BMATH102: Algebra

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The primary objective of this course is to introduce the basic tools of theory of equations, complex numbers, number theory and matrices to understand their connection with the real-world problems. Perform matrix algebra with applications to computer graphics.

Course Learning Outcomes: This course will enable the students to:

i) Employ De Moivre's theorem in a number of applications to solve numerical problems.
ii) Learn about equivalent classes and cardinality of a set.
iii) Use modular arithmetic and basic properties of congruences.
iv) Recognize consistent and inconsistent systems of linear equations by the row echelon form of the augmented matrix.
v) Find eigenvalues and corresponding eigenvectors for a square matrix.

Unit 1: Theory of Equations and Complex Numbers
Polynomials, The remainder and factor theorem, Synthetic division, Factored form of a polynomial, Fundamental theorem of algebra, Relations between the roots and the coefficients of polynomial equations, Theorems on imaginary, integral and rational roots; Polar representation of complex numbers, De Moivre's theorem for integer and rational indices and their applications. The nth roots of unity.

Unit 2: Equivalence Relations and Functions
Equivalence relations, Functions, Composition of functions, Invertibility and inverse of functions, One-to-one correspondence and the cardinality of a set.

Unit 3: Basic Number Theory
Well ordering principle, The division algorithm in $\mathbb{Z}$, Divisibility and the Euclidean algorithm, Fundamental theorem of arithmetic, Modular arithmetic and basic properties of congruences; Principle of mathematical induction.

Unit 4: Row Echelon Form of Matrices and Applications
Systems of linear equations, Row reduction and echelon forms, Vector equations, The matrix equation $A\mathbf{x} = b$, Solution sets of linear systems, The inverse of a matrix; Subspaces, Linear independence, Basis and dimension, The rank of a matrix and applications; Introduction to linear transformations, The matrix of a linear transformation; Applications to computer graphics, Eigenvalues and eigenvectors, The characteristic equation and Cayley–Hamilton theorem.

References:

1. Andreescu, Titu & Andrica Dorin. (2014). *Complex Numbers from A to...Z.* (2nd ed.). Birkhäuser.

2. Dickson, Leonard Eugene (2009). *First Course in the Theory of Equations*. The Project Gutenberg EBook (http://www.gutenberg.org/ebooks/29785)
3. Goodaire, Edgar G., & Parmenter, Michael M. (2005). *Discrete Mathematics with Graph Theory* (3rd ed.). Pearson Education Pvt. Ltd. Indian Reprint 2015.
4. Kolman, Bernard, & Hill, David R. (2001). *Introductory Linear Algebra with Applications* (7th ed.). Pearson Education, Delhi. First Indian Reprint 2003.
5. Lay, David C., Lay, Steven R., & McDonald, Judi J. (2016). *Linear Algebra and its Applications* (5th ed.). Pearson Education.

Additional Readings:

i. Andrilli, Stephen, & Hecker, David (2016). *Elementary Linear Algebr*a (5th ed.). Academic Press, Elsevier India Private Limited.
ii. Burton, David M. (2012). *Elementary Number Theory* (7th ed.). McGraw-Hill Education Pvt. Ltd. Indian Reprint.

Teaching Plan (BMATH102: Algebra):

Weeks 1 and 2: Polynomials, The remainder and factor theorem, Synthetic division, Factored form of a polynomial, Fundamental theorem of algebra, Relations between the roots and the coefficients of polynomial equations, Theorems on imaginary, integral and rational roots.

[2] Chapter II (Sections 12 to 16, 19 to 21, 24 and 27, Statement of the Fundamental theorem of algebra).

Weeks 3 and 4: Polar representation of complex numbers, De Moivre's theorem for integer and rational indices and their applications, The *n*th roots of unity.

[1] Chapter 2 [Section 2.1(2.1.1 to 2.1.3), Section 2.2 (2.2.1, 2.2.2 (up to Page 45, without propositions), 2.2.3].

Weeks 5 and 6. Equivalence relations, Functions, Composition of functions, Invertibility and inverse of functions, One-to-one correspondence and the cardinality of a set.

[3] Chapter 2 (Section 2.4 (2.4.1 to 2.4.4)), and Chapter 3.

Weeks 7 and 8: Well ordering principle, The division algorithm in $\mathbb{Z}$, Divisibility and the Euclidean algorithm, Modular arithmetic and basic properties of congruences, Statements of the fundamental theorem of arithmetic and principle of mathematical induction.

[3] Chapter 4 [Sections 4.1 (4.1.2,4.1.5,4.1.6), 4.2 (4.2.1 to 4.2.11, up to problem 11), 4.3 (4.3.7 to 4.3.9), 4.4 (4.4.1 to 4.4.8)], and Chapter 5 (Section 5.1.1).

Weeks 9 and 10: Systems of linear equations, Row reduction and echelon forms, Vector equations, The matrix equation $A\mathbf{x} = b$, Solution sets of linear systems, The inverse of a matrix.

[5] Chapter 1 (Sections 1.1 to 1.5) and Chapter 2 (Section 2.2).

Week 11 and 12: Subspaces, Linear independence, Basis and dimension, The rank of a matrix and applications.

[4] Chapter 6 (Sections 6.2, 6.3, 6.4, and 6.6).

Weeks 13: Introduction to linear transformations, Matrix of a linear transformation; Applications to computer graphics.

[5] Chapter 1 (Sections 1.8 and 1.9), and Chapter 2 (Section 2.7).

Week 14: Eigenvalues and eigenvectors, The characteristic equation and Cayley–Hamilton theorem.

[5] Chapter 5 (Sections 5.1 and 5.2, Supplementary Exercises 5 and 7, Page 328).

Facilitating the Achievement of Course Learning Outcomes

Unit No.	Course Learning Outcomes	Teaching and Learning Activity	Assessment Tasks
1.	Employ De Moivre's theorem in a number of applications to solve	(i) Each topic to be explained with examples.	• Student

	numerical problems.	(ii) Students to be involved in discussions and encouraged to ask questions. (iii) Students to be given homework/assignments. (iv) Students to be encouraged to give short presentations.	presentations. • Participation in discussions. • Assignments and class tests. • Mid-term examinations. • End-term examinations.
2.	Learn about equivalent classes and cardinality of a set.		
3.	Use modular arithmetic and basic properties of congruences.		
4.	Recognize consistent and inconsistent systems of linear equations by the row echelon form of the augmented matrix. Find eigenvalues and corresponding eigenvectors for a square matrix.		

Keywords: Cardinality of a set, Cayley–Hamilton theorem, De Moivre's theorem, Eigenvalues and eigenvectors, Equivalence relations, Modular arithmetic, Row echelon form, The Fundamental theorem of algebra.