# Semester-III

## BMATH305: Theory of Real Functions

**Total Marks: 100** (Theory: 75, Internal Assessment: 25)
**Workload:** 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

**Course Objectives:** It is a basic course on the study of real valued functions that would develop an analytical ability to have a more matured perspective of the key concepts of calculus, namely, limits, continuity, differentiability and their applications.

**Course Learning Outcomes:** This course will enable the students to:

i) Have a rigorous understanding of the concept of limit of a function.
ii) Learn about continuity and uniform continuity of functions defined on intervals.
iii) Understand geometrical properties of continuous functions on closed and bounded intervals.
iv) Learn extensively about the concept of differentiability using limits, leading to a better understanding for applications.
v) Know about applications of mean value theorems and Taylor's theorem.

**Unit 1: Limits of Functions**
Limits of functions ($\varepsilon-\delta$ approach), Sequential criterion for limits, Divergence criteria, Limit theorems, One-sided limits, Infinite limits and limits at infinity.

**Unit 2: Continuous Functions and their Properties**
Continuous functions, Sequential criterion for continuity and discontinuity, Algebra of continuous functions, Properties of continuous functions on closed and bounded intervals; Uniform continuity, Non-uniform continuity criteria, Uniform continuity theorem.

**Unit 3: Derivability and its Applications**
Differentiability of a function, Algebra of differentiable functions, Carathéodory's theorem, Chain rule; Relative extrema, Interior extremum theorem, Rolle's theorem, Mean- value theorem and applications, Intermediate value property of derivatives, Darboux's theorem.

**Unit 4: Taylor's Theorem and its Applications**
Taylor polynomial, Taylor's theorem with Lagrange form of remainder, Application of Taylor's theorem in error estimation; Relative extrema, and to establish a criterion for convexity; Taylor's series expansions of $e^x$, $\sin x$ and $\cos x$.

**Reference:**

1. Bartle, Robert G., & Sherbert, Donald R. (2015). *Introduction to Real Analysis* (4th ed.). Wiley India Edition. New Delhi.

**Additional Readings:**

i. Ghorpade, Sudhir R. & Limaye, B. V. (2006). *A Course in Calculus and Real Analysis*. Undergraduate Texts in Mathematics, Springer (SIE). First Indian reprint.
ii. Mattuck, Arthur. (1999). *Introduction to Analysis*, Prentice Hall.

iii. Ross, Kenneth A. (2013). *Elementary Analysis*: *The Theory of Calculus* (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian Reprint.

**Teaching Plan (BMATH305: Theory of Real Functions):**

**Week 1:** Definition of the limit, Sequential criterion for limits, Criterion for non-existence of limit.
[1] Chapter 4 (Section 4.1).

**Week 2:** Algebra of limits of functions with illustrations and examples, Squeeze theorem.
[1] Chapter 4 (Section 4.2).

**Week 3:** Definition and illustration of the concepts of one-sided limits, Infinite limits and limits at infinity.
[1] Chapter 4 (Section 4.3).

**Weeks 4 and 5:** Definitions of continuity at a point and on a set, Sequential criterion for continuity, Algebra of continuous functions, Composition of continuous functions.
[1] Sections 5.1 and 5.2.

**Weeks 6 and 7:** Various properties of continuous functions defined on an interval, viz., Boundedness theorem, Maximum-minimum theorem, Statement of the location of roots theorem, Intermediate value theorem and the preservation of intervals theorem.
[1] Chapter 5 (Section 5.3).

**Week 8:** Definition of uniform continuity, Illustration of non-uniform continuity criteria, Uniform continuity theorem.
[1] Chapter 5 [Section 5.4 (5.4.1 to 5.4.3)].

**Weeks 9 and 10:** Differentiability of a function, Algebra of differentiable functions, Carathéodory's theorem and chain rule.
[1] Chapter 6 [Section 6.1 (6.1.1 to 6.1.7)].

**Weeks 11 and 12:** Relative extrema, Interior extremum theorem, Mean value theorem and its applications, Intermediate value property of derivatives - Darboux's theorem.
[1] Section 6.2.

**Weeks 13 and 14:** Taylor polynomial, Taylor's theorem and its applications, Taylor's series expansions of $e^x$, $\sin x$ and $\cos x$.
[1] Chapter 6 (Sections 6.4.1 to 6.4.6), and Chapter 9 (Example 9.4.14, Page 286).

**Facilitating the Achievement of Course Learning Outcomes**

| Unit No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
|---|---|---|---|
| 1. | Have a rigorous understanding of the concept of limit of a function. | (i) Each topic to be explained with examples.<br>(ii) Students to be involved in discussions and encouraged to ask questions.<br>(iii) Students to be given homework/ assignments.<br>(iv) Students to be encouraged to give short presentations.<br>(v) Illustrate the concepts through CAS. | • Presentations and participation in discussions.<br>• Assignments and class tests.<br>• Mid-term examinations.<br>• End-term examinations. |
| 2. | Learn about continuity and uniform continuity of functions defined on intervals.<br>Understand geometrical properties of continuous functions on closed and bounded intervals. | | |
| 3. | Learn extensively about the concept of differentiability using limits, leading to a better understanding for applications. | | |
| 4. | Know about applications of mean value theorems and Taylor's theorem. | | |

**Keywords:** Continuity, Convexity, Differentiability, Limit, Relative extrema, Rolle's theorem, Taylor's theorem, Uniform continuity.