Discipline Specific Elective (DSE) Course -1 (including practicals)

Any *one* of the following (at least *two* shall be offered by the college):

DSE-1 (i): Numerical Analysis
DSE-1 (ii): Mathematical Modeling and Graph Theory
DSE-1 (iii): C++ Programming for Mathematics

DSE-1 (i): Numerical Analysis

Total Marks: 150 (Theory: 75 + Internal Assessment: 25 + Practical: 50)
Workload: 4 Lectures, 4 Periods practical (per week) **Credits:** 6 (4+2)
Duration: 14 Weeks (56 Hrs. Theory + 56 Hrs. practical) **Examination:** 3 Hrs.

Course Objectives: To comprehend various computational techniques to find approximate value for possible root(s) of non-algebraic equations, to find the approximate solutions of system of linear equations and ordinary differential equations. Also, the use of Computer Algebra System (CAS) by which the numerical problems can be solved both numerically and analytically, and to enhance the problem solving skills.

Course Learning Outcomes: The course will enable the students to:

i) Learn some numerical methods to find the zeroes of nonlinear functions of a single variable and solution of a system of linear equations, up to a certain given level of precision.
ii) Know about methods to solve system of linear equations, such as Gauss–Jacobi, Gauss–Seidel and SOR methods.
iii) Interpolation techniques to compute the values for a tabulated function at points not in the table.
iv) Applications of numerical differentiation and integration to convert differential equations into difference equations for numerical solutions.

Unit 1: Methods for Solving Algebraic and Transcendental Equations
Algorithms, Convergence, Bisection method, False position method, Fixed point iteration method, Newton's method and Secant method.

Unit 2:Techniques to Solve Linear Systems
Partial and scaled partial pivoting, LU decomposition and its applications, Iterative methods: Gauss–Jacobi, Gauss–Seidel and SOR methods.

Unit 3: Interpolation
Lagrange and Newton interpolation, Piecewise linear interpolation.

Unit 4: Numerical Differentiation and Integration
First and higher order approximation for first derivative, Approximation for second derivative, Richardson extrapolation method; Numerical integration by closed Newton–Cotes formulae**:** Trapezoidal rule, Simpson's rule and its error analysis; Euler’s method to solve ODE’s, Second order Runge–Kutta Methods**:** Modified Euler’s method, Heun’s method and optimal RK2 method.

Note: Emphasis is to be laid on the algorithms of the above numerical methods. Non programmable scientific calculator may be allowed in the University examination.

Reference:

1. Bradie, Brian. (2006). *A Friendly Introduction to Numerical Analysis*. Pearson Education, India. Dorling Kindersley (India) Pvt. Ltd. Third impression 2011.

Additional Readings:

i. Jain, M. K., Iyengar, S. R. K., & Jain, R. K. (2012). *Numerical Methods for Scientific and Engineering Computation*. (6th ed.). New Age International Publisher, India, 2016.
ii. Gerald, C. F., & Wheatley, P. O. (2008). *Applied Numerical Analysis* (7th ed.). Pearson Education. India.

Practical / Lab work to be performed in Computer Lab:

Use of computer algebra software (CAS), for example Mathematica/MATLAB/Maple/ Maxima/Scilab etc., for developing the following numerical programs:

1. Bisection method
2. Newton–Raphson method
3. Secant method
4. Regula–Falsi method
5. LU decomposition method
6. Gauss–Jacobi method
7. SOR method
8. Gauss–Seidel method
9. Lagrange interpolation
10. Newton interpolation
11. Trapezoidal rule
12. Simpson's rule
13. Euler's method
14. Second order Runge–Kutta methods.

Note: For any of the CAS: Mathematica /MATLAB/ Maple/Maxima/Scilab etc., data types-simple data types, floating data types, character data types, arithmetic operators and operator precedence, variables and constant declarations, expressions, input/output, relational operators, logical operators and logical expressions, control statements and loop statements, Arrays should be introduced to the students.

Teaching Plan (Theory of DSE-l (i): Numerical Analysis):

Week 1: Algorithms, Convergence, Order of convergence and examples.
[1] Chapter 1 (Sections 1.1 and 1.2).

Week 2: Bisection method, False position method and their convergence analysis, Stopping condition and algorithms.
[1] Chapter 2 (Sections 2.1 and 2.2).

Week 3: Fixed point iteration method, its order of convergence and stopping condition.
[1] Chapter 2 (Section 2.3).

Week 4: Newton's method, Secant method, their order of convergence and convergence analysis.
[1] Chapter 2 (Sections 2.4 and 2.5).

Week 5: Examples to understand partial and scaled partial pivoting. LU decomposition.
[1] Chapter 3 (Sections 3.2, and 3.5 up to Example 3.15).

Weeks 6 and 7: Application of LU decomposition to solve system of linear equations. Gauss–Jacobi method, Gauss–Seidel and SOR iterative methods to solve system of linear equations.
[1] Chapter 3 (Sections 3.5 and 3.8).

Week 8: Lagrange interpolation**:** Linear and higher order interpolation, and error in it.
[1] Chapter 5 (Section 5.1).

Weeks 9 and 10: Divided difference and Newton interpolation, Piecewise linear interpolation.
[1] Chapter 5 (Sections 5.3 and 5.5).

Weeks 11 and 12: First and higher order approximation for first derivative and error in the approximation. Second order forward, Backward and central difference approximations for second derivative, Richardson extrapolation method
[1] Chapter 6 (Sections 6.2 and 6.3).

Week 13: Numerical integration**:** Trapezoidal rule, Simpson's rule and its error analysis.
[1] Chapter 6 (Section 6.4).

Week 14: Euler's method to solve ODE's, Second order Runge–Kutta methods**:** Modified Euler's method, Heun's method and optimal RK2 method.
[1] Chapter 7 (Section 7.2 up to Page 562 and Section 7.4, Pages 582-585).

Facilitating the achievement of Course Learning Outcomes

<table>
<tr><th>Unit No.</th><th>Course Learning Outcomes</th><th>Teaching and Learning Activity</th><th>Assessment Tasks</th></tr>
<tr><td>1.</td><td>Learn some numerical methods to find the zeroes of nonlinear functions of a single variable and solution of a system of linear equations, up to a certain given level of precision.</td><td rowspan="4">(i) Each topic to be explained with illustrations.
(ii) Students be encouraged to discover the relevant concepts.
(iii) Students to be given homework/assignments.
(iv) Discuss and solve the theoretical and practical problems in the class.
(v) Students to be encouraged to apply concepts to real world problems.</td><td rowspan="4">• Presentations and class discussions.
• Assignments and class tests.
• Student presentations.
• Mid-term examinations.
• Practical and viva-voce examinations.
• End-term examinations.</td></tr>
<tr><td>2.</td><td>Know about methods to solve system of linear equations, such as Gauss–Jacobi, Gauss–Seidel and SOR methods.</td></tr>
<tr><td>3.</td><td>Interpolation techniques to compute the values for a tabulated function at points not in the table.</td></tr>
<tr><td>4.</td><td>Applications of numerical differentiation and integration to convert differential equations into difference equations for numerical solutions.</td></tr>
</table>

Keywords: Algorithm, Euler's method, Interpolation, Iterative methods, LU decomposition, Newton–Cotes formulae, Order of convergence, Order of a method, Partial pivoting.