Semester-II

BMATH203: Real Analysis

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The course will develop a deep and rigorous understanding of real line $\mathbb{R}$. and of defining terms to prove the results about convergence and divergence of sequences and series of real numbers. These concepts have wide range of applications in real life scenario.

Course Learning Outcomes: This course will enable the students to:

i) Understand many properties of the real line $\mathbb{R}$, including completeness and Archimedean properties.
ii) Learn to define sequences in terms of functions from $\mathbb{N}$ to a subset of $\mathbb{R}$**.**
iii) Recognize bounded, convergent, divergent, Cauchy and monotonic sequences and to calculate their limit superior, limit inferior, and the limit of a bounded sequence.
iv) Apply the ratio, root, alternating series and limit comparison tests for convergence and absolute convergence of an infinite series of real numbers.

Unit 1: Real Number System $\mathbb{R}$
Algebraic and order properties of $\mathbb{R}$, Absolute value of a real number; Bounded above and bounded below sets, Supremum and infimum of a nonempty subset of $\mathbb{R}$.

Unit 2: Properties of $\mathbb{R}$
The completeness property of $\mathbb{R}$, Archimedean property, Density of rational numbers in $\mathbb{R}$; Definition and types of intervals, Nested intervals property; Neighborhood of a point in $\mathbb{R}$, Open and closed sets in $\mathbb{R}$.

Unit 3: Sequences in $\mathbb{R}$
Convergent sequence, Limit of a sequence, Bounded sequence, Limit theorems, Monotone sequences, Monotone convergence theorem, Subsequences, Bolzano–Weierstrass theorem for sequences, Limit superior and limit inferior for bounded sequence, Cauchy sequence, Cauchy's convergence criterion.

Unit 4: Infinite Series
Convergence and divergence of infinite series of real numbers, Necessary condition for convergence, Cauchy criterion for convergence; Tests for convergence of positive term series**:** Integral test, Basic comparison test, Limit comparison test, D'Alembert's ratio test, Cauchy's *n*th root test; Alternating series, Leibniz test, Absolute and conditional convergence.

References:

1. Bartle, Robert G., & Sherbert, Donald R. (2015). *Introduction to Real Analysis* (4th ed.). Wiley India Edition. New Delhi.
2. Bilodeau, Gerald G., Thie, Paul R., & Keough, G. E. (2010). *An Introduction to Analysis* (2nd ed.). Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

3. Denlinger, Charles G. (2011). *Elements of Real Analysis*. Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

Additional Readings:

i. Ross, Kenneth A. (2013). *Elementary Analysis*: *The Theory of Calculus* (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian Reprint.
ii. Thomson, Brian S., Bruckner, Andrew. M., & Bruckner, Judith B. (2001). *Elementary Real Analysis*. Prentice Hall.

Teaching Plan (BMATH203: Real Analysis):

Weeks 1 and 2: Algebraic and order properties of ℝ. Absolute value of a real number; Bounded above and bounded below sets, Supremum and infimum of a nonempty subset of ℝ.
[1] Chapter 2 [Sections 2.1, 2.2 (2.2.1 to 2.2.6) and 2.3 (2.3.1 to 2.3.5)]

Weeks 3 and 4: The completeness property of ℝ, Archimedean property, Density of rational numbers in ℝ, Definition and types of intervals, Nested intervals property; Neighborhood of a point in ℝ, Open and closed sets in ℝ.
[1] Sections 2.3 (2.3.6), 2.4 (2.4.3 to 2.4.9), and 2.5 up to Theorem 2.5.3.
[1] Chapter 11 [Section 11.1 (11.1.1 to 11.1.3)].

Weeks 5 and 6: Sequences and their limits, Bounded sequence, Limit theorems.
[1] Sections 3.1, 3.2.

Week 7: Monotone sequences, Monotone convergence theorem and applications.
[1] Section 3.3.

Week 8: Subsequences and statement of the Bolzano–Weierstrass theorem. Limit superior and limit inferior for bounded sequence of real numbers with illustrations only.
[1] Chapter 3 [Section 3.4 (3.4.1 to 3.4.12), except 3.4.4, 3.4.7, 3.4.9 and 3.4.11].

Week 9: Cauchy sequences of real numbers and Cauchy's convergence criterion.
[1] Chapter 3 [Section 3.5 (3.5.1 to 3.5.6)].

Week 10: Convergence and divergence of infinite series, Sequence of partial sums of infinite series, Necessary condition for convergence, Cauchy criterion for convergence of series.
[3] Section 8.1.

Weeks 11 and 12: Tests for convergence of positive term series**:** Integral test statement and convergence of *p*-series, Basic comparison test, Limit comparison test with applications, D'Alembert's ratio test and Cauchy's *n*th root test.
[3] Chapter 8 (Section 8.2 up to 8.2.19).

Weeks 13 and 14: Alternating series, Leibniz test, Absolute and conditional convergence.
[2] Chapter 6 (Section 6.2).

Facilitating the Achievement of Course Learning Outcomes

Unit No.	Course Learning Outcomes	Teaching and Learning Activity	Assessment Tasks
1. & 2.	Understand many properties of the real line ℝ including, completeness and Archimedean properties.	(i) Each topic to be explained with examples. (ii) Students to be involved in discussions and encouraged to ask questions. (iii) Students to be given homework/assignments. (iv) Students to be	• Presentations and participation in discussions. • Assignments and class tests. • Mid-term examinations. • End-term examinations.
3.	Learn to define sequences in terms of functions from ℕ to a subset of ℝ. Recognize bounded, convergent, divergent, Cauchy and monotonic sequences and to calculate their limit superior, limit inferior, and the limit of a bounded sequence.		

4.	Apply the ratio, root, alternating series and limit comparison tests for convergence and absolute convergence of an infinite series of real numbers.	encouraged to give short presentations. (v) Illustrate the concepts through CAS.	

Keywords: Archimedean property, Absolute and conditional convergence of series, Bolzano−Weierstrass theorem, Cauchy sequence, Convergent sequence, Leibniz test, Limit of a sequence, Nested intervals property, Open and closed sets in $\mathbb{R}$.