Semester-II
Generic Elective (GE) Course -Mathematics

Any *one* of the following:

GE-2: Linear Algebra

GE-2: Discrete Mathematics

GE-2: Linear Algebra

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The objective of the course is to introduce the concept of vectors in $\mathbb{R}^n$. The concepts of linear independence and dependence, rank and linear transformations has been explained through matrices. Various applications of vectors in computer graphics and movements in a plane has also been introduced.

Course Learning Outcomes: This course will enable the students to:

i) Visualize the space $\mathbb{R}^n$ in terms of vectors and the interrelation of vectors with matrices, and their application to computer graphics.
ii) Familiarize with concepts in vector spaces, namely, basis, dimension and minimal spanning sets.
iii) Learn about linear transformations, transition matrix and similarity.
iv) Learn about orthogonality and to find approximate solution of inconsistent system of linear equations.

Unit 1: Euclidean space $\mathbb{R}^n$ and Matrices
Fundamental operation with vectors in Euclidean space $\mathbb{R}^n$, Linear combination of vectors, Dot product and their properties, Cauchy–Schwarz inequality, Triangle inequality, Projection vectors, Some elementary results on vectors in $\mathbb{R}^n$, Matrices**:** Gauss–Jordan row reduction, Reduced row echelon form, Row equivalence, Rank, Linear combination of vectors, Row space, Eigenvalues, Eigenvectors, Eigenspace, Characteristic polynomials, Diagonalization of matrices; Definition and examples of vector spaces, Some elementary properties of vector spaces, Subspace, Span, Spanning set for an eigenspace, Linear independence and linear dependence of vectors, Basis and dimension of a vector space, Maximal linearly independent sets, Minimal spanning sets; Application of rank**:** Homogenous and non-homogenous systems of linear equations; Coordinates of a vector in ordered basis, Transition matrix.

Unit 2: Linear Transformations and Computer Graphics
Linear transformations**:** Definition and examples, Elementary properties, The matrix of a linear transformation, Linear operator and similarity; Application**:** Computer graphics, Fundamental movements in a plane, Homogenous coordinates, Composition of movements; Kernel and range of a linear transformation, Dimension theorem, One to one and onto linear transformations, Invertible linear transformations, Isomorphism, Isomorphic vector spaces (to $\mathbb{R}^n$).

Unit 3: Orthogonality and Least Square Solutions
Orthogonal and orthonormal vectors, Orthogonal and orthonormal bases, Orthogonal complement, Projection theorem, Orthogonal projection onto a subspace; Application**:** Least square solutions for inconsistent systems, Non-unique least square solutions.

References:

1. Andrilli, S., & Hecker, D. (2016). *Elementary Linear Algebr*a (5th ed.). Elsevier India.
2. Kolman, Bernard, & Hill, David R. (2001). *Introductory Linear Algebra with Applications* (7th ed.). Pearson Education, Delhi. First Indian Reprint 2003.

Additional Reading:

i. Lay, David C., Lay, Steven R., & McDonald, Judi J. (2016). *Linear Algebra and its Applications* (5th ed.). Pearson Education.

Teaching Plan (GE-2: Linear Algebra):

Week 1: Fundamental operation with vectors in Euclidean space $\mathbb{R}^n$, Linear combination of vectors, dot product and their properties, Cauchy–Schwarz inequality, Triangle inequality, Projection vectors.
[1] Chapter 1 (Sections 1.1 and 1.2).

Week 2: Some elementary results on vectors in $\mathbb{R}^n$; Matrices**:** Gauss–Jordan row reduction, Reduced row echelon form, Row equivalence, Rank.
[1] Chapter 1 [Section 1.3 (Pages 34 to 44)].
[1] Chapter 2 [Sections 2.2 (up to Page 111), 2.3 (up to Page 122, Statement of Theorem 2.5)].

Week 3: Linear combination of vectors, Row space, Eigenvalues, Eigenvectors, Eigenspace, Characteristic polynomials, Diagonalization of matrices.
[1] Chapter 2 [Section 2.3 (Pages 122-132, Statements of Lemma 2.8, Theorem 2.9)], Chapter 3 (Section 3.4).

Week 4: Definition and examples of vector spaces, Some elementary properties of vector spaces.
[1] Chapter 4 (Section 4.1).

Week 5 and 6: Subspace, Span, Spanning set for an eigenspace, Linear independence and dependence, Basis and dimension of a vector space, Maximal linearly independent sets, Minimal spanning sets.
[1] Chapter 4 (Sections 4.2 to 4.5, Statements of technical Lemma 4.10 and Theorem 4.12).

Week 7: Application of rank**:** Homogenous and non-homogenous systems of linear equations; Coordinates of a vector in ordered basis, Transition matrix.
[2] Chapter 6 [Sections 6.6 (Pages 287 to 291), and 6.7 (Statement of Theorem 6.15 and examples)].

Week 8: Linear transformations**:** Definition and examples, Elementary properties.
[1] Chapter 5 (Section 5.1).

Week 9: The matrix of a linear transformation, Linear operator and similarity.
[1] Chapter 5 [Section 5.2 (Statements of Theorem 5.5 and Theorem 5.6)].

Week 10: Application**:** Computer graphics, Fundamental movements in a plane, Homogenous coordinates, Composition of movements.
[1] Chapter 8 (Section 8.8).

Week 11: Kernel and range of a linear transformation, Statement of the dimension theorem and examples.
[1] Chapter 5 (Sections 5.3).

Week 12: One to one and onto linear transformations, Invertible linear transformations, isomorphism, isomorphic vector spaces (to $\mathbb{R}^n$).
[1] Chapter 5 [Sections 5.4, 5.5 (up to Page 378, Statements of Theorem 5.15, and Theorem 5.16)]

Week 13 and 14: Orthogonal and orthonormal vectors, orthogonal and orthonormal bases, orthogonal complement, statement of the projection theorem and examples. Orthogonal projection onto a subspace. Application: Least square solutions for inconsistent systems, non-unique least square solutions.

[1] Chapter 6 [Sections 6.1 (up to Example 3, Page 416, Statement of Theorem 6.3), 6.2 (up to Page 435, and Pages 439 to 443, and Statement of Theorem 6.12)].

[1] Chapter 8 [Section 8.9 (up to Page 593, Statement of Theorem 8.13).

Facilitating the Achievement of Course Learning Outcomes

Unit No.	Course Learning Outcomes	Teaching and Learning Activity	Assessment Tasks
1.	Visualize the space $\mathbb{R}^n$ in terms of vectors and the interrelation of vectors with matrices, and their application to computer graphics. Familiarize with concepts in vector spaces, namely, basis, dimension and minimal spanning sets.	(i) Each topic to be explained with examples. (ii) Students to be involved in discussions and encouraged to ask questions. (iii) Students to be given homework/assignments. (iv) Students to be encouraged to give short presentations.	• Student presentations. • Participation in discussions. • Assignments and class tests. • Mid-term examinations. • End-term examinations.
2.	Learn about linear transformations, transition matrix and similarity.		
3.	Learn about orthogonality and to find approximate solution of inconsistent system of linear equations.		

Keywords: Cauchy–Schwarz inequality Gauss–Jordan row reduction Basis and dimension of vector spaces, matrix of linear transformations, Orthogonality, Orthonormality, Least square solutions.