150
Theory: 75
Practical: 50
Internal Assessment: 25
5 Lectures, 4 Practicals (each in group of 15-20)

Algorithms, Convergence, Bisection method, False position method, Fixed point iteration method, Newton's method, Secant method, LU decomposition, GaussJacobi, Gauss-Siedel and SOR iterative methods.
[1]: Chapter 1 (Sections 1.1-1.2), Chapter 2 (Sections 2.1-2.5), Chapter 3 (Section 3.5, 3.8).

Lagrange and Newton interpolation: linear and higher order, finite difference operators.
[1]: Chapter 5 (Sections 5.1, 5.3)
[2]: Chapter 4 (Section 4.3).
Numerical differentiation: forward difference, backward difference and central difference. Integration: trapezoidal rule, Simpson's rule, Euler's method.
[1]: Chapter 6 (Sections 6.2, 6.4), Chapter 7 (Section 7.2)
Note: Emphasis is to be laid on the algorithms of the above numerical methods.

## Practical / Lab work to be performed on a computer:

Use of computer aided software (CAS), for example Matlab / Mathematica / Maple / Maxima etc., for developing the following Numerical programs:
(i) Calculate the sum $1 / 1+1 / 2+1 / 3+1 / 4+$ $\qquad$ $+1 / \mathrm{N}$.
(ii) To find the absolute value of an integer.
(iii) Enter 100 integers into an array and sort them in an ascending order.
(iv) Any two of the following
(a) Bisection Method
(b) Newton Raphson Method
(c) Secant Method
(d) Regulai Falsi Method
(v) LU decomposition Method
(vi) Gauss-Jacobi Method
(vii) SOR Method or Gauss-Siedel Method
(viii) Lagrange Interpolation or Newton Interpolation
(ix) Simpson's rule.

Note: For any of the CAS Matlab / Mathematica / Maple / Maxima etc., Data types-simple data types, floating data types, character data types, arithmetic operators and operator precedence, variables and constant declarations, expressions, input/output, relational operators, logical operators and logical expressions, control statements and loop statements, Arrays should be introduced to the students.

## References:

1. B. Bradie, A Friendly Introduction to Numerical Analysis, Pearson Education, India, 2007.
2. M. K. Jain, S. R. K. Iyengar and R. K. Jain, Numerical Methods for Scientific and Engineering Computation, New age International Publisher, India, $5^{\text {th }}$ edition, 2007.

## Suggested Reading:

1. C. F. Gerald and P. O. Wheatley, App;ied Numerical Analysis, Pearson Education, India, $7^{\text {th }}$ edition, 2008

## DSE-1(ii) Mathematical Modeling \& Graph Theory

Total marks: 150
Theory: 75
Practical: 50
Internal Assessment: 25
5 Lectures, 4 Practicals (each in group of 15-20)
Power series solution of a differential equation about an ordinary point, solution about a regular singular point, Bessel's equation and Legendre's equation, Laplace transform and inverse transform, application to initial value problem up to second order.
[2]: Chapter 7 (Sections 7.1-7.3), Chapter 8 (Sections 8.2-8.3).
Monte Carlo Simulation Modeling: simulating deterministic behavior (area under a curve, volume under a surface), Generating Random Numbers: middle square method, linear congruence, Queuing Models: harbor system, morning rush hour, Overview of optimization modeling, Linear Programming Model: geometric solution algebraic solution, simplex method, sensitivity analysis
[3]: Chapter 5 (Sections 5.1-5.2, 5.5), Chapter 7.
Graphs, diagraphs, networks and subgraphs, vertex degree, paths and cycles, regular and bipartite graphs, four cube problem, social networks, exploring and traveling, Eulerian and Hamiltonian graphs, applications to dominoes, diagram tracing puzzles, Knight’s tour problem, gray codes.
[1]: Chapter 1 (Section 1.1), Chapter 2, Chapter 3.
Note: Chapter 1 (Section 1.1), Chapter 2 (Sections 2.1-2.4), Chapter 3 (Sections 3.1-3.3) are to be reviewed only. This is in order to understand the models on Graph Theory.

## Practical / Lab work to be performed on a computer:

Modeling of the following problems using Matlab / Mathematica / Maple etc.
(i) Plotting of Legendre polynomial for $n=1$ to 5 in the interval [0,1]. Verifying graphically that all the roots of $P_{n}(x)$ lie in the interval $[0,1]$.
(ii) Automatic computation of coefficients in the series solution near ordinary points
(iii) Plotting of the Bessel's function of first kind of order 0 to 3 .
(iv) Automating the Frobenius Series Method
(v) Random number generation and then use it for one of the following
(a) Simulate area under a curve
(b) Simulate volume under a surface
(vi) Programming of either one of the queuing model
(a) Single server queue (e.g. Harbor system)
(b) Multiple server queue (e.g. Rush hour)
(vii) Programming of the Simplex method for $2 / 3$ variables

## References:

1. Joan M. Aldous and Robin J. Wilson, Graphs and Applications: An Introductory Approach, Springer, Indian reprint, 2007.
2. Tyn Myint-U and Lokenath Debnath, Linear Partial Differential Equation for Scientists and Engineers, Springer, Indian reprint, 2006.
3. Frank R. Giordano, Maurice D. Weir and William P. Fox, A First Course in Mathematical Modeling, Thomson Learning, London and New York, 2003.

# DSE-1(iii) C++ PROGRAMMING 

Total marks: 150
Theory: 75
Practical: 50
Internal Assessment: 25
5 Lectures, 4 Practicals (each in group of 15-20)

Introduction to structured programming: data types- simple data types, floating data types, character data types, string data types, arithmetic operators and operators precedence, variables and constant declarations, expressions, input using the extraction operator >> and cin, output using the insertion operator $\ll$ and cout, preprocessor directives, increment(++) and decrement(--) operations, creating a C++ program, input/ output, relational operators, logical operators and logical expressions, if and if-else statement, switch and break statements.
[1]Chapter 2(pages 37-95), Chapter3(pages $96-129$ ), Chapter 4(pages 134-178)
"for", "while" and "do-while" loops and continue statement, nested control statement, value returning functions, value versus reference parameters, local and global variables, one dimensional array, two dimensional array, pointer data and pointer variables,.
[1] Chapter 5 (pages 181-236), Chapter 6, Chapter 7(pages 287-304()Chapter 9 (pages 357-390), Chapter 14 (pages 594-600).

## Reference:

[1]D. S. Malik: C++ Programming Language, Edition-2009, Course Technology, Cengage Learning, India Edition

## Suggested Readings:

[2]E. Balaguruswami: Object oriented programming with C++, fifth edition, Tata McGraw Hill Education Pvt. Ltd.
[3]Marshall Cline, Greg Lomow, Mike Girou: C++ FAQs, Second Edition, Pearson
Education.
Note: Practical programs of the following (and similar) typeare suggestive.

1. Calculate the Sum of the series $1 / 1+1 / 2+1 / 3 \ldots \ldots \ldots \ldots \ldots . . .+1 / \mathrm{N}$ for any positive integer N .
2. Write a user defined function to find the absolute value of an integer and use it to evaluate the function $(-1)^{n} /|n|$, for $n=-2,-1,0,1,2$.

Calculate the factorial of any natural number.
4. Read floating numbers and compute two averages: the average of negative numbers and the average of positive numbers.
5. Write a program that prompts the user to input a positive integer. It should then output a message indicating whether the number is a prime number.

Write a program that prompts the user to input the value of $a, b$ and $c$ involved in the equation $a x^{\wedge} 2+b x+c=0$ and outputs the type of the roots of the equation. Also the program should outputs all the roots of the equation.
write a program that generates random integer between 0 and 99 . Given that first two Fibonacci numbers are 0 and 1, generate all Fibonacci numbers less than or equal to generated number.
8. Write a program that does the following:
a. Prompts the user to input five decimal numbers.
b. Prints the five decimal numbers.
c. Converts each decimal number to the nearest integer.
d. Adds these five integers.
e. Prints the sum and average of them.
9. Write a program that uses whileloops to perform the following steps:
a. Prompt the user to input two integers :firstNum and secondNum (firstNum should be less than secondNum).
b. Output all odd and even numbers between firstNum and secondNum.
c. Output the sum of all even numbers between firstNum and secondNum.
d. Output the sum of the square of the odd numbers firstNum and secondNum.
e. Output all uppercase letters corresponding to the numbers between firstNum and secondNum, if any.
10. Write a program that prompts the user to input five decimal numbers. The program should then add the five decimal numbers, convert the sum to the nearest integer, and print the result.
11. Write a program that prompts the user to enter the lengths of three sides of a triangle and then outputs a message indicating whether the triangle is a right triangle or a scalene triangle.

12. Write a value returning function smaller to determine the smallest number from a set of numbers. Use this function to determine the smallest number from a set of 10 numbers.
13. Write a function that takes as a parameter an integer (as a long value) and returns the number of odd, even, and zero digits. Also write a program to test your function.
14. Enter 100 integers into an array and short them in an ascending/ descending order and print the largest/ smallest integers.
15. Enter 10 integers into an array and then search for a particular integer in the array.
16. Multiplication/ Addition of two matrices using two dimensional arrays.
17. Using arrays, read the vectors of the following type: $A=(12345678), B=(0$ 2340156 ) and compute the product and addition of these vectors.
18. Read from a text file and write to a text file.
19. Write a program to create the following grid using for loops:
$\begin{array}{lllll}1 & 2 & 3 & 4 & 5\end{array}$
$\begin{array}{lllll}2 & 3 & 4 & 5 & 6\end{array}$
$\begin{array}{lllll}3 & 4 & 5 & 6 & 7\end{array}$
$\begin{array}{lllll}4 & 5 & 6 & 7 & 8\end{array}$
$\begin{array}{lllll}5 & 6 & 7 & 8 & 9\end{array}$
20. Write a function, reverseDigit, that takes an integer as a parameter and returns the number with its digits reversed. For example, the value of function reverseDigit(12345) is 54321 and the value of reverseDigit(-532) is -235 .


