DSE-2 (ii): Probability Theory and Statistics

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: To provide a foundation in probability theory and statistics in order to solve applied problems and to prepare for providing the solutions that take account of their everyday experiences with their scientific interests.

Course Learning Outcomes: This course will enable the students to learn:

i) Basic probability axioms and familiar with discrete and continuous random variables.
ii) To measure the scale of association between two variables, and to establish a formulation helping to predict one variable in terms of the other, i.e., correlation and linear regression.
iii) Central limit theorem, which helps to understand the remarkable fact that**:** the empirical frequencies of so many natural populations, exhibit a bell-shaped curve.

Unit 1: Univariate Discrete and Continuous Distributions
Sample space, Probability set function, Real random variables - Discrete and continuous, cumulative distribution function, Probability mass/density functions, Mathematical expectation, Moments, Moment generating function, Characteristic function; Discrete distributions**:** Uniform, Bernoulli, Binomial, Negative binomial, Geometric and Poisson; Continuous distributions**:** Uniform, Gamma, Exponential and Normal; Normal approximation to the binomial distribution.

Unit 2: Bivariate Distribution
Joint cumulative distribution function and its properties, Joint probability density function, Marginal distributions, Expectation of function of two random variables, Joint moment generating function, Conditional distributions and expectations.

Unit 3: Correlation, Regression and Central Limit Theorem
Independent random variables, Covariance, Correlation coefficient; Linear regression for two variables and the method of least squares; Chebyshev's theorem, Statement and interpretation of (weak) law of large numbers and strong law of large numbers; Central limit theorem for independent and identically distributed random variables with finite variance.

References:

1. Hogg, Robert V., McKean, Joseph W., & Craig, Allen T. (2013). *Introduction to Mathematical Statistics* (7th ed.), Pearson Education, Inc.
2. Miller, Irwin, & Miller, Marylees (2014). John E. Freund's: *Mathematical Statistics with Applications* (8th ed.). Pearson Education Ltd. Indian Reprint. Dorling Kindersley.
3. Ross, Sheldon M. (2014). *Introduction to Probability Models* (11th ed.). Elsevier Inc.

Additional Reading:

i. Mood, Alexander M., Graybill, Franklin A. & Boes, Duane C. (1974). *Introduction to The Theory of Statistics* (3rd ed.). McGraw-Hill Education, Indian Edition (2017).

Teaching Plan (DSE-2 (ii): Probability Theory and Statistics):
Week 1: Sample space, Probability set function and examples.
[1] Chapter 1 (Sections 1.1, and 1.3).
Week 2: Random variable, Probability mass /density function, Cumulative distribution function and its properties.

[1] Chapter 1 (Section 1.5).

Week 3: Discrete and continuous random variables.

[1] Sections 1.6, and 1.7, except Transformations.

Week 4: Expectation of random variables, and some special expectations**:** Mean, Variance, Standard deviation, Moments and moment generating function, Characteristic function.

[1] Sections 1.8, and 1.9.

Week 5: The discrete distributions**:** Uniform, Bernoulli, Binomial, Negative binomial, Geometric, and Poisson.

[2] Chapter 5 (Sections 5.2 to 5.5, and 5.7).

Week 6: The continuous distributions**:** Uniform, Gamma, and Exponential.

[2] Sections 6.2, and 6.3.

Week 7: Normal distribution, and normal approximation to the binomial distribution.

[2] Sections 6.5, 6.6.

Week 8 and 9: Random vector**:** Discrete and continuous, Joint cumulative distribution function and its properties, Joint probability mass/density function, Marginal probability mass function, and expectation of two random variables, Joint moment generating function.

[1] Chapter 2 (Section 2.1).

Week 10: Conditional distribution and expectations.

[1] Chapter 2 (Section 2.3).

Week 11: Correlation coefficient, Covariance, Calculation of covariance from joint moment generating function, Independent random variables.

[1] Chapter 2 (Sections 2.4, and 2.5).

Weeks 12 and 13: Linear regression for two variables, and the method of least squares; Chebyshev's theorem.

[2] Chapter 14 (Sections 14.1 to 14.3), Chapter 4 (Section 4.4).

Week 14: Statement and interpretation of the strong law of large numbers; Central limit theorem and the weak law of large numbers.

[3] Chapter 2 (Section 2.8, and Exercise 76, Page 89).

Facilitating the Achievement of Course Learning Outcomes

Unit No.	Course Learning Outcomes	Teaching and Learning Activity	Assessment Tasks
1.	Basic probability axioms and familiar with discrete and continuous random variables.	(i) Each topic to be explained with examples. (ii) Students to be involved in discussions and encouraged to ask questions. (iii) Students to be given homework/assignments. (iv) Students to be encouraged to give short presentations.	• Student presentations. • Participation in discussions. • Assignments and class tests. • Mid-term examinations. • End-term examinations.
2.	To measure the scale of association between two variables, and to establish a formulation helping to predict one variable in terms of the other, i.e., correlation and linear regression.		
3.	Central limit theorem, which helps to understand the remarkable fact that**:** the empirical frequencies of so many natural populations, exhibit a bell-shaped curve.		

Keywords: Moments, Moment generating functions, Discrete and continuous distributions, Chebyshev's theorem, Central limit theorem, Weak law of large numbers.