Semester-III

Paper III: Algebra

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The objective of this course is to introduce the fundamental theory of groups, rings and vector spaces, a major part of abstract algebra, which is an essential tool in number theory, geometry, topology and has applications in cryptography, coding theory, quantum chemistry and physics.

Course Learning Outcomes: The course will enable the students to:

i) Recognize the mathematical objects that are groups, and classify them as abelian, cyclic and permutation groups etc.
ii) Explain the significance of the notion of cosets, normal subgroups, and factor groups.
iii) Understand the fundamental concepts of rings, fields and integral domains.
iv) Know about vector spaces over a field, and linear transformations.

Unit 1: Groups

Definition and examples of groups, Abelian and non-abelian groups, The group $\mathbb{Z}_n$ of integers under addition modulo n and the group $U(n)$ of units under multiplication modulo n; Cyclic groups from sets of numbers, Group of nth roots of unity, The general linear group; Elementary properties of groups; Groups of symmetries of (i) an isosceles triangle, (ii) an equilateral triangle, (iii) a rectangle, and (iv) a square; The permutation group Sym(n), and properties of permutations; Order of an element, Subgroups and its examples, Subgroup tests, Cyclic subgroup, Center of a group, Properties of cyclic groups; Cosets and its properties, Lagrange's theorem, Index of a subgroup; Definition and examples of normal subgroups.

Unit 2: Rings, Integral Domains and Fields

Definition and examples of rings, Commutative and noncommutative rings, Properties of rings, Subrings and ideals; Integral domains and fields, Examples of fields: $\mathbb{Z}_p$, $\mathbb{Q}$, $\mathbb{R}$ and $\mathbb{C}$.

Unit 3: Vector Spaces and Linear Transformations

Definition and examples of vector spaces, Subspaces, Linear independence, Basis and dimension of a vector space; Linear transformations, Null spaces, Ranges and illustrations of the rank-nullity theorem.

References:

1. Gallian, Joseph. A. (2013). *Contemporary Abstract Algebra* (8th ed.). Cengage Learning India Private Limited, Delhi. Fourth impression, 2015.
2. Friedberg, Stephen H., Insel, Arnold J., & Spence, Lawrence E. (2003). *Linear Algebra* (4th ed.). Prentice-Hall of India Pvt. Ltd. New Delhi.

Additional Readings:

i. Beachy, John A., & Blair, William D. (2006). *Abstract Algebra* (3rd ed.). Waveland Press.
ii. Lay, David C., Lay, Steven, R., & McDonald Judi, J. (2016). *Linear Algebra and its Applications* (5th ed.). Pearson.

Teaching Plan (Paper III: Algebra):

Weeks 1 and 2: Groups**:** Definition and examples of Abelian and non-abelian groups, The group $\mathbb{Z}_n$ of integers under addition modulo n and the group $U(n)$ of units under multiplication modulo n; Cyclic groups from sets of numbers, Group of nth roots of unity, The general linear group; Elementary properties of groups.

[1] Chapter 2.

Week 3: Groups of symmetries of (i) an isosceles triangle, (ii) an equilateral triangle, (iii) a rectangle, and (iv) a square; The permutation group Sym(n), and properties of permutations.

[1] Chapter 1, Chapter 5 (Examples 1 to 7 and illustrations of Theorems 5.1 to 5.7 without proofs).

Weeks 4 and 5: Order of an element, Subgroups and its examples, Subgroup tests, Cyclic Subgroup, Center of a group, Properties of cyclic groups.

[1] Chapters 3, and 4.

Week 6: Cosets and its properties, Lagrange's Theorem, Index of a subgroup.

[1] Chapter 7 up to Corollary 4, Page 149.

Week 7: Normal subgroups: Definition, examples and characterizations, Factor groups.

[1] Chapter 9 (Theorem 9.1, and Theorem 9.2 (Statement only) up to Examples 11, Page 189.

Weeks 8 and 9: Definition and examples of rings, commutative and noncommutative rings, Properties of rings, Subrings and ideals.

[1] Chapter 12, and Chapter 14 up to Example 4, Page 268.

Week 10: Integral domains and fields, Examples of fields: $\mathbb{Z}_p$, $\mathbb{Q}$, $\mathbb{R}$ and $\mathbb{C}$.

[1] Chapter 13 up to Example 10, Page 258.

Weeks 11 and 12: Definition and examples of vector spaces, Subspaces, Linear independence, Basis and dimension of a vector space.

[1] Chapter 19.

Weeks 13 and 14: Linear transformations, Null spaces, Ranges and illustrations of the rank-nullity theorem.

[2] Chapter 2 (Section 2.1).

Facilitating the Achievement of Course Learning Outcomes

Unit No.	Course Learning Outcomes	Teaching and Learning Activity	Assessment Tasks
1.	Recognize the mathematical objects that are groups, and classify them as abelian, cyclic and permutation groups etc. Explain the significance of the notion of cosets, normal subgroups, and factor groups.	(i) Each topic to be explained with examples. (ii) Students to be involved in discussions and encouraged to ask questions. (iii) Students to be given homework/assignments. (iv) Students to be encouraged to give short presentations.	• Student presentations. • Participation in discussions. • Assignments and class tests. • Mid-term examinations. • End-term examinations.
2.	Understand the fundamental concepts of rings, fields and integral domains.		
3.	Know about vector spaces over a field, and linear transformations.		

Keywords: Groups, Lagrange's theorem, Normal subgroups, Rings, Ideals, Integral domains, Fields, Vector spaces, Basis, Linear transformations.