BMATH408: Partial Differential Equations

Total Marks: 150 (Theory: 75, Internal Assessment: 25 and Practical: 50)
Workload: 4 Lectures, 4 Practicals (per week) **Credits:** 6 (4+2)
Duration: 14 Weeks (56 Hrs. Theory + 56 Hrs. Practical) **Examination:** 3 Hrs.

Course Objectives: The main objectives of this course are to teach students to form and solve partial differential equations and use them in solving some physical problems.

Course Learning Outcomes: The course will enable the students to:
i) Formulate, classify and transform first order PDEs into canonical form.
ii) Learn about method of characteristics and separation of variables to solve first order PDE's.
iii) Classify and solve second order linear PDEs.
iv) Learn about Cauchy problem for second order PDE and homogeneous and nonhomogeneous wave equations.
v) Apply the method of separation of variables for solving many well-known second order PDEs.

Unit 1: First Order PDE and Method of Characteristics
Introduction, Classification, Construction and geometrical interpretation of first order partial differential equations (PDE), Method of characteristic and general solution of first order PDE, Canonical form of first order PDE, Method of separation of variables for first order PDE.

Unit 2: Mathematical Models and Classification of Second Order Linear PDE
Gravitational potential, Conservation laws and Burger's equations, Classification of second order PDE, Reduction to canonical forms, Equations with constant coefficients, General solution.

Unit 3: The Cauchy Problem and Wave Equations
Mathematical modeling of vibrating string and vibrating membrane, Cauchy problem for second order PDE, Homogeneous wave equation, Initial boundary value problems, Nonhomogeneous boundary conditions, Finite strings with fixed ends, Non-homogeneous wave equation, Goursat problem.

Unit 4: Method of Separation of Variables
Method of separation of variables for second order PDE, Vibrating string problem, Existence and uniqueness of solution of vibrating string problem, Heat conduction problem, Existence and uniqueness of solution of heat conduction problem, Non-homogeneous problem.

Reference:
1. Myint-U, Tyn & Debnath, Lokenath. (2007). *Linear Partial Differential Equation for Scientists and Engineers* (4th ed.). Springer, Third Indian Reprint, 2013.

Additional Readings:
i. Sneddon, I. N. (2006). *Elements of Partial Differential Equations*, Dover Publications. Indian Reprint.
ii. Stavroulakis, Ioannis P & Tersian, Stepan A. (2004). *Partial Differential Equations: An Introduction with Mathematica and* MAPLE (2nd ed.). World Scientific.

Practical / Lab work to be performed in a Computer Lab:
Modeling of the following similar problems using Mathematica/MATLAB/Maple/Maxima/Scilab etc.

1. Solution of Cauchy problem for first order PDE.
2. Plotting the characteristics for the first order PDE.
3. Plot the integral surfaces of a given first order PDE with initial data.
4. Solution of wave equation $u_{tt} = c^2 u_{xx}$ for any two of the following associated conditions:

(i) $u(x,0) = \phi(x), u(x,0) = \varphi(x)$, x ϵ $R, t > 0$
(ii) $u(x,0) = \phi(x), u_t(x,0) = \varphi(x), u(0,t) = 0,$ x ϵ $R, t > 0$
(iii) $u(x,0) = \phi(x), u_t(x,0) = \varphi(x), u_x(0,t) = 0,$ x ϵ $R, t > 0$
(iv) $u(x,0) = \phi(x), u(x,0) = \varphi(x), u(0,t) = 0,$ x ϵ $R, t > 0$

5. Solution of one-dimensional heat equation $u_t = k\, u_{xx}$ for a homogeneous rod of length l.
That is - solve the IBVP:

$$u_t = k\, u_{xx},\ 0 < x < l\ , t \geq 0$$
$$u(0,t) = f(x),\ 0 \leq x \leq l,\ u(0,t) = 0,$$
$$u(l,t) = 0,\ t > 0$$

6. Solving systems of ordinary differential equations.
7. Draw the following sequence of functions on the given interval and discuss the pointwise convergence:

(i) $\boldsymbol{f_n(x)} = \mathbf{x^2}$ for x ∈ $\mathbb{R}$,
(ii) $\boldsymbol{f_n(x)} = \frac{\boldsymbol{x}}{\boldsymbol{n}}$ for x ∈ $\mathbb{R}$,
(iii) $\boldsymbol{f_n(x)} = \frac{\boldsymbol{sinx+x}}{\boldsymbol{n}}$ for x ∈ $\mathbb{R}$,
(iv) $\boldsymbol{f_n(x)} = \frac{\mathbf{x^2}+\boldsymbol{nx}}{\boldsymbol{n}}$ for x ∈ $\mathbb{R}$,
(v) $\boldsymbol{f_n(x)} = \frac{\boldsymbol{x}}{\boldsymbol{x+n}}$ for x ∈ $\mathbb{R}$,
(vi) $\boldsymbol{f_n(x)} = \frac{\boldsymbol{nx}}{\mathbf{1+nx^2}}$ for x ∈ $\mathbb{R}$,
(vii) $\boldsymbol{f_n(x)} = \frac{\boldsymbol{xn}}{\mathbf{1+nx}}$ for x ∈ $\mathbb{R}$,
(viii) $\boldsymbol{f_n(x)} = \frac{\boldsymbol{x^n}}{\boldsymbol{1+x^n}}$ for x ∈ $\mathbb{R}$,

8. Discuss the uniform convergence of sequence of functions (i) to (viii) given above in (7).