# DSE-2: Any one of the following ( at least two shall be offered by the college): 

## DSE-2(i) Mathematical Finance

Total Marks: 100
Theory: 75
Internal Assessment: 25
5 Lectures, 1 Tutorial (per week per student)

Basic principles: Comparison, arbitrage and risk aversion, Interest (simple and compound, discrete and continuous), time value of money, inflation, net present value, internal rate of return (calculation by bisection and Newton-Raphson methods), comparison of NPV and IRR. Bonds, bond prices and yields, Macaulay and modified duration, term structure of interest rates: spot and forward rates, explanations of term structure, running present value, floating-rate bonds, immunization, convexity, putable and callable bonds.
[1]: Chapter 1, Chapter 2, Chapter 3, Chapter 4.
Asset return, short selling, portfolio return, (brief introduction to expectation, variance, covariance and correlation), random returns, portfolio mean return and variance, diversification, portfolio diagram, feasible set, Markowitz model (review of Lagrange multipliers for 1 and 2 constraints), Two fund theorem, risk free assets, One fund theorem, capital market line, Sharpe index. Capital Asset Pricing Model (CAPM), betas of stocks and portfolios, security market line, use of CAPM in investment analysis and as a pricing formula, Jensen's index.
[1]: Chapter 6, Chapter 7, Chapter 8 (Sections 8.5--8.8).
[3]: Chapter 1 (for a quick review/description of expectation etc.)
Forwards and futures, marking to market, value of a forward/futures contract, replicating portfolios, futures on assets with known income or dividend yield, currency futures, hedging (short, long, cross, rolling), optimal hedge ratio, hedging with stock index futures, interest rate futures, swaps. Lognormal distribution, Lognormal model / Geometric Brownian Motion for stock prices, Binomial Tree model for stock prices, parameter estimation, comparison of the models. Options, Types of options: put / call, European / American, pay off of an option, factors affecting option prices, put call parity.
[1]: Chapter 10 (except 10.11, 10.12), Chapter 11 (except 11.2 and 11.8)
[2]: Chapter 3, Chapter 5, Chapter 6, Chapter 7 (except 7.10 and 7.11), Chapter 8, Chapter 9
[3]: Chapter 3

## References:

1. David G. Luenberger, Investment Science, Oxford University Press, Delhi, 1998.
2. John C. Hull, Options, Futures and Other Derivatives (6th Edition), PrenticeHall India, Indian reprint, 2006.
3. Sheldon Ross, An Elementary Introduction to Mathematical Finance (2nd Edition), Cambridge University Press, USA, 2003.

## DSE-2(ii) Discrete Mathematics

Total Marks: 100
Theory: 75
Internal Assessment: 25
5 Lectures, 1 Tutorial (per week per student)

Definition, examples and basic properties of ordered sets, maps between ordered sets, duality principle, lattices as ordered sets, lattices as algebraic structures, sublattices, products and homomorphisms.
[1]: Chapter 1 (till the end of 1.18), Chapter 2 (Sections 2.1-2.13), Chapter 5 (Sections 5.1-5.11).
[3]: Chapter 1 (Section 1).
Definition, examples and properties of modular and distributive lattices, Boolean algebras, Boolean polynomials, minimal forms of Boolean polynomials, Quinn-McCluskey method, Karnaugh diagrams, switching circuits and applications of switching circuits.
[1]: Chapter 6.
[3]: Chapter 1 (Sections 3-4, 6), Chapter 2 (Sections 7-8).
Definition, examples and basic properties of graphs, pseudographs, complete graphs, bipartite graphs, isomorphism of graphs, paths and circuits, Eulerian circuits, Hamiltonian cycles, the adjacency matrix, weighted graph, travelling salesman's problem, shortest path, Dijkstra's algorithm, Floyd-Warshall algorithm.
[2]: Chapter 9, Chapter 10.

## REFERENCES:

1. B A. Davey and H. A. Priestley, Introduction to Lattices and Order, Cambridge University Press, Cambridge, 1990.
2. Edgar G. Goodaire and Michael M. Parmenter, Discrete Mathematics with Graph Theory (2nd Edition), Pearson Education (Singapore) Pte. Ltd., Indian Reprint 2003.
3. Rudolf Lidl and Günter Pilz, Applied Abstract Algebra (2nd Edition), Undergraduate Texts in Mathematics, Springer (SIE), Indian reprint, 2004.

## DSE-2(iii) CRYPTOGRAPHY AND NETWORK SECURITY_

Total Marks: 100
Theory: 75
Internal Assessment: 25
5 Lectures, 1 Tutorial (per week per student)

Definition of a cryptosystem, Symmetric cipher model, Classical encryption techniquesSubstitution and transposition ciphers, caesar cipher, Playfair cipher. Block cipher Principles, Shannon theory of diffusion and confusion, Data encryption standard (DES).
[1] 2.1-2.3, 3.1, 3.2, 3.3.
Polynomial and modular arithmetic, Introduction to finite field of the form $\operatorname{GF}(\mathrm{p})$ and $\operatorname{GF}\left(2^{n}\right)$, Fermat theorem and Euler's theorem(statement only), Chinese Remainder theorem, Discrete logarithm.
[1] 4.2, 4.3, 4.5, 4.6, 4.7, 8.2, 8.4, 8.5
Advanced Encryption Standard(AES), Stream ciphers . Introduction to public key cryptography, RSA algorithm and security of RSA, Introduction to elliptic curve cryptography.
[1] 5.2-5.5(tables 5.5, 5.6 excluded), $7.4,9.1,9.2,10.3,10.4$
Information/Computer Security: Basic security objectives, security attacks, security services, Network security model,
[1]1.1, 1.3, 1.4, 1.6
Cryptographic Hash functions, Secure Hash algorithm, SHA-3.
[1] 11.1, 11.5, 11.6
Digital signature, Elgamal signature, Digital signature standards, Digital signature algorithm
[1] 13.1, 13.2, 13.4
E-mail security: Pretty Good Privacy (PGP)
[1] 18.1 Page 592-596(Confidentiality excluded)

## REFERENCE:

[1] William Stallings, "Cryptography and Network Security", Principles and Practise, Fifth Edition, Pearson Education, 2012.
SUGGESTED READING:
[1] Douglas R. Stinson, "Cryptography theory and practice", CRC Press, Third edition, 2005.

# DSE-3: Any one of the following ( at least two shall be offered by the college): 

DSE-3(i) Probability Theory and Statistics
Total marks: 100
Theory: 75
Internal Assessment: 25
5 Lectures, 1 Tutorial (per week per student)

Sample space, probability axioms, real random variables (discrete and continuous), cumulative distribution function, probability mass/density functions, mathematical expectation, moments, moment generating function, characteristic function, discrete distributions: uniform, binomial, Poisson, geometric, negative binomial, continuous distributions: uniform, normal, exponential.
[1]: Chapter 1 (Sections 1.1, 1.3, 1.5-1.9).
[2]: Chapter 5 (Sections 5.1-5.5, 5.7), Chapter 6 (Sections 6.2-6.3, 6.5-6.6).
Joint cumulative distribution function and its properties, joint probability density functions, marginal and conditional distributions, expectation of function of two random variables, conditional expectations, independent random variables, bivariate normal distribution, correlation coefficient, joint moment generating
function (jmgf) and calculation of covariance (from jmgf), linear regression for two variables.
[1]: Chapter 2 (Sections 2.1, 2.3-2.5).
[2]: Chapter 4 (Exercise 4.47), Chapter 6 (Section 6.7), Chapter 14 (Sections 14.1, 14.2).

Chebyshev's inequality, statement and interpretation of (weak) law of large numbers and strong law of large numbers, Central Limit theorem for independent and identically distributed random variables with finite variance, Markov Chains, Chapman-Kolmogorov equations, classification of states.
[2]: Chapter 4 (Section 4.4).
[3]: Chapter 2 (Section 2.7), Chapter 4 (Sections 4.1-4.3).

## References:

1. Robert V. Hogg, Joseph W. McKean and Allen T. Craig, Introduction to Mathematical Statistics, Pearson Education, Asia, 2007.
2. Irwin Miller and Marylees Miller, John E. Freund's Mathematical Statistics with Applications (7th Edition), Pearson Education, Asia, 2006.
3. Sheldon Ross, Introduction to Probability Models (9th Edition), Academic Press, Indian Reprint, 2007.

## Suggested Reading:

1. Alexander M. Mood, Franklin A. Graybill and Duane C. Boes, Introduction to the Theory of Statistics, ( $3_{\text {rd }}$ Edition), Tata McGraw- Hill, Reprint 2007

## DSE-3(ii) Mechanics

Total Marks: 100
Theory: 75
Internal Assessment: 25
5 Lectures, 1 Tutorial (per week per student)

Moment of a force about a point and an axis, couple and couple moment, Moment of a couple about a line, resultant of a force system, distributed force system, free body diagram, free body involving interior sections, general equations of equilibrium, two point equivalent loading, problems arising from structures, static indeterminacy.
[1]: Chapter 3, Chapter 4, Chapter 5.
Laws of Coulomb friction, application to simple and complex surface contact friction problems, transmission of power through belts, screw jack, wedge, first moment of an area and the centroid, other centers, Theorem of Pappus-Guldinus, second moments and the product of area of a plane area, transfer theorems, relation between second moments and products of area, polar moment of area, principal axes.
[1]: Chapter 6 (Sections 6.1-6.7), Chapter 7

Conservative force field, conservation for mechanical energy, work energy equation, kinetic energy and work kinetic energy expression based on center of mass, moment of momentum equation for a single particle and a system of particles, translation and rotation of rigid bodies, Chasles' theorem, general relationship between time derivatives of a vector for different references, relationship between velocities of a particle for different references, acceleration of particle for different references.
[1]: Chapter 11, Chapter 12 (Sections 12.5-12.6), Chapter 13.

