

Unit – 3**(18 hours)****First and Second Order Partial Differential Equations**

Classification and Construction of first-order partial differential equations, Method of characteristics and general solutions of first-order partial differential equations, Canonical forms and method of separation of variables for first order partial differential equations; Classification and reduction to canonical forms of second-order linear partial differential equations and their general solutions.

Essential Readings

1. Myint-U, Tyn and Debnath, Lokenath (2007). Linear Partial Differential Equations for Scientist and Engineers (4th ed.). Birkhäuser. Indian Reprint.
2. Ross, Shepley L. (1984). Differential Equations (3rd ed.). John Wiley & Sons.

Suggestive Readings

- Edwards, C. Henry, Penney, David E., & Calvis, David T. (2015). Differential Equations and Boundary Value Problems: Computing and Modeling (5th ed.). Pearson Education.
- Kreyszig, Erwin. (2011). Advanced Engineering Mathematics (10th ed.). Wiley India.
- Sneddon I. N. (2006). Elements of Partial Differential Equations. Dover Publications.

GENERIC ELECTIVES-GE-3(ii): LATTICES AND NUMBER THEORY**CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Lattices and Number Theory	4	3	1	0	Class XII pass with Mathematics	Nil

Learning Objectives

The primary objective of this course is to introduce:

- The concepts of ordered sets, lattices, sublattices and homomorphisms between lattices.
- Distributive lattices along with Boolean algebra and their applications in the real-world.
- Divisibility theory of congruences along with some applications.
- The number-theoretic functions and quadratic reciprocity law.

Learning Outcomes

This course will enable the students to:

- Understand the notion of ordered sets. Learn about lattices, distributive lattices, sublattices and homomorphisms between lattices.
- Become familiar with Boolean algebra, Boolean polynomials, switching circuits and their applications.
- Learn the concept of Karnaugh diagrams and Quinn–McCluskey method which gives an aid to apply truth tables in real-world problems.

- Learn about some fascinating properties of prime numbers, and some of the open problems in number theory, viz., Goldbach conjecture etc.
- Know about modular arithmetic and number-theoretic functions like Euler's Phi-function.
- Find quadratic residues and nonresidues modulo primes using Gauss's Quadratic Reciprocity Law.

SYLLABUS OF GE-3(ii)

Unit – 1

(21 hours)

Partially Ordered Sets and Lattices

Definitions, Examples and basic properties of partially ordered sets, Order isomorphism, Hasse Diagram, Maximal and minimal elements, Dual of an ordered set, Duality principle; Statements of Well Ordering Principle and Zorn's Lemma; Lattices as ordered sets, Lattices as algebraic structures, Sublattices, Products and homomorphisms, Distributive lattices, Boolean algebras, Boolean polynomials, Minimal forms of Boolean polynomials, Quinn-McCluskey method, Karnaugh diagrams, Switching circuits and applications.

Unit – 2

(12 hours)

Divisibility and Theory of Congruences

The division algorithm: GCD, The Euclidean algorithm, Diophantine equation $ax + by = c$
Primes: The Fundamental Theorem of Arithmetic, Infinitude of primes, Twin primes and Goldbach conjecture.

The theory of congruences: Basic properties and applications, Linear congruences and the Chinese Remainder Theorem, Fermat's Little Theorem and Wilson's Theorem.

Unit – 3

(12 hours)

Number-Theoretic Functions, Primitive roots and Quadratic Reciprocity Law

Number-Theoretic Functions: Sum and number of divisors, Euler's Phi-function and Euler's generalization of Fermat's Little Theorem.

Primitive roots: The order of an integer modulo n , and primitive roots of an integer.

Quadratic Reciprocity Law: Quadratic residue and nonresidue, Euler's Criterion, The Legendre symbol and its properties and Quadratic Reciprocity Law.

Essential Readings

1. Davey, B A., & Priestley, H. A. (2002). Introduction to Lattices and Order (2nd ed.), Cambridge University Press, Cambridge.
2. Lidl, Rudolf & Pilz, Günter. (1998). Applied Abstract Algebra (2nd ed.), Undergraduate Texts in Mathematics, Springer. (SIE), Indian Reprint 2004.
3. Burton, David M. (2012). Elementary Number Theory (7th ed.), Mc-Graw Hill Education Pvt. Ltd. Indian Reprint.

Suggestive Readings

- Rosen, Kenneth H. (2019). Discrete Mathematics and its Applications (8th ed.), Indian adaptation by Kamala Krithivasan. McGraw-Hill Education. Indian Reprint 2021.
- Goodaire, Edgar G., & Parmenter, Michael M. (2006). Discrete Mathematics with Graph Theory (3rd ed.). Pearson Education Pvt. Ltd. Indian Reprint 2018.
- Jones, G. A., & Jones, J. Mary. (2005). Elementary Number Theory. Springer Undergraduate Mathematics Series (SUMS). Indian Reprint.

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