# DSE-3 (iii): Biomathematics

**Total Marks:** 100 (Theory: 75 + Internal Assessment: 25)
**Workload:** 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

**Course Objectives:** The focus of the course is on scientific study of normal functions in living systems. The emphasis is on exposure to nonlinear differential equations with examples such as heartbeat, chemical reactions and nerve impulse transmission. The basic concepts of the probability to understand molecular evolution and genetics have also been applied.

**Course Learning outcomes:** Apropos conclusion of the course will empower the student to:

i) Learn the development, analysis and interpretation of bio mathematical models such as population growth, cell division, and predator-prey models.
ii) Learn about the mathematics behind heartbeat model and nerve impulse transmission model.
iii) Appreciate the theory of bifurcation and chaos.
iv) Learn to apply the basic concepts of probability to molecular evolution and genetics.

**Unit 1: Modeling Biological Phenomenon**
Population growth, Administration of drugs, Cell division, Systems of linear ordinary differential equations, Heartbeat, Nerve impulse transmission, Chemical reactions, Predator-prey models.

**Unit 2: Mathematics of Heart Physiology and Nerve Impulse Transmission**
Stability and oscillations**:** Epidemics, Phase plane and Jacobian matrix, Local stability, Stability, Limit cycles, Forced oscillations; Mathematics of heart physiology**:** local model, threshold effect, phase plane analysis and heartbeat model, A model of the cardiac pacemaker; Mathematics of nerve impulse transmission**:** excitability and repetitive firing, travelling waves.

**Unit 3: Bifurcation and Chaos**
Bifurcation, Bifurcation of a limit cycle, Discrete bifurcation and period-doubling, Chaos, Stability of limit cycles, Poincaré plane.

**Unit 4: Modeling Molecular Evolution and Genetics**
Modelling Molecular Evolution**:** Matrix models of base substitutions for DNA sequences, Jukes–Cantor model, Kimura models, Phylogenetic distances; Constructing Phylogenetic Trees**:** Phylogenetic trees, Unweighted pair-group method with arithmetic means (UPGMA), Neighbor joining method; Genetics**:** Mendelian genetics, Probability distributions in genetics.

**References:**

1. Allman, Elizabeth S., & Rhodes, John A. (2004). *Mathematical Models in Biology: An Introduction*. Cambridge University Press.
2. Jones, D. S., Plank, M. J., & Sleeman, B. D. (2009). *Differential Equations and Mathematical Biology* (2nd ed.). CRC Press, Taylor & Francis Group, LLC.

**Additional Readings:**

i. Murray, J. D. (2002). *An Introduction to Mathematical Biology* (3rd ed.). Springer.
ii. Myint-U, Tyn (1977). *Ordinary Differential Equations*. Elsevier North-Holland, Inc.
iii. Simmons, George F., & Krantz, Steven G. (2015). *Differential Equations*. McGraw-Hill Education. Indian Reprint.
iv. Strogatz, Steven H. (2009). *Nonlinear Dynamics and Chaos* (2nd ed.). Perseus Book Publishing. LLC. Sarat Publication, Kolkata, India.

**Teaching Plan (DSE-3 (iii): Biomathematics):**

**Week 1:** Population growth, Administration of drugs, Cell division, Systems of linear ordinary differential equations.
[2] Chapter 1 (Sections 1.1 to 1.3) and Chapter 3 (An overview of the methods in Sections 3.1 to 3.6).

**Week 2:** Heartbeat, Nerve impulse transmission.
[2] Chapter 4 (Sections 4.2, and 4.3).

**Week 3:** Chemical reactions, Predator-prey models, Epidemics (mathematical model).
[2] Chapter 4 (Sections 4.4 and 4.5) and Chapter 5 (Section 5.2)

**Week 4:** The phase plane and Jacobian matrix, Local stability.
[2] Chapter 5 (Sections 5.3 and 5.4).

**Week 5:** Stability, Limit cycles.
[2] Chapter 5 [Sections 5.5, and 5.6 (up to Page number 137)].

**Week 6:** Limit cycle criterion and Poincaré−Bendixson Theorem (interpretation only, with Example 5.6.1), Forced oscillations.
[2] Chapter 5 [Section 5.6 (Page number 137 to 138) and Section 5.7).

**Week 7:** Mathematics of heart physiology: local model, threshold effect, phase plane analysis and heartbeat model.
[2] Chapter 6 (Sections 6.1 to 6.3).

**Week 8:** A model of the cardiac pacemaker, Excitability and repetitive firing.
[2] Chapter 6 (Section 6.5) and Chapter 7 (Section 7.1).

**Week 9:** Travelling waves, Bifurcation, Bifurcation of a limit cycle.
[2] Chapter 7 (Section 7.2), and Chapter 13 (Sections 13.1 and 13.2).

**Weeks 10 and 11:** Discrete bifurcation and period-doubling, Chaos, Stability of limit cycles, Poincaré plane.
[2] Chapter 13 (Sections 13.3 to 13.6).

**Week 12:** Matrix models of base substitutions for DNA sequences, Jukes−Cantor model, Kimura models, Phylogenetic distances.
[1] Chapter 4 (Sections 4.4 and 4.5).

**Week 13:** Constructing phylogenetic trees: phylogenetic trees, unweighted pair-group method with arithmetic means (UPGMA), Neighbor joining method.
[1] Chapter 5 (Sections 5.1 to 5.3).

**Week 14:** Genetics**:** Mendelian genetics, probability distributions in genetics.
[1] Chapter 6 [Sections 6.1 and 6.2 (up to Equation 6.2 only)].

## Facilitating the Achievement of Course Learning Outcomes

| Unit No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
|---|---|---|---|
| 1. | Learn the development, analysis and interpretation of bio mathematical models such as | (i) Each topic to be explained with examples.<br>(ii) Students to be involved in | • Student presentations. |

<table>
<tr><td></td><td>population growth, cell division, and predator-prey models.</td><td rowspan="4">discussions and encouraged to ask questions.<br>(iii) Students to be given homework/assignments.<br>(iv) Students to be encouraged to give short presentations.</td><td rowspan="4">• Participation in discussions.<br>• Assignments and class tests.<br>• Mid-term examinations.<br>• End-term examinations.</td></tr>
<tr><td>2.</td><td>Learn about the mathematics behind heartbeat model and nerve impulse transmission model.</td></tr>
<tr><td>3.</td><td>Appreciate the theory of bifurcation and chaos.</td></tr>
<tr><td>4.</td><td>Learn to apply the basic concepts of probability to molecular evolution and genetics.</td></tr>
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**Keywords:** Bifurcation and chaos, Forced oscillations, Jukes–Cantor model, Kimura model, Limit cycles, Phase plane, Phylogenetic distances, Stability, UPGMA.