# Semester-IV

## Paper IV: Analysis

**Total Marks:** 100 (Theory: 75 and Internal Assessment: 25)
**Workload:** 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hours.

**Course Objectives:** The course aims at building an understanding of convergence of sequence and series of real numbers and various methods/tools to test their convergence. The course also aims at building understanding of the theory of Riemann integration.

**Course Learning Outcomes:** The course will enable the students to:
  i)   Understand basic properties of the field of real numbers.
  ii)  Examine continuity and uniform continuity of functions using sequential criterion.
  iii) Test convergence of sequence and series of real numbers.
  iv)  Distinguish between the notion of integral as anti-derivative and Riemann integral.

**Unit 1: Real numbers and Real Valued Functions**
Algebraic and order properties of $\mathbb{R}$, Absolute value and the real line, Suprema and infima, The completeness and Archimedean property of $\mathbb{R}$; Limit of functions, Sequential criterion for limits, Algebra of limits, Continuous functions, Sequential criterion for continuity and discontinuity, Properties of continuous functions, Uniform continuity.

**Unit 2: Sequence and Series**
Sequences and their limits, Convergent sequences, Limit theorems, Monotone sequences and their convergence, Subsequences, Cauchy sequence and convergence criterion; Infinite series and their convergence, Cauchy criterion for series, Positive term series, Comparison tests, Absolute and conditional convergence, Cauchy's $n$th root test, D'Alembert's ratio test, Raabe's test, Alternating series, Leibnitz test.

**Unit 3: Riemann Integral**
Riemann integral, Integrability of continuous and monotonic functions.

**References:**
  1. Bartle, Robert G., & Sherbert, Donald R. (2015). *Introduction to Real Analysis* (4th ed.). Wiley India Edition.
  2. Ross, Kenneth A. (2013). *Elementary Analysis*: *The Theory of Calculus* (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian Reprint.

**Additional Readings:**
  i.  Bilodeau, Gerald G., Thie, Paul R., & Keough, G. E. (2010). *An Introduction to Analysis* (2nd ed.). Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.
  ii. Denlinger, Charles G. (2011). *Elements of Real Analysis*. Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

**Teaching Plan (Paper IV: Analysis):**
**Week 1:** Algebraic and order properties of $\mathbb{R}$, Absolute value and the real line.
    [1] Chapter 2 (Sections 2.1 and 2.2)
**Weeks 2 and 3:** Suprema and infima, The completeness properties of $\mathbb{R}$, Archimedean property of $\mathbb{R}$.
    [1] Chapter 2 (Sections 2.3 and 2.4).

**Weeks 4 and 5:** Sequences and their limits, Convergent sequences, Limit theorems.
    [1] Chapter 3 (Sections 3.1 and 3.2).
**Week 6:** Monotone sequences and monotone convergence theorem.
    [1] Chapter 3 (Section 3.3).
**Week 7:** Subsequences, Cauchy sequence and Cauchy convergence criterion.
    [1] Chapter 3 (Sections 3.4 [3.4.1, 3.4.2, 3.4.3, 3.4.5, 3.4.6{(a), (b)}, 3.4.8 (Statement only) and 3.5 [up to 3.5.6]).
**Weeks 8 and 9:** Infinite series, Convergence of a series, $n$th term test, Cauchy's criterion for series, The $p$-series, Positive term series, Comparison tests, Absolute and conditional convergence.
    [1] Chapter 3 (Section 3.7), Chapter 9 [Section 9.1 (9.1.1 and 9.1.2)].
**Week 10:** Cauchy's $n$th root test, D'Alembert's ratio test, Raabe's test, Alternating series, Leibnitz test.
    [1] Chapter 9 [Sections 9.2 (Statements of tests only) and 9.3 (9.3.1 and 9.3.2)].
**Week 11:** Limit of functions, Sequential criterion for limits, Algebra of limits.
    [1] Chapter 4 (Sections 4.1 and 4.2).
**Week 12:** Continuous functions, Sequential criterion for continuity and discontinuity, Boundedness theorem, Intermediate value theorem, Uniform continuity.
    [1] Chapter 5 (Sections 5.1, 5.3, and 5.4 excluding continuous extension and approximation)
**Week 13:** Riemann integral**:** Upper and lower integrals, Riemann integrable functions.
    [2] Chapter 6 (Section 32, only statement of the results up to Page 274, with Examples 1, and 2)
**Week 14:** Riemann integrability of continuous and monotone functions.
    [2] Chapter 6 [Section 33 (33.1 and 33.2)].

## Facilitating the Achievement of Course Learning Outcomes

| Unit No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
|---|---|---|---|
| 1. | Understand basic properties of the field of real numbers. Examine continuity and uniform continuity of functions using sequential criterion. | (i) Each topic to be explained with examples. (ii) Students to be involved in discussions and encouraged to ask questions. (iii) Students to be given homework/assignments. (iv) Students to be encouraged to give short presentations. | • Student presentations. • Participation in discussions. • Assignments and class tests. • Mid-term examinations. • End-term examinations. |
| 2. | Test convergence of sequence and series of real numbers. | | |
| 3. | Distinguish between the notion of integral as anti-derivative and Riemann integral. | | |

**Keywords:** Continuity, Cauchy convergence criterion, Convergence, Cauchy's $n$th root test, D'Alembert's ratio test, Intermediate value theorem, Riemann integral, Supremum, Uniform continuity.