## References:

1. I.H. Shames and G. Krishna Mohan Rao, Engineering Mechanics: Statics and Dynamics (4 ${ }^{\text {th }}$ Edition), Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi, 2009.
2. R.C. Hibbeler and Ashok Gupta, Engineering Mechanics: Statics and Dynamics (11 ${ }^{\text {th }}$ Edition), Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi.

DSE-3(iii) Bio-Mathematics
Total Marks: 100
Theory: 75
Internal Assessment: 25
5 Lectures, 1 Tutorial (per week per student)


Population growth, Administration of drugs, Cell division. Modelling Biological Phenomena: Heart beat, Blood Flow, Nerve Impulse transmission, Chemical Reactions, Predator-prey models. Stability and oscillations: Epidemics, the phase plane, Local Stability, Stability, Limit Cycles, Forced oscillations, Computing trajectories. Mathematics of Heart Physiology: The local model, The Threshold effect, The phase plane analysis and the heart beat model, Physiological considerations of the heart beat model, A model of the cardiac pace-maker. Mathematics of Nerve Impulse transmission: Excitability and repetitive firing, travelling waves. Bifurcation and chaos: Bifurcation, Bifurcation of a limit cycle, Discrete bifurcation, Chaos, Stability, The Poincare plane, Computer programs for Iteration Schemes.

References: Relevant sections of chapters 1, 3, 4, 5, 6, 7 and 13 of [4]

Mathematics of imaging of the Brain: Modelling of computerized tomography (CT, Magnetic resonance Imaging (MRI), Positron emission Tomography (PET), Single Photon Emission Computerized Tomography(SPECT), Discrete analogues and Numerical Implementation. Networks in Biological Sciences: Dynamics of Small world networks, scale-free networks, complex networks, cellular automata.

References: Relevant parts of [2] and [3]
Modelling Molecular Evolution: Matrix models of base substitutions for DNA sequences, The Jukes-Cantor Model, the Kimura Models, Phylogenetic distances. Constructing Phylogenetic trees: Unweighted pair-group method with arithmetic means (UPGMA), Neighbour- Joining Method, Maximum Likelihood approaches. Genetics: Mendelian Genetics, Probability distributions in Genetics, Linked genes and Genetic Mapping, Statistical Methods and Prediction techniques.

References: Relevant sections of Chapters 4, 5 and 6 of [1] and chapters 3, 4, 6 and 8 of [5].

## Recommended Books:

1. Elizabeth S. Allman and John a. Rhodes, Mathematical Models in Biology, Cambridge University Press, 2004.
2. C. Epstein, The Mathematics of Medical Imaging, Prentice Hall, 2003 ( copyright Pearson Education, 2005).
3. S. Helgason, The Radon transform, Second Edition, Birkhauser, 1997.
4. D. S. Jones and B. D. Sleeman, Differential Equations and Mathematical Biology, Cahapman \& Hall, CRC Press, London, UK, 2003.
5. James Keener and James Sneyd, Mathematical Physiology, Springer Verlag, 1998, Corrected $2^{\text {nd }}$ printing, 2001.

DSE-4: Any one of the following ( at least two shall be offered by the college):
DSE-4(i) Number Theory
Total Marks: 100
Theory: 75
Internal Assessment: 25
5 Lectures, 1 Tutorial (per week per student)

Linear Diophantine equation, prime counting function, statement of prime number theorem, Goldbach conjecture, linear congruences, complete set of residues, Chinese remainder theorem, Fermat's little theorem, Wilson's theorem.

References:

[1]: Chapter 2 (Section 2.5), Chapters 3 (Section 3.3), Chapter 4 (Sections 4.2 and 4.4), Chapter 5 (Section 5.2 excluding pseudoprimes, Section 5.3). [2]: Chapter 3 (Section 3.2).

Number theoretic functions, sum and number of divisors, totally multiplicative functions, definition and properties of the Dirichlet product, the Möbius inversion formula, the greatest integer function, Euler's phi-function, Euler's theorem, reduced set of residues, some properties of Euler's phi-function.

References:
[1]: Chapter 6 (Sections 6.1-6.3), Chapter 7.
[2]: Chapter 5 (Section 5.2 (Definition 5.5-Theorem 5.40), Section 5.3 (Theorem 5.15-Theorem 5.17, Theorem 5.19)).

Order of an integer modulo $n$, primitive roots for primes, composite numbers having primitive roots, Euler's criterion, the Legendre symbol and its properties, quadratic reciprocity, quadratic congruences with composite moduli. Public key encryption, RSA encryption and decryption, the equation $x^{2}+y^{2}=z^{2}$, Fermat's Last Theorem.

Reference:
[1]: Chapters 8 (Sections 8.1-8.3), Chapter 9, Chapter 10 (Section 10.1), Chapter 12.

## References:

1. David M. Burton, Elementary Number Theory (6th Edition), Tata McGraw-Hill Edition, Indian reprint, 2007.
2. Neville Robinns, Beginning Number Theory (2nd Edition), Narosa Publishing House Pvt. Limited, Delhi, 2007.

## DSE-4 (ii) Linear Programming and Theory of Games

Total Marks: 100
Theory: 75
Internal Assessment: 25
5 Lectures, 1 Tutorial (per week per student)

Introduction to linear programming problem, Theory of simplex method, optimality and unboundedness, the simplex algorithm, simplex method in tableau format, introduction to artificial variables, two-phase method, Big-M method and their comparison.
[1]: Chapter 3 (Sections 3.2-3.3, 3.5-3.8), Chapter 4 (Sections 4.1-4.4).

Duality, formulation of the dual problem, primal-dual relationships, economic interpretation of the dual.
[1]: Chapter 6 (Sections 6.1-6.3).
Transportation problem and its mathematical formulation, northwest-corner method least cost method and Vogel approximation method for determination of starting basic solution, algorithm for solving transportation problem, assignment problem and its mathematical formulation, Hungarian method for solving assignment problem.
[3]: Chapter 5 (Sections 5.1, 5.3-5.4).
Game theory: formulation of two person zero sum games, solving two person zero sum games, games with mixed strategies, graphical solution procedure, linear programming solution of games.
[2]: Chapter 14.

## References:

1. Mokhtar S. Bazaraa, John J. Jarvis and Hanif D. Sherali, Linear Programming and Network Flows (2nd edition), John Wiley and Sons, India, 2004.
2. F. S. Hillier and G. J. Lieberman, Introduction to Operations ResearchConcepts and Cases (9th Edition), Tata McGraw Hill, 2010.
3. Hamdy A. Taha, Operations Research, An Introduction (9th edition), Prentice-Hall, 2010.

Suggested Reading:

1. G. Hadley, Linear Programming, Narosa Publishing House, New Delhi, 2002.

## DSE-4(iii) Applications of Algebra

Total Marks: 100
Theory: 75
Internal Assessment: 25
5 Lectures, 1 Tutorial (per week per student)

Balanced incomplete block designs (BIBD): definitions and results, incidence matrix of a BIBD, construction of BIBD from difference sets, construction of BIBD using quadratic residues, difference set families, construction of BIBD from finite fields.
[2]: Chapter 2 (Sections 2.1-2.4,2.6).


Coding Theory: introduction to error correcting codes, linear cods, generator and parity check matrices, minimum distance, Hamming Codes, decoding and cyclic codes.
[2]: Chapter 4 (Sections 4.1-4.3.17).
Symmetry groups and color patterns: review of permutation groups, groups of symmetry and action of a group on a set; colouring and colouring patterns, Polya theorem and pattern inventory, generating functions for non-isomorphic graphs.
[2]: Chapter 5.
Application of linear transformations: Fibonacci numbers, incidence models, and differential equations. Lease squares methods: Approximate solutions of system of linear equations, approximate inverse of an $m^{\mathrm{x}} \mathrm{n}$ matrix, solving a matrix equation using its normal equation, finding functions that approximate data. Linear algorithms: LDU factorization, the row reduction algorithm and its inverse, backward and forward substitution, approximate substitution, approximate inverse and projection algorithms.
[1]: Chapter 9-11.

## Reference:

2. I.N. Herstein and D.J. Winter, Primer on Linear Algebra, Macmillan Publishing Company, New York, 1990.
3. S.R. Nagpaul and S.K. Jain, Topics in Applied Abstract Algebra, Thomson Brooks and Cole, Belmont, 2005.

# UNIVERSITY OF DELHI <br> DEPARTMENT OF MATHEMATICS <br> GENERIC ELECTIVE (GE) Courses for Honours Courses (For students other than B.Sc. (Hons.) Mathematics) 

Learning Outcomes based Curriculum Framework (LOCF) 2019


## GENERIC ELECTIVE (GE) COURSES OFFERED TO

## B.Sc. (Hons.) / B.A. (Hons.) / B.Com (Hons.)

(Other than B.Sc. (Hons.) Mathematics)

| Semester | Core <br> Cour <br> se <br> (14) | Ability Enhancement Compulsory Course (AECC)(2) | Skill <br> Enhancement Course (SEC) (2) | Discipline Specific Elective (DSE)(4) | Generic Elective (GE) (4) Credits: 6 each |
| :---: | :---: | :---: | :---: | :---: | :---: |
| I |  |  |  |  | GE-1 <br> Calculus <br> OR <br> Analytic Geometry and Theory of Equations |
| II |  |  |  |  | GE-2 <br> Linear Algebra <br> OR <br> Discrete Mathematics |
| III |  |  |  |  | GE-3 <br> Differential Equations (with Practicals) OR <br> Linear Programming and Game Theory |
| IV |  |  |  |  | GE-4 Numerical Methods (with Practicals) OR Elements of Analysis |
| V |  |  |  |  |  |
| VI |  |  |  |  |  |

# Semester-I <br> Generic Elective (GE) Course -Mathematics 

Any one of the following:
GE-1: Calculus
GE-1: Analytic Geometry and Theory of Equations

## GE-1: Calculus

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) Credits: $6(5+1)$
Duration: 14 Weeks ( 70 Hrs.) Examination: 3 Hrs.
Course Objectives: The main aim of this course is to learn about applications of derivatives for sketching of curves and conics and applications of definite integrals for calculating volumes of solids of revolution, length of plane curves and surface areas of revolution. Various notions related to vector-valued functions and functions of several variables are discussed in this course.

