BMATH306: Group Theory-I

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The objective of the course is to introduce the fundamental theory of groups and their homomorphisms. Symmetric groups and group of symmetries are also studied in detail. Fermat's Little theorem as a consequence of the Lagrange's theorem on finite groups.

Course Learning Outcomes: The course will enable the students to:

i) Recognize the mathematical objects that are groups, and classify them as abelian, cyclic and permutation groups, etc.
ii) Link the fundamental concepts of groups and symmetrical figures.
iii) Analyze the subgroups of cyclic groups and classify subgroups of cyclic groups.
iv) Explain the significance of the notion of cosets, normal subgroups and factor groups.
v) Learn about Lagrange's theorem and Fermat's Little theorem.
vi) Know about group homomorphisms and group isomorphisms.

Unit 1: Groups and its Elementary Properties
Symmetries of a square, Dihedral groups, Definition and examples of groups including permutation groups and quaternion groups (illustration through matrices), Elementary properties of groups.

Unit 2: Subgroups and Cyclic Groups
Subgroups and examples of subgroups, Centralizer, Normalizer, Center of a group, Product of two subgroups; Properties of cyclic groups, Classification of subgroups of cyclic groups.

Unit 3: Permutation Groups and Lagrange's Theorem
Cycle notation for permutations, Properties of permutations, Even and odd permutations, Alternating groups; Properties of cosets, Lagrange's theorem and consequences including Fermat's Little theorem; Normal subgroups, Factor groups, Cauchy's theorem for finite abelian groups.

Unit 4: Group Homomorphisms
Group homomorphisms, Properties of homomorphisms, Group isomorphisms, Cayley's theorem, Properties of isomorphisms, First, Second and Third isomorphism theorems for groups.

Reference:

1. Gallian, Joseph. A. (2013). *Contemporary Abstract Algebra* (8th ed.). Cengage Learning India Private Limited, Delhi. Fourth impression, 2015.

Additional Reading:

i. Rotman, Joseph J. (1995). *An Introduction to The Theory of Groups* (4th ed.). Springer-Verlag, New York.

Teaching Plan (BMATH306: Group Theory-I):

Week 1: Symmetries of a square, Dihedral groups, Definition and examples of groups including permutation groups and quaternion groups (illustration through matrices).

[1] Chapter 1.

Week 2: Definition and examples of groups, Elementary properties of groups.

[1] Chapter 2.

Week 3: Subgroups and examples of subgroups, Centralizer, Normalizer, Center of a Group, Product of two subgroups.

[1] Chapter 3.

Weeks 4 and 5: Properties of cyclic groups. Classification of subgroups of cyclic groups.

[1] Chapter 4

Weeks 6 and 7: Cycle notation for permutations, Properties of permutations, Even and odd permutations, Alternating group.

[1] Chapter 5 (up to Page 110).

Weeks 8 and 9: Properties of cosets, Lagrange's theorem and consequences including Fermat's Little theorem.

[1] Chapter 7 (up to Example 6, Page 150).

Week 10: Normal subgroups, Factor groups, Cauchy's theorem for finite abelian groups.

[1] Chapters 9 (Theorem 9.1, 9.2, 9.3 and 9.5, and Examples 1 to 12).

Weeks 11 and 12: Group homomorphisms, Properties of homomorphisms, Group isomorphisms, Cayley's theorem.

[1] Chapter 10 (Theorems 10.1 and 10.2, Examples 1 to 11).

[1] Chapter 6 (Theorem 6.1, and Examples 1 to 8).

Weeks 13 and 14: Properties of isomorphisms, First, Second and Third isomorphism theorems.

[1] Chapter 6 (Theorems 6.2 and 6.3), Chapter 10 (Theorems 10.3, 10.4, Examples 12 to 14, and Exercises 41 and 42 for second and third isomorphism theorems for groups).

Facilitating the Achievement of Course Learning Outcomes

Unit No.	Course Learning Outcomes	Teaching and Learning Activity	Assessment Tasks
1.	Recognize the mathematical objects that are groups, and classify them as abelian, cyclic and permutation groups, etc. Link the fundamental concepts of groups and symmetrical figures.	(i) Each topic to be explained with examples. (ii) Students to be involved in discussions and encouraged to ask questions. (iii) Students to be given homework/assignments. (iv) Students to be encouraged to give short presentations.	• Presentations and participation in discussions. • Assignments and class tests. • Mid-term examinations. • End-term examinations.
2.	Analyze the subgroups of cyclic groups and classify subgroups of cyclic groups.		
3.	Explain the significance of the notion of cosets, normal subgroups and factor groups. Learn about Lagrange's theorem and Fermat's Little theorem.		
4.	Know about group homomorphisms and group isomorphisms.		

Keywords: Cauchy's theorem for finite Abelian groups, Cayley's theorem, Centralizer, Cyclic group, Dihedral group, Group homomorphism, Lagrange's theorem, Normalizer, Permutations.