BMATH410: Ring Theory & Linear Algebra-I

Total Marks: 100 (Theory: 75 and Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The objective of this course is to introduce the fundamental theory of two objects, namely - rings and vector spaces, and their corresponding homomorphisms.

Course Learning Outcomes: The course will enable the students to:
  i) Learn about the fundamental concept of rings, integral domains and fields.
  ii) Know about ring homomorphisms and isomorphisms theorems of rings.
  iii) Learn about the concept of linear independence of vectors over a field, and the dimension of a vector space.
  iv) Basic concepts of linear transformations, dimension theorem, matrix representation of a linear transformation, and the change of coordinate matrix.

Unit 1: Introduction of Rings
Definition and examples of rings, Properties of rings, Subrings, Integral domains and fields, Characteristic of a ring, Ideals, Ideal generated by a subset of a ring, Factor rings, Operations on ideals, Prime and maximal ideals.

Unit 2: Ring Homomorphisms
Ring homomorphisms, Properties of ring homomorphisms, First, Second and Third Isomorphism theorems for rings, The Field of quotients.

Unit 3: Introduction of Vector Spaces
Vector spaces, Subspaces, Algebra of subspaces, Quotient spaces, Linear combination of vectors, Linear span, Linear independence, Basis and dimension, Dimension of subspaces.

Unit 4: Linear Transformations
Linear transformations, Null space, Range, Rank and nullity of a linear transformation, Matrix representation of a linear transformation, Algebra of linear transformations, Isomorphisms, Isomorphism theorems, Invertibility and the change of coordinate matrix.

References:
  1. Gallian, Joseph. A. (2013). *Contemporary Abstract Algebra* (8th ed.). Cengage Learning India Private Limited. Delhi. Fourth impression, 2015.
  2. Friedberg, Stephen H., Insel, Arnold J., & Spence, Lawrence E. (2003). *Linear Algebra* (4th ed.). Prentice-Hall of India Pvt. Ltd. New Delhi.

Additional Readings:
  i. Dummit, David S., & Foote, Richard M. (2016). *Abstract Algebra* (3rd ed.). Student Edition. Wiley India.
  ii. Herstein, I. N. (2006). *Topics in Algebra* (2nd ed.). Wiley Student Edition. India.
  iii. Hoffman, Kenneth, & Kunze, Ray Alden (1978). *Linear Algebra* (2nd ed.). Prentice-Hall of India Pvt. Limited. Delhi. Pearson Education India Reprint, 2015.

Teaching Plan (BMATH410: Ring Theory & Linear Algebra-I):
Week 1: Definition and examples of rings, Properties of rings, Subrings.
      [1] Chapter 12.
Week 2: Integral domains and fields, Characteristic of a ring.
      [1] Chapter 13.
Week 3 and 4: Ideals, Ideal generated by a subset of a ring, Factor rings, Operations on ideals, Prime and maximal ideals.
      [1] Chapter 14.
Week 5: Ring homomorphisms, Properties of ring homomorphisms.
      [1] Chapter 15 (up to Theorem 15.2).
Week 6: First, Second and Third Isomorphism theorems for rings, The field of quotients.
      [1] Chapter 15 (Theorems 15.3 to 15.6, Examples 10 to 12), and Exercises 3 and 4 on Page 347.
      Week 7: Vector spaces, Subspaces, Algebra of subspaces.
      [2] Chapter 1 (Sections 1.2 and 1.3).
Week 8: Linear combination of vectors, Linear span, Linear independence.
      [2] Chapter 1 (Sections 1.4 and 1.5).
Weeks 9 and 10: Bases and dimension. Dimension of subspaces.
      [2] Chapter 1 (Section 1.6).
Week 11: Linear transformations, Null space, Range, Rank and nullity of a linear transformation.
      [2] Chapter 2 (Section 2.1).
Weeks 12 and 13: Matrix representation of a linear transformation, Algebra of linear transformations.
      [2] Chapter 2 (Sections 2.2 and 2.3).
Week 14: Isomorphisms, Isomorphism theorems, Invertibility and the change of coordinate matrix.
      [2] Chapter 2 (Sections 2.4 and 2.5).

Facilitating the achievement of Course Learning Outcomes

Unit No.	Course Learning Outcomes	Teaching and Learning Activity	Assessment Tasks
1.	Learn about the fundamental concept of rings, integral domains and fields.	(i) Each topic to be explained with examples. (ii) Students to be involved in discussions and encouraged to ask questions. (iii) Students to be given homework/assignments. (iv) Students to be encouraged to give short presentations.	• Presentations and participation in discussions. • Assignments and class tests. • Mid-term examinations. • End-term examinations.
2.	Know about ring homomorphisms and isomorphisms theorems of rings.		
3.	Learn about the concept of linear independence of vectors over a field, and the dimension of a vector space.		
4.	Basic concepts of linear transformations, dimension theorem, matrix representation of a linear transformation, and the change of coordinate matrix.		

Keywords: Basis and dimension of a vector space, Characteristic of a ring, Integral domain, Isomorphism theorems for rings, Linear transformations, Prime and maximal ideals, Quotient field, Vector space.