Course Learning Outcomes: This course will enable the students to:
i) Sketch the curves in Cartesian and polar coordinates as well as learn techniques of sketching the conics.
ii) Visualize three dimensional figures and calculate their volumes and surface areas.
iii) Understand limits, continuity and derivatives of functions of several variable and vector-valued functions.

## Unit 1: Applications of Derivatives and Limits

The first derivative test, Concavity and inflection points, Second derivative test, Curve sketching using first and second derivative test; Limits at infinity, Horizontal asymptotes, Vertical asymptotes, Graphs with asymptotes; L'Hôpital's rule.

## Unit 2: Applications of Definite Integrals

Volumes by slicing, Volumes of solids of revolution by the disk method, Volumes of solids of revolution by the washer method, Volume by cylindrical shells, Length of plane curves, Arc length of parametric curve, Area of surface of revolution.

## Unit 3: Conics, Vector-Valued Functions and Partial Derivatives

Techniques of sketching conics, Reflection properties of conics; Polar coordinates, graphing in polar coordinates; Vector-valued functions: Limits, Continuity, Derivatives, Integrals, Arc length, Unit tangent vector, Curvature, Unit normal vector; Functions of several variables: Graphs and level curves, Limits and continuity, Partial derivatives and differentiability, The chain rule, Directional derivatives and gradient vectors, Tangent plane and normal line, Extreme values and saddle points.

## References:

1. Anton, Howard, Bivens, Irl, \& Davis, Stephen (2013). Calculus (10th ed.). John Wiley \& Sons Singapore Pvt. Ltd. Reprint (2016) by Wiley India Pvt. Ltd. Delhi.
2. Strauss, M. J., Bradley, G. L., \& Smith, K. J. (2007). Calculus (3rd ed.). Dorling Kindersley (India) Pvt. Ltd. (Pearson Education). Delhi. Sixth impression 2011.


## Additional Reading:

i. Thomas, Jr. George B., Weir, Maurice D., \& Hass, Joel (2014). Thomas' Calculus (13th ed.). Pearson Education, Delhi. Indian Reprint 2017.

## Teaching Plan (GE-1: Calculus):

Weeks 1 and 2: The first derivative test, Concavity and inflection points, Second derivative test, Curve sketching using first and second derivative test.
[2] Chapter 4 (Section 4.3).
Weeks 3 and 4: Limits at infinity, Horizontal asymptotes, Vertical asymptotes, Graphs with asymptotes; L'Hôpital's rule.
[2] Chapter 4 (Sections 4.4, and 4.5).
[1] Chapter 3 (Section 3.3), and Chapter 6 (Section 6.5).
Weeks 5 and 6: Volumes by slicing, Volumes of solids of revolution by the disk method, Volumes of solids of revolution by the washer method, Volume by cylindrical shells.
[1] Chapter 5 (Sections 5.2, and 5.3).
Week 7: Length of plane curves, Arc length of parametric curves, Area of surface of revolution.
[1] Chapter 5 (Sections 5.4, and 5.5).
Week 8: Techniques of sketching conics, Reflection properties of conics.
[1] Chapter 10 (Section 10.4).
Week 9: Polar coordinates, Graphing in polar coordinates.
[1] Chapter 10 (Section 10.2).
Week 10: Vector-valued functions: Limit, continuity, Derivatives, Integrals, Arc length, Unit tangent vector, Curvature, Unit normal vector.
[1] Chapter 12 (Sections 12.1 to 12.5).
Weeks 11 and 12: Functions of several variables: Graphs, Level curves, Limits and continuity, Partial derivatives and differentiability.
[1] Chapter 13 (Section 13.1 to 13.4).
Week 13: Functions of several variables: The chain rule, Directional derivatives and gradient vectors.
[1] Chapter 13 (Sections 13.5, and 13.6).
Week 14: Functions of several variables: Tangent plane and normal line, Extreme values and saddle points.
[1] Chapter 13 (Sections 13.7, and 13.8).
Facilitating the Achievement of Course Learning Outcomes

| Unit <br> No. | Course Learning Outcomes | Teaching and Learning <br> Activity | Assessment Tasks |
| :---: | :--- | :--- | :--- |
| 1. | Sketch the curves in Cartesian and <br> polar coordinates as well as learn <br> techniques of sketching the conics. | (i) Each topic to be explained <br> with examples. <br> (ii) Students to be involved in <br> discussions and encouraged <br> to ask questions. | • Student <br> presentations. <br> • Participation in <br> discussions. |
| 2. | Visualize three dimensional <br> figures and calculate their volumes <br> and surface areas. | Assignments and <br> (iii) Students to be given <br> class tests. |  |
| 3. | Understand limits, continuity and <br> derivatives of functions of several <br> variable and vector-valued <br> functions. | (iv) Students to be encouraged <br> to give short presentations. | Mid-term <br> examinations. <br> end-term <br> Enaminations. |

Keywords: Concavity, Asymptotes, Curve sketching, L'Hôpital's rule, Volumes of solids of revolution, Sketching of conics, Vector-valued functions, Functions of several variables.


## GE-1: Analytic Geometry and Theory of Equations

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) Credits: 6 (5+1)
Duration: 14 Weeks (70 Hrs.) Examination: 3 Hrs.
Course Objectives: The goal of this paper is to acquaint students with certain ideas about conic sections, vectors in coordinate system and general properties of roots of polynomial equations with some applications.

Course Learning Outcomes: After completion of this paper, the students will be able to:
i) Classify and sketch conics four different types of conic sections - the circle, the ellipse, the hyperbola and the parabola - in Cartesian and polar coordinates.
ii) Visualize three dimensional objects - spheres and cylinders - using vectors.
iii) Understand the properties of roots of polynomial equations.

## Unit 1: Conic Sections, Parametrized Curves, and Polar Coordinates

Conic sections and quadratic equations: Circle, Parabola, Ellipse, and hyperbola; Techniques for sketching: Parabola, Ellipse, and Hyperbola; Reflection properties of parabola, ellipse, and hyperbola, Classifying conic sections by eccentricity, Classification of quadratic equations representing lines, parabola, ellipse, and hyperbola; Parameterization of plane curves, Conic sections in polar coordinates and their sketching.

## Unit 2: Three-Dimensional Space: Vectors

Rectangular coordinates in 3-space, Spheres and cylindrical surfaces, Vectors viewed geometrically, Vectors in coordinate systems, Vectors determined by length and angle, Dot product, Cross product and their geometrical properties, Parametric equations of lines in 2-space and 3-space.

## Unit 3: Theory of Equations

General properties of polynomials and equations, Descartes' rule of signs for positive and negative roots, Relation between the roots and the coefficients of equations, Applications, Depression of an equation when a relation exists between two of its roots, Symmetric functions of the roots and its applications, Transformation of equations (multiplication, reciprocal, increase/diminish in the roots by a given quantity), Removal of terms; Graphical representation of derived function, Rolle's theorem, Multiple roots of the equation.

## References:

1. Anton, Howard, Bivens, Irl, \& Davis, Stephen (2013). Calculus (10th ed.). John Wiley \& Sons Singapore Pvt. Ltd. Reprint (2016) by Wiley India Pvt. Ltd. Delhi.
2. Burnside, W.S., \& Panton, A.W. (1979), The Theory of Equations, Vol. 1. Eleventh Edition, Fourth Indian Reprint. S. Chand \& Co. New Delhi.
3. Thomas, Jr. George B., Weir, Maurice D., \& Hass, Joel (2014). Thomas' Calculus (13th ed.). Pearson Education, Delhi. Indian Reprint 2017.

## Additional Readings:

i. Dickson, Leonard Eugene (2009). First Course in the Theory of Equations. The Project Gutenberg EBook (http://www.gutenberg.org/ebooks/29785).


