Discipline Specific Elective (DSE) Course - 2

Any *one* of the following (at least *two* shall be offered by the college):

DSE-2 (i): Probability Theory and Statistics
DSE-2 (ii): Discrete Mathematics
DSE-2 (iii): Cryptography and Network Security

DSE-2 (i): Probability Theory and Statistics

Total Marks: 100 (Theory: 75 + Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: To make the students familiar with the basic statistical concepts and tools which are needed to study situations involving uncertainty or randomness. The course intends to render the students to several examples and exercises that blend their everyday experiences with their scientific interests.

Course Learning Outcomes: This course will enable the students to:

i) Learn about probability density and moment generating functions.
ii) Know about various univariate distributions such as Bernoulli, Binomial, Poisson, gamma and exponential distributions.
iii) Learn about distributions to study the joint behavior of two random variables.
iv) Measure the scale of association between two variables, and to establish a formulation helping to predict one variable in terms of the other, i.e., correlation and linear regression.
v) Understand central limit theorem, which helps to understand the remarkable fact that**:** the empirical frequencies of so many natural populations, exhibit a bell-shaped curve, i.e., a normal distribution.

Unit 1: Probability Functions and Moment Generating Function
Sample space, Probability set function, Real random variables - Discrete and continuous, Cumulative distribution function, Probability mass/density functions, Transformations, Mathematical expectation, Moments, Moment generating function, Characteristic function.

Unit 2: Univariate Discrete and Continuous Distributions
Discrete distributions**:** Uniform, Bernoulli, Binomial, Negative binomial, Geometric and Poisson; Continuous distributions**:** Uniform, Gamma, Exponential, Chi-square, Beta and normal; Normal approximation to the binomial distribution.

Unit 3: Bivariate Distribution
Joint cumulative distribution function and its properties, Joint probability density function, Marginal distributions, Expectation of function of two random variables, Joint moment generating function, Conditional distributions and expectations.

Unit 4: Correlation, Regression and Central Limit Theorem

The Correlation coefficient, Covariance, Calculation of covariance from joint moment generating function, Independent random variables, Linear regression for two variables, Method of least squares, Bivariate normal distribution, Chebyshev's theorem, Strong law of large numbers, Central limit theorem and weak law of large numbers.

References:

1. Hogg, Robert V., McKean, Joseph W., & Craig, Allen T. (2013). *Introduction to Mathematical Statistics* (7th ed.). Pearson Education, Inc.
2. Miller, Irwin & Miller, Marylees. (2014). John E. Freund's *Mathematical Statistics with*
   Applications (8th ed.). Pearson. Dorling Kindersley (India).
3. Ross, Sheldon M. (2014). *Introduction to Probability Models* (11th ed.). Elsevier Inc.

Additional Reading:

i. Mood, A. M., Graybill, F. A. & Boes, D. C. (1974). *Introduction to the Theory of Statistics* (3rd ed.). McGraw-Hill Education Pvt. Ltd. Indian Edition (2017).

Teaching Plan (DSE-2 (i): Probability Theory and Statistics):

Weeks 1 and 2: Sample space, Probability set function and examples, Random variable, Probability mass/density function, Cumulative distribution function and its properties.

[1] Chapter 1 (Sections 1.1, 1.3 and 1.5).

Week 3 and 4: Discrete and continuous random variables, and Transformations. Expectation of random variables, and some special expectations**:** Mean, Variance, Standard deviation, Moments and moment generating function, Characteristic function.

[1] Chapter 1 (Sections 1.6 to 1.9).

Week 5: The discrete distributions - Uniform, Bernoulli and binomial.

[2] Chapter 5 (Sections 5.2 to 5.4).

Week 6: The discrete distributions - negative Binomial, Geometric and Poisson.

[2] Chapter 5 (Sections 5.5 and 5.7).

Week 7: The continuous distributions - Uniform, Gamma, Exponential, Chi-square and Beta.

[2] Chapter 6 (Sections 6.2 to 6.4).

Week 8: Normal distribution, and normal approximation to the binomial distribution.

[2] Chapter 6 (Sections 6.5 and 6.6).

Weeks 9 and 10: Random vector**:** Discrete and continuous, Joint cumulative distribution function and its properties, Joint probability mass/density function, Marginal probability mass function, and expectation of two random variables, Joint moment generating function, Conditional distributions and expectations.

[1] Chapter 2 (Sections 2.1 and 2.3).

Week 11: Correlation coefficient, Covariance, Calculation of covariance from joint moment generating function, Independent random variables.

[1] Chapter 2 (Sections 2.4 and 2.5).

Week 12: Linear regression for two variables, and the method of least squares.

[2] Chapter 14 (Sections 14.1 to 14.3).

Week 13: Bivariate normal distribution; Chebyshev's theorem.

[2] Chapter 6 (Section 6.7), and Chapter 4 (Section 4.4).

Week 14: Statement and interpretation of the strong law of large numbers, Central limit theorem and the weak law of large numbers.

[3] Chapter 2 (Section 2.8, and Exercise 76, Page 89).

Facilitating the Achievement of Course Learning Outcomes

Unit No.	Course Learning Outcomes	Teaching and Learning Activity	Assessment Tasks
1.	Learn about probability density and moment generating functions.	(i) Each topic to be explained with examples. (ii) Students to be involved in discussions and encouraged to ask questions. (iii) Students to be given homework/ assignments. (iv) Students to be encouraged to give short presentations.	• Presentations and participation in discussions. • Assignments and class tests. • Mid-term examinations. • End-term examinations.
2.	Know about various univariate distributions such as Bernoulli, Binomial, Poisson, gamma and exponential distributions.		
3.	Learn about distributions to study the joint behavior of two random variables.		
4.	Measure the scale of association between two variables, and to establish a formulation helping to predict one variable in terms of the other, i.e., correlation and linear regression. Understand central limit theorem, which helps to understand the remarkable fact that**:** the empirical frequencies of so many natural populations, exhibit a bell-shaped curve, i.e., a normal distribution.		

Keywords: Chebyshev's theorem, Correlation, Distributions, Distribution functions, Expectation, moments, Random variable, Regression.