BMATH409: Riemann Integration & Series of Functions

Total Marks: 100 (Theory: 75 and Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: To understand the integration of bounded functions on a closed and bounded interval and its extension to the cases where either the interval of integration is infinite, or the integrand has infinite limits at a finite number of points on the interval of integration. The sequence and series of real valued functions, and an important class of series of functions (i.e., power series).

Course Learning Outcomes: The course will enable the students to:

i) Learn about some of the classes and properties of Riemann integrable functions, and the applications of the Fundamental theorems of integration.
ii) Know about improper integrals including, beta and gamma functions.
iii) Learn about Cauchy criterion for uniform convergence and Weierstrass M-test for uniform convergence.
iv) Know about the constraints for the inter-changeability of differentiability and integrability with infinite sum.
v) Approximate transcendental functions in terms of power series as well as, differentiation and integration of power series.

Unit 1: Riemann Integration
Definition of Riemann integration, Inequalities for upper and lower Darboux sums, Necessary and sufficient conditions for the Riemann integrability, Definition of Riemann integration by Riemann sum and equivalence of the two definitions, Riemann integrability of monotone functions and continuous functions, Properties of Riemann integrable functions, Definitions of piecewise continuous and piecewise monotone functions and their Riemann integrability, intermediate value theorem for integrals, Fundamental theorems (I and II) of calculus, and the integration by parts.

Unit 2: Improper Integral
Improper integrals of Type-I, Type-II and mixed type, Convergence of beta and gamma functions, and their properties.

Unit 3: Sequence and Series of Functions
Pointwise and uniform convergence of sequence of functions, Theorem on the continuity of the limit function of a sequence of functions, Theorems on the interchange of the limit and derivative, and the interchange of the limit and integrability of a sequence of functions. Pointwise and uniform convergence of series of functions, Theorems on the continuity, derivability and integrability of the sum function of a series of functions, Cauchy criterion and the Weierstrass M-test for uniform convergence.

Unit 4: Power Series
Definition of a power series, Radius of convergence, Absolute convergence (Cauchy–Hadamard theorem), Uniform convergence, Differentiation and integration of power series, Abel's theorem.

References:

1. Bartle, Robert G., & Sherbert, Donald R. (2015). *Introduction to Real Analysis* (4th ed.). Wiley India Edition. Delhi.
2. Denlinger, Charles G. (2011). *Elements of Real Analysis*. Jones & Bartlett (Student Edition). First Indian Edition. Reprinted 2015.
3. Ghorpade, Sudhir R. & Limaye, B. V. (2006). *A Course in Calculus and Real Analysis*. Undergraduate Texts in Mathematics, Springer (SIE). First Indian reprint.
4. Ross, Kenneth A. (2013). *Elementary Analysis*: *The Theory of Calculus* (2nd ed.). Undergraduate Texts in Mathematics, Springer.

Additional Reading:

i. Bilodeau, Gerald G., Thie, Paul R., & Keough, G. E. (2010). *An Introduction to Analysis* (2nd ed.). Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

Teaching Plan (BMATH409: Riemann Integration & Series of Functions):

Week 1: Definition of Riemann integration, Inequalities for upper and lower Darboux sums.
[4] Chapter 6 [Section 32 (32.1 to 32.4)].

Week 2: Necessary and sufficient conditions for the Riemann integrability, Definition of Riemann integration by Riemann sum and equivalence of the two definitions.
[4] Chapter 6 [Section 32 (32.5 to 32.10)].

Week 3: Riemann integrability of monotone functions and continuous functions, Algebra and properties of Riemann integrable functions.
[4] Chapter 6 [Section 33 (33.1 to 33.6)].

Week 4: Definitions of piecewise continuous and piecewise monotone functions and their Riemann integrability, Intermediate value theorem for integrals.
[4] Chapter 6 [Section 33 (33.7 to 33.10)].

Week 5: First and second fundamental theorems of integral calculus, and the integration by parts.
[4] Chapter 6 [Section 34 (34.1 to 34.3)].

Week 6: Improper integrals of Type-I, Type-II and mixed type.
[2] Chapter 7 [Section 7.8 (7.8.1 to 7.8.18)].

Week 7: Convergence of beta and gamma functions, and their properties.
[3] Pages 405-408.

Week 8: Definitions and examples of pointwise and uniformly convergent sequence of functions.
[1] Chapter 8 [Section 8.1 (8.1.1 to 8.1.10)].

Week 9: Motivation for uniform convergence by giving examples, Theorem on the continuity of the limit function of a sequence of functions.
[1] Chapter 8 [Section 8.2 (8.2.1 to 8.2.2)].

Week 10: The statement of the theorem on the interchange of the limit function and derivative, and its illustration with the help of examples, The interchange of the limit function and integrability of a sequence of functions.
[1] Chapter 8 [Section 8.2 (Theorems 8.2.3 and 8.2.4)].

Week 11: Pointwise and uniform convergence of series of functions, Theorems on the continuity, derivability and integrability of the sum function of a series of functions.
[1] Chapter 9 [Section 9.4 (9.4.1 to 9.4.4)].

Week 12: Cauchy criterion for the uniform convergence of series of functions, and the Weierstrass M-test for uniform convergence.
[2] Chapter 9 [Section 9.4 (9.4.5 to 9.4.6)].

Week 13: Definition of a power series, Radius of convergence, Absolute and uniform convergence of a power series.
[4] Chapter 4 (Section 23).

Week 14: Differentiation and integration of power series, Statement of Abel's theorem and its illustration with the help of examples.

[4] Chapter 4 [Section 26 (26.1 to 26.6)].

Facilitating the Achievement of Course Learning Outcomes

Unit No.	Course Learning Outcomes	Teaching and Learning Activity	Assessment Tasks
1.	Learn about some of the classes and properties of Riemann integrable functions, and the applications of the fundamental theorems of integration.	(i) Each topic to be explained with examples. (ii) Students to be involved in discussions and encouraged to ask questions. (iii) Students to be given homework/assignments. (iv) Students to be encouraged to give short presentations.	• Presentations and participation in discussions. • Assignments and class tests. • Mid-term examinations. • End-term examinations.
2.	Know about improper integrals including, beta and gamma functions.		
3.	Learn about Cauchy criterion for uniform convergence and Weierstrass M-test for uniform convergence. Know about the constraints for the inter-changeability of differentiability and integrability with infinite sum.		
4.	Approximate transcendental functions in terms of power series as well as, differentiation and integration of power series.		

Keywords: Beta function, Gamma function, Improper integral, Power series, Radius of convergence, Riemann integration, Uniform convergence, Weierstrass M-test.