## ii. Ferrar, W. L. (1956). Higher Algebra. Oxford University Press.

## Teaching Plan (GE-I: Analytical Geometry and Theory of Equations):

Weeks 1 and 2: Conic sections and quadratic equations: circle, parabola, ellipse, and hyperbola; Techniques for sketching: parabola, ellipse, and hyperbola; Reflection properties of parabola, ellipse, and hyperbola.
[3] Chapter 11 (Section 11.6).
[1] Chapter 10 (Section 10.4).
Week 3: Classifying conic sections by eccentricity.
[3] Chapter 11 (Section 11.7).
Weeks 4 and 5: Classification of quadratic equations representing lines, parabola, ellipse, and hyperbola; Parameterization of plane curves, Conic sections in polar coordinates and their sketching.
[1] Chapter 10 (Section 10.2).
[3] Chapter 11 (Sections 11.1, and 11.4).
Weeks 6 and 7: Rectangular coordinates in 3-space, Spheres and cylindrical surfaces, Vectors viewed geometrically, Vectors in coordinate systems, Vectors determined by length and angle.
[1] Chapter 11 (11.1, and 11.2)
Weeks 8 and 9: Dot product, Cross product and their geometrical properties, Parametric equations of lines in 2 -space and 3 -space.
[1] Chapter 11 (Sections 11.3 to 11.5 ).
Weeks 10 and 11: General properties of polynomials and equations, Descartes' rule of signs for positive and negative roots, Relation between the roots and the coefficients of equations, Applications.
[2] Chapter 2 (Sections 12 to 22), Chapter 3 (Sections 23 and 24).
Weeks 12 and 13: Depression of an equation when a relation exists between two of its roots, Symmetric functions of the roots and its applications, Transformation of equations (multiplication, reciprocal, increase/diminish in the roots by a given quantity), Removal of terms.
[2] Chapter 3 (Sections 25 to 28), Chapter 4 (Sections 29 to 34).
Week 14: Graphical representation of derived function, Rolle's theorem, Multiple roots of the equation.
[2] Chapter 7 (Sections 69, 71, 73 and 74).
[1] Chapter 3 (Section 3.8.1).
Facilitating the Achievement of Course Learning Outcomes

| Unit <br> No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
| :---: | :---: | :---: | :---: |
| 1. | Classify and sketch conics four different types of conic sections - the circle, the ellipse, the hyperbola and the parabola - in Cartesian and polar coordinates. | (i) Each topic to be explained with examples. <br> (ii) Students to be involved in discussions and encouraged to ask questions. <br> (iii) Students to be given homework/assignments. <br> (iv) Students to be encouraged to give short presentations. | - Student presentations. <br> - Participation in discussions. <br> - Assignments and class tests. <br> - Mid-term examinations. <br> - End-term examinations. |
| 2. | Visualize three-dimensional objects - spheres and cylinders using vectors. |  |  |
| 3. | Understand the properties of roots of polynomial equations. |  |  |

Keywords: Circle, Parabola, Ellipse, Hyperbola, Spheres, Cylindrical surfaces, Vectors, Roots of equations, Coefficients of equations.


# Semester-II <br> Generic Elective (GE) Course -Mathematics 

Any one of the following:
GE-2: Linear Algebra
GE-2: Discrete Mathematics

## GE-2: Linear Algebra

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) Credits: 6 (5+1)
Duration: 14 Weeks ( 70 Hrs.) Examination: 3 Hrs.
Course Objectives: The objective of the course is to introduce the concept of vectors in $\mathbb{R}^{n}$. The concepts of linear independence and dependence, rank and linear transformations has been explained through matrices. Various applications of vectors in computer graphics and movements in a plane has also been introduced.

Course Learning Outcomes: This course will enable the students to:
i) Visualize the space $\mathbb{R}^{n}$ in terms of vectors and the interrelation of vectors with matrices, and their application to computer graphics.
ii) Familiarize with concepts in vector spaces, namely, basis, dimension and minimal spanning sets.
iii) Learn about linear transformations, transition matrix and similarity.
iv) Learn about orthogonality and to find approximate solution of inconsistent system of linear equations.

## Unit 1: Euclidean space $\mathbb{R}^{n}$ and Matrices

Fundamental operation with vectors in Euclidean space $\mathbb{R}^{n}$, Linear combination of vectors, Dot product and their properties, Cauchy-Schwarz inequality, Triangle inequality, Projection vectors, Some elementary results on vectors in $\mathbb{R}^{n}$, Matrices: Gauss-Jordan row reduction, Reduced row echelon form, Row equivalence, Rank, Linear combination of vectors, Row space, Eigenvalues, Eigenvectors, Eigenspace, Characteristic polynomials, Diagonalization of matrices; Definition and examples of vector spaces, Some elementary properties of vector spaces, Subspace, Span, Spanning set for an eigenspace, Linear independence and linear dependence of vectors, Basis and dimension of a vector space, Maximal linearly independent sets, Minimal spanning sets; Application of rank: Homogenous and non-homogenous systems of linear equations; Coordinates of a vector in ordered basis, Transition matrix.

## Unit 2: Linear Transformations and Computer Graphics

Linear transformations: Definition and examples, Elementary properties, The matrix of a linear transformation, Linear operator and similarity; Application: Computer graphics, Fundamental movements in a plane, Homogenous coordinates, Composition of movements; Kernel and range of a linear transformation, Dimension theorem, One to one and onto linear transformations, Invertible linear transformations, Isomorphism, Isomorphic vector spaces (to $\mathbb{R}^{n}$ ).


## Unit 3: Orthogonality and Least Square Solutions

Orthogonal and orthonormal vectors, Orthogonal and orthonormal bases, Orthogonal complement, Projection theorem, Orthogonal projection onto a subspace; Application: Least square solutions for inconsistent systems, Non-unique least square solutions.

## References:

1. Andrilli, S., \& Hecker, D. (2016). Elementary Linear Algebra (5th ed.). Elsevier India.
2. Kolman, Bernard, \& Hill, David R. (2001). Introductory Linear Algebra with Applications (7th ed.). Pearson Education, Delhi. First Indian Reprint 2003.

## Additional Reading:

i. Lay, David C., Lay, Steven R., \& McDonald, Judi J. (2016). Linear Algebra and its Applications (5th ed.). Pearson Education.

## Teaching Plan (GE-2: Linear Algebra):

Week 1: Fundamental operation with vectors in Euclidean space $\mathbb{R}^{n}$, Linear combination of vectors, dot product and their properties, Cauchy-Schwarz inequality, Triangle inequality, Projection vectors.
[1] Chapter 1 (Sections 1.1 and 1.2).
Week 2: Some elementary results on vectors in $\mathbb{R}^{n}$; Matrices: Gauss-Jordan row reduction, Reduced row echelon form, Row equivalence, Rank.
[1] Chapter 1 [Section 1.3 (Pages 34 to 44)].
[1] Chapter 2 [Sections 2.2 (up to Page 111), 2.3 (up to Page 122, Statement of Theorem 2.5)].
Week 3: Linear combination of vectors, Row space, Eigenvalues, Eigenvectors, Eigenspace, Characteristic polynomials, Diagonalization of matrices.
[1] Chapter 2 [Section 2.3 (Pages 122-132, Statements of Lemma 2.8, Theorem 2.9)], Chapter 3 (Section 3.4).
Week 4: Definition and examples of vector spaces, Some elementary properties of vector spaces. [1] Chapter 4 (Section 4.1).
Week 5 and 6: Subspace, Span, Spanning set for an eigenspace, Linear independence and dependence, Basis and dimension of a vector space, Maximal linearly independent sets, Minimal spanning sets.
[1] Chapter 4 (Sections 4.2 to 4.5, Statements of technical Lemma 4.10 and Theorem 4.12).
Week 7: Application of rank: Homogenous and non-homogenous systems of linear equations; Coordinates of a vector in ordered basis, Transition matrix.
[2] Chapter 6 [Sections 6.6 (Pages 287 to 291), and 6.7 (Statement of Theorem 6.15 and examples)].
Week 8: Linear transformations: Definition and examples, Elementary properties.
[1] Chapter 5 (Section 5.1).
Week 9: The matrix of a linear transformation, Linear operator and similarity.
[1] Chapter 5 [Section 5.2 (Statements of Theorem 5.5 and Theorem 5.6)].
Week 10: Application: Computer graphics, Fundamental movements in a plane, Homogenous coordinates, Composition of movements.
[1] Chapter 8 (Section 8.8).
Week 11: Kernel and range of a linear transformation, Statement of the dimension theorem and examples.
[1] Chapter 5 (Sections 5.3).
Week 12: One to one and onto linear transformations, Invertible linear transformations, isomorphism, isomorphic vector spaces (to $\mathbb{R}^{n}$ ).
[1] Chapter 5 [Sections 5.4, 5.5 (up to Page 378, Statements of Theorem 5.15, and Theorem 5.16)]


Week 13 and 14: Orthogonal and orthonormal vectors, orthogonal and orthonormal bases, orthogonal complement, statement of the projection theorem and examples. Orthogonal projection onto a subspace. Application: Least square solutions for inconsistent systems, non-unique least square solutions.
[1] Chapter 6 [Sections 6.1 (up to Example 3, Page 416, Statement of Theorem 6.3), 6.2 (up to Page 435, and Pages 439 to 443, and Statement of Theorem 6.12)].
[1] Chapter 8 [Section 8.9 (up to Page 593, Statement of Theorem 8.13).
Facilitating the Achievement of Course Learning Outcomes

| $\begin{aligned} & \hline \text { Unit } \\ & \text { No. } \\ & \hline \end{aligned}$ | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
| :---: | :---: | :---: | :---: |
| 1. | Visualize the space $\mathbb{R}^{n}$ in terms of vectors and the interrelation of vectors with matrices, and their application to computer graphics. Familiarize with concepts in vector spaces, namely, basis, dimension and minimal spanning sets. | (i) Each topic to be explained with examples. <br> (ii) Students to be involved in discussions and encouraged to ask questions. <br> (iii) Students to be given homework/assignments. <br> (iv) Students to be encouraged to give short presentations. | - Student presentations. <br> - Participation in discussions. <br> - Assignments and class tests. <br> - Mid-term examinations. <br> - End-term examinations. |
| 2. | Learn about linear transformations, transition matrix and similarity. |  |  |
| 3. | Learn about orthogonality and to find approximate solution of inconsistent system of linear equations. |  |  |

Keywords: Cauchy-Schwarz inequality Gauss-Jordan row reduction Basis and dimension of vector spaces, matrix of linear transformations, Orthogonality, Orthonormality, Least square solutions.


## GE-2: Discrete Mathematics

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) Credits: 6 (5+1)
Duration: 14 Weeks ( 70 Hrs.) Examination: 3 Hrs.
Course Objectives: The course introduces formal logic notation, methods of proof, mathematical induction, set theory, permutations and combinations and counting principles. One can learn the concepts of lattices and Boolean algebra in analysis of various applications.
Course Learning Outcomes: This course will enable the students to:
i) Understand the basic principles of logic, set theory, lattices and Boolean algebra.
ii) Understand the ideas of mathematical induction and basic counting techniques.
iii) Proficiently construct logical arguments and rigorous proofs.

