DISCIPLINE SPECIFIC CORE COURSE– 9: DISCRETE MATHEMATICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Discrete Mathematics	4	3	0	1	Class XII pass with Mathematics	DSC-I and IV: Algebra and Linear Algebra

Learning Objectives

The primary objective of the course is to:

- Make students embark upon a journey of enlightenment, starting from the abstract concepts in mathematics to practical applications of those concepts in real life.
- Make the students familiar with the notion of partially ordered set and a level up with the study of lattice, Boolean algebra and related concepts.
- Culminate the journey of learning with practical applications using the knowledge attained from the abstract concepts learnt in the course.

Learning Outcomes

This course will enable the students to:

- Understand the notion of partially ordered set, lattice, Boolean algebra with applications.
- Handle the practical aspect of minimization of switching circuits to a great extent with the methods discussed in this course.
- Apply the knowledge of Boolean algebras to logic, set theory and probability theory.

SYLLABUS OF DSC - 9

Unit – 1 **(15 hours)**

Cardinality and Partially Ordered Sets

The cardinality of a set; Definitions, examples and basic properties of partially ordered sets, Order-isomorphisms, Covering relations, Hasse diagrams, Dual of an ordered set, Duality principle, Bottom and top elements, Maximal and minimal elements, Zorn's lemma, Building new ordered sets, Maps between ordered sets.

Unit – 2 **(15 hours)**

Lattices

Lattices as ordered sets, Lattices as algebraic structures, Sublattices, Products, Lattice isomorphism; Definitions, examples and properties of modular and distributive lattices; The $M_3 – N_5$ theorem with applications, Complemented lattice, Relatively complemented lattice, Sectionally complemented lattice.

Unit – 3 **(15 hours)**

Boolean Algebras and Applications

Boolean algebras, De Morgan's laws, Boolean homomorphism, Representation theorem, Boolean polynomials, Boolean polynomial functions, Equivalence of Boolean polynomials, Disjunctive normal form and conjunctive normal form of Boolean polynomials; Minimal forms of Boolean polynomials, Quine-McCluskey method, Karnaugh diagrams, Switching circuits and applications, Applications of Boolean algebras to logic, set theory and probability theory.

Practical (30 hours):

Practical/Lab work to be performed in a computer Lab using any of the Computer Algebra System Software such as Mathematica/MATLAB /Maple/Maxima/Scilab/SageMath etc., for the following problems based on:

1) Expressing relations as ordered pairs and creating relations.
2) Finding whether or not, a given relation is:
   i. Reflexive ii. Antisymmetric iii. Transitive iv. Partial order
3) Finding the following for a given partially ordered set
   i. Covering relations.
   ii. The corresponding Hasse diagram representation.
   iii. Minimal and maximal elements.
4) Finding the following for a subset *S* of a given partially ordered set *P*
   i. Whether a given element in *P* is an upper bound (lower bound) of *S* or not.
   ii. Set of all upper bounds (lower bounds) of *S*.
   iii. The least upper bound (greatest lower bound) of *S*, if it exists.
5) Creating lattices and determining whether or not, a given partially ordered set is a lattice.
6) Finding the following for a given Boolean polynomial function:
   i. Representation of Boolean polynomial function and finding its value when the Boolean variables in it take particular values over the Boolean algebra {0,1}.
   ii. Display in table form of all possible values of Boolean polynomial function over the Boolean algebra {0,1}.
7) Finding the following:
   i. Dual of a given Boolean polynomial/expression.
   ii. Whether or not two given Boolean polynomials are equivalent.
   iii. Disjunctive normal form (Conjunctive normal form) from a given Boolean expression.
   iv. Disjunctive normal form (Conjunctive normal form) when the given Boolean polynomial function is expressed by a table of values.
8) Representing a given circuit diagram (expressed using gates) in the form of Boolean expression.
9) Minimizing a given Boolean expression to find minimal expressions.

Essential Readings

1. Davey, B. A., & Priestley, H. A. (2002). Introduction to Lattices and Order (2nd ed.). Cambridge University press, Cambridge.
2. Goodaire, Edgar G., & Parmenter, Michael M. (2006). Discrete Mathematics with Graph Theory (3rd ed.). Pearson Education Pvt. Ltd. Indian Reprint.
3. Lidl, Rudolf & Pilz, Gunter. (2004). Applied Abstract Algebra (2nd ed.), Undergraduate Texts in Mathematics. Springer (SIE). Indian Reprint.

Suggested Readings

- Donnellan, Thomas. (1999). Lattice Theory (1st ed.). Khosla Pub. House. Indian Reprint.
- Rosen, Kenneth H. (2019). Discrete Mathematics and its Applications (8th ed.), Indian adaptation by Kamala Krithivasan. McGraw-Hill Education. Indian Reprint 2021.