## Unit 1: Logical Mathematics

Compound statements (and, or, implication, negation, contrapositive, quantifiers), Truth tables, Basic logical equivalences and its consequences, Logical arguments, Set theory, Operation on sets, Types of binary relations, Equivalence relations, Congruences and its properties, Partial and total ordering, Lattices, Properties of integers, Division algorithm, Divisibility and Euclidean algorithm, GCD, LCM, Relatively prime.

## Unit 2: Applications of Numbers

Prime numbers, Statement of fundamental theorem of arithmetic, Fermat primes, Mathematical induction, Recursive relations and its solution (characteristics polynomial and generating function), Principles of counting (inclusion/exclusion, pigeon-hole), Permutation and combinations (with and without repetition).

## Unit 3: Lattices and its Properties

Duality principle, Lattices as ordered sets, Lattices as algebraic structures, Sublattices, Products and homomorphisms, Distributive lattices, Boolean algebras, Boolean polynomials, Minimal forms of Boolean polynomials, Quinn-McCluskey method, Karnaugh diagrams, Switching circuits and applications of switching circuits.

## References:

1. Davey, B A., \& Priestley, H. A. (2002). Introduction to Lattices and Order (2nd ed.). Cambridge University Press. Cambridge. 7th Printing 2012.
2. Goodaire, Edgar G., \& Parmenter, Michael M. (2003). Discrete Mathematics with Graph Theory (2nd ed.). Pearson Education (Singapore) Pte. Ltd. Indian Reprint.
3. Lidl, Rudolf \& Pilz, Günter. (1998). Applied Abstract Algebra (2nd ed.). Undergraduate Texts in Mathematics. Springer (SIE). Indian Reprint 2004.

## Additional Reading:

i. Rosen, Kenneth H. (2012) Discrete Mathematics and its Applications (7th ed.). McGraw-Hill Education (India) Pvt. Ltd.


## Teaching Plan (GE-2: Discrete Mathematics):

Week 1: Compound Statements (and, or, implication, negation, contrapositive, quantifiers), Truth tables.
[2] Chapter 1 (Sections 1.1, and 1.3).
Week 2: Basic logical equivalences and its consequences, Logical arguments, Set theory.
[2] Chapter 1 (Sections 1.4, and 1.5), and Chapter 2 (Section 2.1).
Week 3: Operation on sets, types of binary relations, Equivalence relations, Congruences and its properties.
[2] Chapter 2 [Sections 2.2, 2.3, and 2.4 (left for convergence)], and Chapter 4 (Section 4.4).
Week 4: Partial and total ordering, Lattices.
[2] Chapter 2 (Section 2.5).
Week 5: Properties of integers, Division algorithm, Divisibility.
[2] Chapter 4 (Sections 4.1 to 4.1.6).
Week 6: Euclidean algorithm, GCD, LCM, Relatively prime.
[2] Chapter 4 (Section 4.2).
Week 7: Prime numbers, statement of fundamental theorem of arithmetic, Fermat primes.
[2] Chapter 4 (Sections 4.3 up to 4.3.11, Page 119).
Week 8: Mathematical induction, Recursive relations and its solution (characteristics polynomial and generating function).
[2] Chapter 5 (Sections5.1, 5.3, and 5.4).
Week 9: Principles of counting (inclusion /exclusion, pigeon-hole), permutation and combinations (with and without repetition).
[2] Chapter 6 (Section 6.1), Chapter 7 (Sections 7.1 to 7.3).
Week 10: Duality principle, lattices as ordered sets.
[1] Sections 1.20, and 2.1 to 2.7.
Week 11: Lattices as algebraic structures, Sublattices, Products and Homomorphisms, Distributive lattices.
[1] Chapter 2 (Sections 2.8 to 2.19), Chapter 4 (Sections 4.1 to 4.11)
Week 12: Boolean algebras, Boolean polynomials, Minimal forms of Boolean polynomials.
[3] Chapter 1 (Section 2)
Weeks 13 and 14: Quinn-McCluskey method, Karnaugh diagrams, Switching circuits and applications of switching circuits.
[3] Chapter 2 (Section 1).
Facilitating the Achievement of Course Learning Outcomes
$\left.\begin{array}{|c|l|l|l|}\hline \begin{array}{l}\text { Unit } \\ \text { No. }\end{array} & \begin{array}{l}\text { Course Learning } \\ \text { Outcomes }\end{array} & \text { Teaching and Learning Activity } & \text { Assessment Tasks } \\ \hline 1 . & \begin{array}{l}\text { Understand the basic } \\ \text { principles of logic, set } \\ \text { theory, lattices and Boolean } \\ \text { algebra. }\end{array} & \begin{array}{l}\text { (i) Each topic to be explained with } \\ \text { examples. } \\ \text { (ii) Students to be involved in } \\ \text { discussions and encouraged to } \\ \text { ask questions. }\end{array} & \begin{array}{l}\text { • Student presentations. } \\ \text { - Participation in } \\ \text { discussions. }\end{array} \\ \text { • Assignments and } \\ \text { class tests. }\end{array}\right\}$

Keywords: Truth tables, Set theory, Division algorithm, Fermat primes, Lattices, Boolean polynomials, Switching circuits.


# Semester-III Generic Elective (GE) Course -Mathematics 

## Any one of the following:

GE-3: Differential Equations (with Practicals)
GE-3: Linear Programming and Game Theory

## GE-3: Differential Equations (with Practicals)

Total Marks: 150 (Theory: 75, Internal Assessment: 25, and Practical: 50)
Workload: 4 Lectures, 4 Practicals (per week) Credits: 6 (4+2)
Duration: 14 Weeks ( 56 Hrs. Theory +56 Hrs. Practical) Examination: 3 Hrs.
Course Objectives: This course includes a variety of methods to solve ordinary and partial differential equations with basic applications to real life problems. It provides a solid foundation to further in mathematics, sciences and engineering through mathematical modeling.

Course Learning Outcomes: The student will be able to:
i) Solve the exact, linear and Bernoulli equations and find orthogonal trajectories.
ii) Apply the method of variation of parameters to solve linear differential equations.
iii) Formulate and solve various types of first and second order partial differential equations.

## Unit 1: Ordinary Differential Equations and Applications

First order exact differential equations, Integrating factors and rules to find integrating factors, Linear equations and Bernoulli equations, Orthogonal trajectories and oblique trajectories, Basic theory of higher order linear differential equations, Wronskian and its properties; Solving differential equation by reducing its order.

Unit 2. Explicit Methods of Solving Higher-Order Linear Differential Equations
Linear homogenous equations with constant coefficients, Linear non-homogenous equations, Method of undetermined coefficients, Method of variation of parameters, Cauchy-Euler equations; Simultaneous differential equations.

## Unit 3. First and Second Order Partial Differential Equations

Partial differential equations: Basic concepts and definitions. Mathematical problems; First order equations: Classification, Construction, Geometrical interpretation; Method of characteristics, General solutions of first order partial differential equations; Canonical forms and method of separation of variables for first order partial differential equations; Classification of second order partial differential equations; Reduction to canonical forms; Second order partial differential equations with constant coefficients, General solutions.

## References:

1. Kreyszig, Erwin. (2011). Advanced Engineering Mathematics (10th ed.). Wiley India.
2. Myint-U, Tyn and Debnath, Lokenath (2007). Linear Partial Differential Equations for Scientist and Engineers (4th ed.). Birkkäuser Boston. Indian Reprint.
3. Ross, Shepley. L. (1984). Differential Equations (3rd ed.). John Wiley \& Sons.


## Additional reading:

i. Sneddon I. N. (2006). Elements of Partial Differential Equations. Dover Publications.

## Practical / Lab work to be performed in a Computer Lab:

Use of Computer Algebra Systems (CAS), for example MATLAB/Mathematica /Maple/Maxima/Scilab etc., for developing the following programs:

1. Solution of first order differential equation.
2. Plotting of second order solution family of differential equation.
3. Plotting of third order solution family of differential equation.
4. Solution of differential equation by variation of parameter method.
5. Solution of system of ordinary differential equations.
6. Solution of Cauchy problem for first order partial differential equations.
7. Plotting the characteristics of the first order partial differential equations.
8. Plot the integral surfaces of first order partial differential equations with initial data.

## Teaching Plan (GE-3: Differential Equations):

Weeks 1 and 2: First order ordinary differential equations: Basic concepts and ideas, First order exact differential equation, Integrating factors and rules to find integrating factors.
[3] Chapter 1 (Sections 1.1, and 1.2), and Chapter 2 (Sections 2.1, and 2.2).
[1] Chapter 1 (Sections 1.1, 1.2, and 1.4).
Week 3: Linear equations and Bernoulli equations, Orthogonal trajectories and oblique trajectories.
[3] Chapter 2 (Sections 2.3, and 2.4), and Chapter 3 (Section 3.1).
Weeks 4 and 5: Basic theory of higher order linear differential equations, Wronskian and its properties, Solving a differential equation by reducing its order.
[3] Chapter 4 (Section 4.1).
Weeks 6 and 7: Linear homogenous equations with constant coefficients, Linear non-homogenous equations, Method of undetermined coefficients.
[3] Chapter 4 (Sections 4.2, and 4.3), and
[1] Chapter 2 (Section 2.2).
Weeks 8 and 9: Method of variation of parameters, Cauchy-Euler equations, Simultaneous differential equations.
[3] Chapter 4 (Sections 4.4, and 4.5), and Chapter 7 (Sections 7.1, and 7.3)
Week 10: Partial differential equations: Basic concepts and definitions, Mathematical problems; First order equations: Classification and construction.
[2] Chapter 2 (Sections 2.1 to 2.3).
Weeks 11 and 12: Geometrical interpretation, Method of characteristics, General solutions of first order partial differential equations.
[2] Chapter 2 (Sections 2.4, and 2.5).
Week 13: Canonical forms and method of separation of variables for first order partial differential equations.
[2] Chapter 2 (Sections 2.6, and 2.7)
Week 14: Second order partial differential equations: Classification, Reduction to canonical forms, With constant coefficients, General solutions.
[2] Chapter 4 (Sections 4.1 to 4.4).


## Facilitating the Achievement of Course Learning Outcomes

| Unit <br> No. | Course Learning <br> Outcomes | Teaching and Learning Activity | Assessment Tasks |
| :---: | :--- | :--- | :--- |
| 1. | Solve the exact, linear and <br> Bernoulli equations and find <br> orthogonal trajectories. | (i) Each topic to be explained with <br> illustrations. <br> (ii) Students to be encouraged to <br> discover the relevant concepts. | - Presentations and <br> class discussions. <br> - Assignments and <br> class tests. |
| 2. | Apply the method of <br> variation of parameters to <br> solve linear differential <br> equations. | Student <br> (iii) Students to be given <br> homework/assignments. <br> (iv) Discuss and solve the <br> theoretical and practical <br> problems in the class. | - Mid-term <br> examinations. |
| 3. | Formulate and solve various <br> types of first and second <br> order partial differential <br> equations. | (v) Students to be encouraged to <br> apply concepts to real world <br> problems. | Pralical and viva- <br> - <br> voce examinations. <br> End-term |
| examinations. |  |  |  |

Keywords: Integrating factors, Bernoulli equations, Wronskian, Cauchy-Euler equation, First and second order PDE's.

## GE-3: Linear Programming and Game Theory

Total Marks: 100 (Theory: 75 and Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) Credits: 6 (5+1)
Duration: 14 Weeks (70 Hrs.) Examination: 3 Hrs.
Course Objectives: This course develops the ideas behind the solution of linear programming problem using simplex method, as well as, the solution of transportation and assignment problems. This course also provides an introduction to game theory which makes possible the analysis of the decision making process of two interdependent subjects.

Course Learning Outcomes: This course will enable the students to:
i) Learn about the simplex method used to find optimal solutions of linear optimization problems subject to certain constraints.
ii) Write the dual of a linear programming problem.
iii) Solve the transportation and assignment problems.
iv) Learn about the solution of rectangular games using graphical method and using the solution of a pair of associated prima-dual linear programming problems.

## Unit 1. Linear Programming Problem, Simplex Method and Duality

Graphical method of solution, Basic feasible solutions, Linear programming and convexity; Introduction to the simplex method: Theory of the simplex method, Optimality and unboundedness; Simplex tableau and examples, Artificial variables; Introduction to duality, Formulation of the dual problem with examples and interpretations, Duality theorem.

## Unit 2. Transportation and Assignment Problems

Definition and mathematical formulation of transportation problems, Methods of finding initial basic feasible solutions, North West corner rule, Least-cost method, Vogel's approximation method, Algorithm for solving transportation problems; Mathematical formulation and Hungarian method of solving assignment problems.

## Unit 3. Two-Person, Zero-Sum Games

Introduction to game theory, Formulation of two-person zero-sum rectangular game, Solution of rectangular games with saddle points, Mixed strategies, Dominance principle, Rectangular games without saddle points, Graphical and linear programming solution of rectangular games.

## References:

1. Taha, Hamdy A. (2010). Operations Research: An Introduction (9th ed.). Pearson.
2. Thie, Paul R., \& Keough, G. E. (2014). An Introduction to Linear Programming and Game Theory. (3rd ed.). Wiley India Pvt. Ltd.

## Additional Readings:

i. Hadley, G. (1997). Linear Programming. Narosa Publishing House. New Delhi.
ii. Hillier, F. S., \& Lieberman, G. J. (2017). Introduction to Operations Research (10th ed.). McGraw-Hill Education (India) Pvt. Ltd.

iii. Kolman. B. and Hill, D.R. (2003). Introductory Linear Algebra with Applications. (7th ed.). Pearson Education. First Indian Reprint 2003.

## Teaching Plan (GE-3: Linear Programming and Game Theory):

Week 1: Introduction to linear programming problem: Graphical method of solution, Basic feasible solutions, Linear programming and convexity.
[2] Chapter 2 (Section 2.2), and Chapter 3 (Sections 3.1, 3.2, and 3.9).
Weeks 2 and 3: Introduction to the simplex method: Theory of the simplex method, Optimality and unboundedness.
[2] Chapter 3 (Sections 3.3, and 3.4).
Weeks 4 and 5: Simplex tableau and examples, Artificial variables.
[2] Chapter 3 (Sections 3.5, and 3.6).
Weeks 6 and 7: Introduction to duality, Formulation of the dual problem with examples and interpretations, Statement of the duality theorem with examples.
[2] Chapter 4 (Sections 4.1 to 4.4).
Weeks 8 and 9: Definition and mathematical formulation of transportation problems, Methods of finding initial basic feasible solutions, North West corner rule, Least-cost method, Vogel's approximation method, Algorithm for solving transportation problems.
[1] Chapter 5 (Sections 5.1, and 5.3).
Week 10: Mathematical formulation and Hungarian method of solving assignment problems.
[1] Chapter 5 (Section 5.4).
Weeks 11 and 12: Introduction to game theory, Formulation of two-person zero-sum rectangular game, Solution of rectangular games with saddle points.
[2] Chapter 9 (Sections 9.1 to 9.3).
Weeks 13 and 14: Mixed strategies, Dominance principle, Rectangular games without saddle points, Graphical and linear programming solution of rectangular games.
[2] Chapter 9 (Sections 9.4 to 9.6).
Facilitating the Achievement of Course Learning Outcomes

| Unit <br> No. | Course Learning Outcomes | Teaching and Learning <br> Activity | Assessment Tasks |
| :---: | :--- | :--- | :--- |
| 1. | Learn about the simplex method <br> used to find optimal solutions of <br> linear optimization problems subject <br> to certain constraints. <br> Write the dual of a linear <br> programming problem. | (i) Each topic to be explained <br> with examples. <br> (ii) Students to be involved <br> in discussions and <br> encouraged to ask <br> questions. | • Student <br> presentations. <br> Participation in <br> discussions. |
| 2. | Solve the transportation and <br> assignment problems. | Assignments and <br> class tests. |  |
| 3. | Lii) Students to be given <br> homework/assignments. about the solution of <br> hectangular games using graphical <br> method and using the solution of a <br> mair of associated prima-dual linear <br> programming problems. | Miderm <br> (iv) Students to be <br> encouraged to give short <br> presentations. | • End-term <br> examinations. |

Keywords: Basic feasible solutions, Duality, Transportation problems, Assignment problems, Rectangular games, Dominanace.


# Semester-IV Generic Elective (GE) Course -Mathematics <br> <br> Any one of the following: <br> <br> Any one of the following: <br> GE-4: Numerical Methods (with Practicals) <br> GE-4: Elements of Analysis 

## GE-4: Numerical Methods (with Practicals)

Total Marks: 150 (Theory: 75, Internal Assessment: 25, and Practical: 50)
Workload: 4 Lectures, 4 Practicals (per week) Credits: 6 (4+2)
Duration: 14 Weeks ( 56 Hrs. Theory +56 Hrs. Practical) Examination: 3 Hrs.
Course Objectives: The goal of this paper is to acquaint students' various topics in Numerical Analysis such as solutions of nonlinear equations in one variable, interpolation and approximation, numerical differentiation and integration, direct methods for solving linear systems, numerical solution of ordinary differential equations using Computer Algebra System (CAS).

Course Learning Outcomes: After completion of this course, students will be able to:
i) Find the consequences of finite precision and the inherent limits of numerical methods.
ii) Appropriate numerical methods to solve algebraic and transcendental equations.
iii) Solve first order initial value problems of ODE's numerically using Euler methods.

## Unit 1: Errors and Roots of Transcendental and Polynomial Equations

Floating point representation and computer arithmetic, Significant digits; Errors: Roundoff error, Local truncation error, Global truncation error; Order of a method, Convergence and terminal conditions; Bisection method, Secant method, Regula-Falsi method, Newton-Raphson method.

## Unit 2: Algebraic Linear Systems and Interpolation

Gaussian elimination method (with row pivoting), Gauss-Jordan method; Iterative methods: Jacobi method, Gauss-Seidel method; Interpolation: Lagrange form, Newton form, Finite difference operators, Gregory-Newton forward and backward difference interpolations, Piecewise polynomial interpolation (linear and quadratic).

## Unit 3: Numerical Differentiation, Integration and ODE

Numerical differentiation: First and second order derivatives, Richardson extrapolation method; Numerical integration: Trapezoidal rule, Simpson's rule; Ordinary differential equation: Euler's method, Modified Euler's methods (Heun's and midpoint).

## References:

1. Chapra, Steven C. (2018). Applied Numerical Methods with MATLAB for Engineers and Scientists (4th ed.). McGraw-Hill Education.
2. Fausett, Laurene V. (2009). Applied Numerical Analysis Using MATLAB. Pearson. India.
3. Jain, M. K., Iyengar, S. R. K., \& Jain R. K. (2012). Numerical Methods for Scientific and Engineering Computation (6th ed.). New Age International Publishers. Delhi.

## Additional Reading:

i. Bradie, Brian (2006). A Friendly Introduction to Numerical Analysis. Pearson Education India. Dorling Kindersley (India) Pvt. Ltd. Third Impression, 2011.

## Practical /Lab work to be performed in the Computer Lab:

Use of Computer Algebra System (CAS), for example MATLAB/Mathematica/Maple/Maxima/ Scilab etc., for developing the following Numerical Programs:

1. Bisection method
2. Secant method and Regula-Falsi method
3. Newton-Raphson method
4. Gaussian elimination method and Gauss-Jordan method
5. Jacobi method and Gauss-Seidel method
6. Lagrange interpolation and Newton interpolation
7. Trapezoidal and Simpson's rule.
8. Euler methods for solving first order initial value problems of ODE's.

## Teaching Plan (Theory of GE-4: Numerical Methods):

Weeks 1 and 2: Floating point representation and computer arithmetic, Significant digits; Errors: Roundoff error, Local truncation error, Global truncation error; Order of a method, Convergence and terminal conditions.
[2] Chapter 1 (Sections 1.2.3, 1.3.1, and 1.3.2).
[3] Chapter 1 (Sections 1.2, and 1.3).
Week 3 and 4: Bisection method, Secant method, Regula-Falsi method, Newton-Raphson method.
[2] Chapter 2 (Sections 2.1 to 2.3).
[3] Chapter 2 (Sections 2.2 and 2.3).
Week 5: Gaussian elimination method (with row pivoting), Gauss-Jordan method;
Iterative methods: Jacobi method, Gauss-Seidel method.
[2] Chapter 3 (Sections 3.1, and 3.2), Chapter 6 (Sections 6.1, and 6.2).
[3] Chapter 3 (Sections 3.2, and 3.4).
Week 6: Interpolation: Lagrange form, and Newton form.
[2] Chapter 8 (Section 8.1). [3] (Section 4.2).
Weeks 7 and 8: Finite difference operators, Gregory-Newton forward and backward difference interpolations.
[3] Chapter 4 (Sections 4.3, and 4.4).
Week 9: Piecewise polynomial interpolation: Linear, and quadratic.
[2] Chapter 8 [Section 8.3 (8.3.1, and 8.3.2)].
[1] Chapter 18 (Sections 18.1 to 18.3).
Weeks 10 and 11: Numerical differentiation: First and second order derivatives, Richardson extrapolation method.
[2] Chapter 11 [Sections 11.1 (11.1.1, 11.1.2 and 11.1.4)]
Weeks 12 and 13: Numerical integration: Trapezoidal rule, Simpson's rule; Ordinary differential equations: Euler's method.
[2] Chapter 11 [Section 11.2 (11.2.1, 11.2.2)].
[1] Chapter 22 (Sections 22.1, and 22.2 (up to Page 583)
Weeks 14: Modified Euler's methods: Heun's method, The midpoint method.
[1] Chapter 22 (Section 22.3)


## Facilitating the Achievement of Course Learning Outcomes

| Unit No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
| :---: | :---: | :---: | :---: |
| 1. | Find the consequences of finite precision and the inherent limits of numerical methods. | (i) Each topic to be explained with illustrations. <br> (ii) Students to be encouraged to discover the relevant concepts. <br> (iii) Students to be given homework/assignments. <br> (iv) Discuss and solve the theoretical and practical problems in the class. <br> (v) Students to be encouraged to apply concepts to real world problems. | - Presentations and class discussions. <br> - Assignments and class tests. <br> - Student presentations. <br> - Mid-term examinations. <br> - Practical and vivavoce examinations. <br> - End-term examinations. |
| 2. | Appropriate numerical methods to solve algebraic and transcendental equations. |  |  |
| 3. | Solve first order initial value problems of ordinary differential equations numerically using Euler methods. |  |  |

Keywords: Bisection method, Secant method, Regula-Falsi method, Newton-Raphson method, Gauss-Seidel method, Piecewise polynomial interpolation, Richardson extrapolation method, Simpson's rule.


## GE-4: Elements of Analysis

Total Marks: 100 (Theory: 75 and Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) Credits: 6 (5+1)
Duration: 14 Weeks (70 Hrs.) Examination: 3 Hrs.
Course Objectives: Real analysis provides tools to lay the foundation for further study in subfields, such as calculus, differential equations, and probability. To study this course one needs a background in calculus and a facility with logic and proofs. This course deals with the analytic properties of real numbers, sequences and series, including convergence and limits of sequences of real numbers, the calculus of the real numbers, and convergence of power series.

Course Learning Outcomes: This course will enable the students to:
i) Understand the real numbers and their basic properties.
ii) Be familiar with convergent and Cauchy sequences.
iii) Test the convergence and divergence of infinite series of real numbers.
iv) Learn about power series expansion of some elementary functions.

## Unit 1. Real Numbers and Sequences

Finite and infinite sets, Examples of countable and uncountable sets; Absolute value and the Real line, Bounded sets, Suprema and infima, The completeness property of $\mathbb{R}$, Archimedean property of $\mathbb{R}$; Real sequences, Convergence, sum and product of convergent sequences, Order preservation and squeeze theorem; Monotone sequences and their convergence; Proof of convergence of some simple sequences such as $\frac{(-1)^{n}}{n}, \frac{1}{n^{2}},\left(1+\frac{1}{n}\right)^{n}, x^{n}$ with $|x|<1, a_{n} / n$, where $a_{n}$ is a bounded sequence. Subsequences and the Bolzano-Weierstrass theorem; Limit superior and limit inferior of a bounded sequence; Cauchy sequences, Cauchy convergence criterion for sequences.

## Unit 2. Infinite Series of Real Numbers

Definition and a necessary condition for convergence of an infinite series, Geometric series, Cauchy convergence criterion for series; Positive term series, Integral test, Convergence of $p$ series, Comparison test, Limit comparison test, D'Alembert's ratio test, Cauchy's root test; Alternating series, Leibniz test; Absolute and conditional convergence.

## Unit 3. Power Series and Elementary Functions

Definition of power series, Radius and interval of convergence, Cauchy-Hadamard theorem, Statement and illustration of term-by-term differentiation, Integration of power series, and Abel's theorem, Power series expansions for $e^{x}, \sin x, \cos x, \log (1+x)$ and their properties.

## References:

1. Bartle, Robert G., \& Sherbert, Donald R. (2015). Introduction to Real Analysis (4th ed.). Wiley India Edition.
2. Denlinger, Charles G. (2015). Elements of Real Analysis. Jones \& Bartlett India Pvt. Ltd. Ross, Kenneth A. (2013). Elementary Analysis: The Theory of Calculus (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian Reprint.


## Additional Reading:

i. Bilodeau, Gerald G., Thie, Paul R., \& Keough, G. E. (2010). An Introduction to Analysis (2nd ed.). Jones \& Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

## Teaching Plan (GE-4: Elements of Analysis):

Weeks 1 and 2: Finite and infinite sets, Examples of countable and uncountable sets; Absolute value of the real line, bounded sets, suprema and infima; Statement of order completeness property of $\mathbb{R}$, Archimedean property of $\mathbb{R}$.
[1] Chapter 1 (Section 1.3), and Chapter 2 (Sections 2.2 to 2.4).
Weeks 3 and 4: Real sequences, Convergence, Sum and product of convergent sequences, Order preservation and squeeze theorem.
[1] Chapter 3 (Sections 3.1 and 3.2).
Week 5: Monotone sequences and their convergence, Proof of convergence of some simple sequences such as $\frac{(-1)^{n}}{n}, \frac{1}{n^{2}},\left(1+\frac{1}{n}\right)^{n}, x^{n}$ with $|x|<1, a_{n} / n$, where $a_{n}$ is a bounded sequence.
[1] Chapter 3 (Section 3.3)
Weeks 6 and 7: Subsequences and the Bolzano-Weierstrass theorem (statement and examples), Limit superior and limit inferior of a bounded sequence (definition and examples), Statement and illustrations of Cauchy convergence criterion for sequences.
[1] Chapter 3 (Sections 3.4, and 3.5).
Weeks 8 and 9: Definition and a necessary condition for convergence of an infinite series, Geometric series, Cauchy convergence criterion for series, positive term series, State the integral test and prove the convergence of $p$-series, Comparison test, Limit comparison test and examples.
[2] Chapter 8 (Section 8.1).
[1] Chapter 3 (Section 3.7).
Week 10: D'Alembert's ratio test, Cauchy's root test.
[2] Chapter 8 (Section 8.2).
Week 11: Alternating series, Leibnitz test; Absolute and conditional convergence.
[2] (Section 8.3).
Week 12: Definition of power series, Radius and interval of convergence, Cauchy-Hadamard theorem.
[3] Chapter 4 [Article 23, 23.1 (without proof)].
[1] Chapter 9 [9.4.7 to 9.4 .9 (without proof)].
Week 13: Statement and illustration of term-by-term differentiation, Integration of power series and Abel's theorem.
[3] Chapter 4 (Article 26).
Week 14: Power series expansions for $e^{x}, \sin x, \cos x, \log (1+x)$ and their properties.
[3] Chapter 7 (Article 37).
[1] Chapter 9 (9.4.14).

## Facilitating the Achievement of Course Learning Outcomes

| $\begin{aligned} & \mathrm{U} \\ & \mathrm{~N} \\ & \hline \end{aligned}$ | Outcomes |  | Assessment Tasks |
| :---: | :---: | :---: | :---: |
| 1. | Understand the real numbers and their basic properties. Be familiar with convergent and Cauchy sequences. | (i) Each topic to be explained with examples. <br> (ii) Students to be involved in discussions and encouraged to ask questions. <br> (iii) Students to be given homework/assignments. <br> (iv) Students to be encouraged to give short presentations. | - Student presentations. <br> - Participation in discussions. <br> - Assignments and class tests. <br> - Mid-term examinations. <br> - End-term examinations. |
| 2. | Test the convergence and divergence of infinite series of real numbers. |  |  |
| 3. | Learn about power series expansion of some elementary functions. |  |  |

Keywords: Countable sets, Completeness property, Bolzano-Weierstrass theorem, Cauchy sequence, Cauchy's root test, D'Alembert's ratio test, Cauchy-Hadamard theorem, Abel's theorem.


## Acknowledgments

The following members were actively involved in drafting the LOCF syllabus of Mathematics of Generic Elective Courses for Honours Courses, University of Delhi.

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